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A New Three Objectives Criterion to Optimize Thermomechanical Engines Model

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Abstract: Regarding thermomechanical engines, the first law efficiency has been until now, the most used criterion, with the famous equilibrium thermodynamics upperbound given by the CARNOT formula. Most recently (second part of the past century), a new appraisal appears, regarding the maximum power objective for the engines. We propose here to reconsider these two main approaches due to the fact that the heat expenses first, and the heat rejected secondly are also involved in the complete characterization of the engine. These three quantities (energy consumption to minimize associated to the energy cost; useful effect to maximize; heat rejection to minimize, in order to protect the environment) are combined through a weighting procedure allowing, to discuss and choose the best scheme. Particular previous studies results are recovered and synthesized. Other new possibilities are proposed to be explored in the near future.

Keywords: thermomechanical engine; optimization; multiobjective; weighting method.

1. Introduction (state of art)

For a long time the main criterion used in physics was the considered system efficiency. The most known criterion is due to Carnot, with the famous Carnot efficiency for an engine in the frame of equilibrium thermodynamics.

Most recently was introduced a new optimization criterion regarding engines. It was maximum of power [1][2]. Consequently the corresponding efficiency $\eta_I(\text{MAX}|\dot{W}|)$ drops to a smaller value: the well known "nice radical" illustrates this situation [3].

Since that time many other situations have been explored, particularly for engines. It corresponds a lot of results regarding specifically efficiency, with precised boundary conditions (or constraints). Some recent review papers intend to summarize these [4].

The same considerations have been developed for reverse cycle machines. It starts probably with the paper of Leff and Teeters [5] that differentiates clearly EER, COP and second law efficiency for air conditioners. Similarly to engines, a maximum of refrigerating effect could be sought or maximum of hot heat rate for heat pump. This has been reviewed recently for vapor compression machines [6], or for three or four heat reservoirs reverse cycles machines [7]. More complex energy systems configurations have been explored too. Books have been published in this direction [8].

To summarize it appears from an engineering point of view that two main types of optimization are developed regarding energy systems:

- the first is related to design optimization for a given purpose
- the second one consists to operate the designed system in an optimal way : optimal control and command of the system [9]. It needs to consider in that case transient conditions. Therefore, the model depends explicitly on time; the corresponding optimizations are more difficult, and out of the scope of the present paper focused on a new appraisal regarding the first point to say optimization of the design [10].

Effectively with the beginning of the century new optimization criterions become more and more considered. For example thermoeconomical optimization applied to various systems: powerplants [11], solar systems [12], even if they start to be considered during the 50's.

Now the tendency is to develop multicriterion optimization of thermal systems design considering simultaneously energy, economy and environment as objectives [13]. These studies are developed for engines and plants, but also for refrigeration systems [14], as well as for combined cooling, heating and power systems [15], even more at a national level [16]. New heuristic optimization methods (evolutionary algorithms) are used too, for example applied to heat pump [17] or solar heat engine [18]. These algorithms appear strongly connected to second law.

The purpose of the present paper is to reconsider the multiobjective optimization problem, in order to have a unified view of physical optimization criterions for energy converters [19]. The new proposal is based on the fundamental energy triangle proposed for the converter (Figure 3). The relation between this criterion triangle, efficiency, economy, and environment is enlightened. Application is developed on the Novikov-Curzon-Ahlborn thermomechanical engine. New results are proposed. Previous results are recovered. The general methodology is to be applied to various other systems and processes in the near future.

2. Extended Model of the NCA thermomechanical engine

2.1. The enhanced NCA thermomechanical engine

The paper is based on the Novikov-Curzon-Ahlborn model of a thermomechanical engine, but in an extended form (see Figure 1 and Figure 2). The objective of the proposed paper is to consider simultaneously the three main criterions related to:

- a) the power of the engine: to maximize (it is a revenue)
- b) the heat expense at the hot source: to minimize (it is an energy cost)
- c) the rejected heat at the cold sink: to minimize (it is losses, or thermal pollution of the environment).

The natural multiobjective function OF appears as a weighted one.

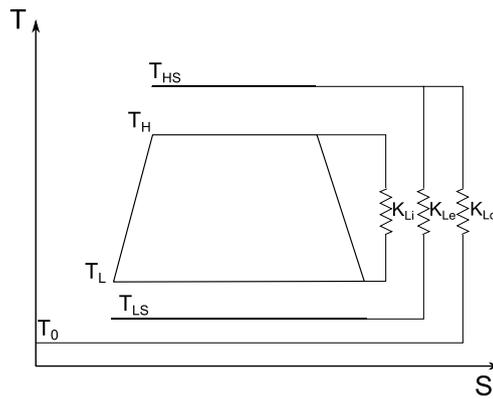


Figure 1 : schema of Novikov-Curzon-Ahlborn thermomechanical engine

The optimal state vector of the system is characterized by a set of optimal temperatures T_H^* , T_L^* , for a given design (K_H , K_L), the two heat transfer conductances at the hot and respectively the cold side.

A sensitivity analysis to parameters (mainly weighting factors) is reported. Particular results are recovered. A focus is done on the corresponding values of created entropy rates of the system, related to the proposed optimization.

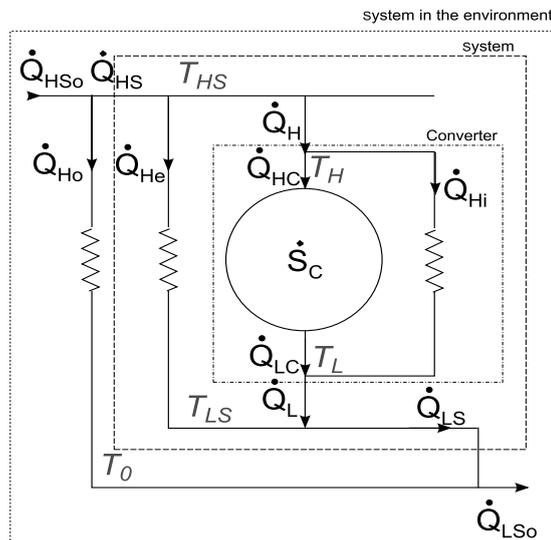


Figure 2 : entropy diagram of Novikov-Curzon-Ahlborn thermomechanical engine

2.2. Hypothesis

- heat source and sink: constant temperature reservoirs (T_{HS} , hot side; T_{LS} , cold side)
- steady state assumption
- linear heat transfer law (first step in modeling)
- converter irreversibilities supposed function of T_H , T_L internal temperatures of the working fluid at the hot and cold side: $\dot{S}_c(T_H, T_L)$
- accounting of the thermal losses through successively K_{li}, K_{le}, K_{lo} (3 heat loss conductances represented on Figure 1 and Figure 2).

2.3. Model equations

The heat transfer rates are linear ones:

$$\dot{Q}_H = K_H(T_{HS} - T_H) \quad (1)$$

$$\dot{Q}_L = K_L(T_{LS} - T_L) \quad (2)$$

The Table 1 summarizes the energy and entropy balances relative to the different control volumes, the single converter (circular control volume of Figure 2), the converter with thermal contacts (internal square control volume of Figure 2), the system (middle square control volume of Figure 2) and the system in the environment (external square volume control of Figure 2). The thermodynamics sign convention is used and the consumed and rejected heat transfer rate expressions are given with the thermal losses associated.

	Single converter	Converter (with thermal contacts)	System	System in the environment
Thermal losses		$\dot{Q}_{Hi} = K_{li}(T_H - T_L)$	$\dot{Q}_{He} = K_{le}(T_{HS} - T_{LS})$	$\dot{Q}_{Ho} = K_{lo}(T_{HS} - T_0)$
consumed and rejected heat transfer rates	$\begin{cases} \dot{Q}_{HC} \\ \dot{Q}_{LC} \end{cases}$	$\begin{cases} \dot{Q}_H = \dot{Q}_{HC} + \dot{Q}_{Hi} \\ \dot{Q}_L = \dot{Q}_{LC} - \dot{Q}_{Hi} \end{cases}$	$\begin{cases} \dot{Q}_{HS} = \dot{Q}_H + \dot{Q}_{He} \\ \dot{Q}_{LS} = \dot{Q}_L - \dot{Q}_{He} \end{cases}$	$\begin{cases} \dot{Q}_{HSO} = \dot{Q}_{HS} + \dot{Q}_{Ho} \\ \dot{Q}_{LSO} = \dot{Q}_{LS} - \dot{Q}_{Ho} \end{cases}$
Energy balances	$\dot{Q}_{HC} + \dot{Q}_{LC} + \dot{W} = 0$	$\dot{Q}_H + \dot{Q}_L + \dot{W} = 0$	$\dot{Q}_{HS} + \dot{Q}_{LS} + \dot{W} = 0$	$\dot{Q}_{HSO} + \dot{Q}_{LSO} + \dot{W} = 0$
Entropy balances	$\frac{\dot{Q}_{HC}}{T_H} + \frac{\dot{Q}_{LC}}{T_L} + \dot{S}_c(T_H, T_L) = 0$	$\frac{\dot{Q}_H}{T_H} + \frac{\dot{Q}_L}{T_L} + \dot{S}_i = 0$	$\frac{\dot{Q}_{HS}}{T_{HS}} + \frac{\dot{Q}_{LS}}{T_{LS}} + \dot{S}_s = 0$	$\frac{\dot{Q}_{HSO}}{T_{HS}} + \frac{\dot{Q}_{LSO}}{T_0} + \dot{S}_o = 0$

Table 1 : Energy and entropy balances relative to the different control volumes

3. The generalized three objectives function of the NCA thermomechanical engine

3.1. Engine simple objective optimization

The three natural objective functions have been introduced in section 1 of the paper. They appear on the criterion triangle (Figure 3).

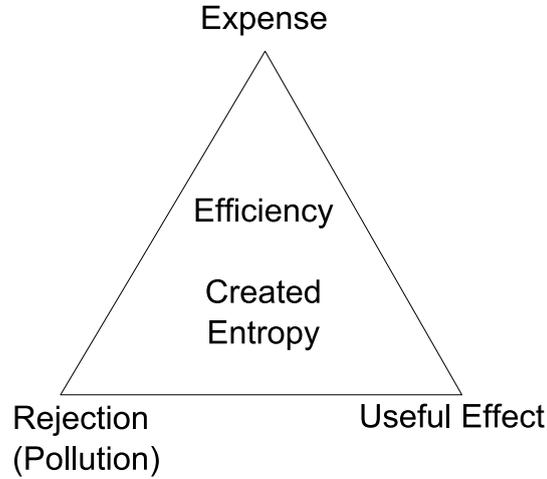


Figure 3 : The fundamental energy criteria for any energy system: criterion triangle

In case of a thermomechanical engine or plant, the heat expense to minimize corresponds to \dot{Q}_H for the converter, \dot{Q}_{HS} for the system, and \dot{Q}_{HS0} for the system in the environment (Table 1). Due to the thermodynamics sign convention, all these values are positive.

Considering the useful effect, whatever is the control volume, the power $|\dot{W}|$ (\dot{W} , negative) remains the same. The most convenient form of $|\dot{W}|$ is given by:

$$|\dot{W}| = \dot{Q}_H + \dot{Q}_L \quad (3)$$

In any case, the objective regarding power is the maximization of $|\dot{W}|$.

The rejected heat \dot{Q}_L ($\dot{Q}_{LC}, \dot{Q}_{LS}, \dot{Q}_{LS0}$; all negative values) is considered at the end a heat pollution of the surrounding (\dot{Q}_{LS0}). So, whatever is the control volume chosen, we have to minimize the corresponding rejection $|\dot{Q}_L|$ or other (see Table 2).

The basic objectives OF of the thermomechanical engine are summarized in Table 2.

OF \ Control volume	Heat expense	Power output	Heat rejection
Converter	$\min(\dot{Q}_H)$	$\text{MAX}(-\dot{W})$	$\min(-\dot{Q}_L)$
System	$\min(\dot{Q}_{HS})$	$\text{MAX}(-\dot{W})$	$\min(-\dot{Q}_{LS})$
System in environment	$\min(\dot{Q}_{HS0})$	$\text{MAX}(-\dot{W})$	$\min(-\dot{Q}_{LS0})$

Table 2 : Various fundamental objectives relative to a thermomechanical engine

It is clear that in all the presented cases, T_H, T_L are dependent control variables through the entropy constraint (Table 1): only one degree of freedom exists for the temperatures.

For a given design $K_H, K_L, K_{li}, K_{le}, K_{lo}$ are fixed parameters, as T_{HS}, T_{LS} the two infinite reservoirs temperatures.

We note here that to the fundamental objectives of Table 2 could be added another standard one usually considered in equilibrium Thermodynamics; the converter efficiency:

$$\eta_{IC} = \frac{\dot{W}}{\dot{Q}_H} \quad (4)$$

Similar equations could be deduced from (4) regarding system efficiency, or efficiency of the system in the surrounding, or others, but it is not the purpose here (see references: [20][21]).

We note also that equations of the entropy balance (Table 1) allow to calculate the various entropy production rates regarding successively the converter (\dot{S}_i), the system (\dot{S}_s), the system in the surrounding (\dot{S}_o). These entropy production rates depend, on the internal entropy production rate of the converter (\dot{S}_c) (working fluid irreversibilities mainly).

Lastly, if a technical constraint is added to the problem, the degree of freedom vanishes at given design. The optimization becomes a simulation relative to T_H, T_L temperatures. But, if we release the parameters K_H, K_L as new variables, with a finite size constraint ($K_H + K_L = K_T$), we recover one degree of freedom: optimization is again possible and furnishes the optimal allocation of the two heat conductances between source and sink.

Table 3 represents the equivalence between various optimization cases adding efficiency of the thermomechanical engine as a fourth objective, and a technical constraint more (imposed power $|\dot{W}_o|$; imposed heat expense \dot{Q}_{Ho} ; imposed heat rejection \dot{Q}_{Lo} , in the case of the converter optimization).

It appears clearly in columns 1 and 3 of Table 3 that optimization with respect to heat rejection (column 1) or with respect to heat expense (column 3) insures maximum power jointly to maximum efficiency: Fundamental objectives are sufficient for these cases. If power output is imposed the problem has to be considered differently, if we intent to optimize efficiency. Only the minimum of heat expense is equivalent to minimum of heat rejection.

In conclusion the three fundamental objectives are essential in an overall optimization of the engine.

Technical Constraint	Imposed heat expense \dot{Q}_{Ho}	Imposed power output $ \dot{W}_o $	Imposed heat rejection \dot{Q}_{Lo}
Control volume			
Converter	$MAX \dot{W} \leftrightarrow \min \dot{Q}_L $ $MAX \eta_c \leftrightarrow \min \dot{Q}_L $	$\min \dot{Q}_H \leftrightarrow \min \dot{Q}_L $ $MAX \eta_c \leftrightarrow \min \left \frac{\dot{Q}_L}{\dot{Q}_H} \right $	$MAX \dot{W} \leftrightarrow MAX \dot{Q}_H$ $MAX \eta_c \leftrightarrow MAX \dot{Q}_H$
System	$MAX \dot{W} \leftrightarrow \min \dot{Q}_{LS} $ $MAX \eta_s \leftrightarrow \min \dot{Q}_{LS} $	$\min \dot{Q}_{HS} \leftrightarrow \min \dot{Q}_{LS} $ $MAX \eta_s \leftrightarrow \min \left \frac{\dot{Q}_{LS}}{\dot{Q}_{HS}} \right $	$MAX \dot{W} \leftrightarrow MAX \dot{Q}_{HS}$ $MAX \eta_s \leftrightarrow MAX \dot{Q}_{HS}$
System in environment	$MAX \dot{W} \leftrightarrow \min \dot{Q}_{LSO} $ $MAX \eta_{sO} \leftrightarrow \min \dot{Q}_{LSO} $	$\min \dot{Q}_{HSO} \leftrightarrow \min \dot{Q}_{LSO} $ $MAX \eta_{sO} \leftrightarrow \min \left \frac{\dot{Q}_{LSO}}{\dot{Q}_{HSO}} \right $	$MAX \dot{W} \leftrightarrow MAX \dot{Q}_{HSO}$ $MAX \eta_{sO} \leftrightarrow MAX \dot{Q}_{HSO}$

Table 3 : Equivalence between fundamental objectives of a thermomechanical engine with added technical constraint

3.2. Multiobjective optimization

3.2.1. General results

The general objective function OF_G is constructed, through a thermoeconomical reasoning. The useful effect has an assumed unitary value v_u . Identically the energy expense and the heat pollution are supposed to have assumed values v_e , v_p , such that:

$$v_T = v_u + v_e + v_p \quad (5)$$

All these values are positive ones.

The resulting thermoeconomical balance of the converter is expressed as a gain V :

$$V = v_u |\dot{W}| - v_e \dot{Q}_H - v_p |\dot{Q}_L| \quad (6)$$

Remark: if \dot{Q}_L is a valuable rejection, $v_p = v_r$. In this case: $V = v_u |\dot{W}| - v_e \dot{Q}_H + v_r |\dot{Q}_L|$ (ex: CHP, TFP)

This balance could be rearranged on a nondimensional form dividing by v_T :

$$OF_G = \alpha |\dot{W}| - \beta \dot{Q}_H - \gamma |\dot{Q}_L| \quad (7)$$

With α, β, γ weighting factors such that $\alpha + \beta + \gamma = 1$

Combining (3) and (7) it comes easily for the multi objective function:

$$OF_G = (\alpha - \beta) \dot{Q}_H + (\alpha + \gamma) \dot{Q}_L \quad (8)$$

The variational method applied to (8) and constraints (Table 1) gives the lagrangian $L_G(T_H, T_L)$ to optimize the converter (respectively the system, or the system in the environment). For example, it comes for the converter:

$$L_G = (\alpha - \beta) K_H (T_{HS} - T_H) + (\alpha + \gamma) K_L (T_{LS} - T_L) + \lambda_G \left(\frac{K_H (T_{HS} - T_H)}{T_H} + \frac{K_L (T_{LS} - T_L)}{T_L} + K_{Li} (T_H - T_L) \left(\frac{1}{T_L} - \frac{1}{T_H} \right) + \dot{S}_C(T_H, T_L) \right) \quad (9)$$

The derivation with respect to T_H , T_L and λ_G , Lagrange parameter, gives after some calculations, the optimal temperatures vector T_{HG}^* , T_{LG}^* through a set of two non linear equations to solve. For the case where:

$$\frac{\partial \dot{S}_C}{\partial T_H} = 0; \quad \frac{\partial \dot{S}_C}{\partial T_L} = 0; \quad K_{Li} = 0$$

The particular analytical solution is:

$$T_{HG}^* = \sqrt{T_{HS}} \frac{K_L \sqrt{T_{LS}} \sqrt{\alpha + \gamma} + K_H \sqrt{T_{HS}} \sqrt{\alpha - \beta}}{\sqrt{\alpha - \beta} (K_H + K_L - \dot{S}_C)} \quad (10)$$

$$T_{LG}^* = \sqrt{T_{LS}} \frac{K_L \sqrt{T_{LS}} \sqrt{\alpha + \gamma} + K_H \sqrt{T_{HS}} \sqrt{\alpha - \beta}}{\sqrt{\alpha + \gamma} (K_H + K_L - \dot{S}_C)} \quad (11)$$

This analytical vector (T_{HG}^*, T_{LG}^*) induces a generalized “nice radical” associated to the endoreversible case ($\dot{S}_C = 0$):

$$\eta_I(MAx OF_G) = 1 - \sqrt{\frac{T_{LS}}{T_{HS}}} \sqrt{\frac{\alpha + \gamma}{\alpha - \beta}} \quad (12)$$

3.2.2. Particular cases of two objectives

	$\left(\begin{array}{l} \alpha = 1; \beta = 0; \\ \gamma = 0 \end{array} \right)$	$\left(\begin{array}{l} \alpha = 0; \beta = 1; \\ \gamma = 0 \end{array} \right)$	$\left(\begin{array}{l} \alpha = 0; \beta = 0; \\ \gamma = 1 \end{array} \right)$
	$MAX(-\dot{W})$	$MAX(-\dot{Q}_{HS})$	$MAX(\dot{Q}_{LS})$
T_{Hopt}	$\sqrt{T_{HS}} \frac{K_L \sqrt{T_{LS}} + K_H \sqrt{T_{HS}}}{K_H + K_L - \dot{S}_C}$	T_{HS}	T_{HS}
T_{Lopt}	$\sqrt{T_{LS}} \frac{K_L \sqrt{T_{LS}} + K_H \sqrt{T_{HS}}}{K_H + K_L - \dot{S}_C}$	T_{LS}	T_{LS}

Table 4 : Particular cases of two objectives

Table 4 reports the corresponding results when only two objectives are considered, without added technical constraint.

This table is an original result, because until now, to our knowledge, only the mixing of maximization of power, with minimization of entropy production has been proposed by Angulo-Brown [22] and Yan [23], but entropy criterion does not appear as a fundamental criterion in the proposed triangle.

3.2.3. Physical conditions to fulfill for three objectives criterion

We see regarding the particular optimal temperatures vector, that these mathematical solutions must fulfill physical conditions. Namely, for an engine $T_{HG}^* < T_{HS}$, $T_{LG}^* > T_{LS}$, and $|\dot{W}| \geq 0$.

The first condition imposes:

$$\frac{1 - \beta}{\alpha - \beta} < \frac{1}{1 - \eta_{CS}} \left(\frac{K_L - \dot{S}_C}{K_L} \right)^2 \quad (13)$$

with:

$$\eta_{CS} = 1 - \frac{T_{HS}}{T_{LS}}$$

The second condition imposes:

$$\frac{\alpha - \beta}{1 - \beta} > (1 - \eta_{CS}) \left(\frac{K_H - \dot{S}_C}{K_H} \right)^2 \quad (14)$$

α, β, γ weighting factors appear related to η_{CS} Carnot efficiency of the system, converter irreversibility \dot{S}_C and the design (K_H, K_L) .

3.2.4. Some example of obtained results

Numerical results are given for nondimensionnal system where:

$$k_j = \frac{K_j}{K_T}; t_j = \frac{T_j}{T_0}; s_j = \frac{\dot{S}_j}{K_T}; q_j = \frac{\dot{Q}_j}{K_T T_0}; w = \frac{\dot{W}}{K_T T_0} \quad (15)$$

Dimensional parameters values of the problem are:

$$K_T = 5000 \text{ W/K}; T_0 = 293 \text{ K}; k_h = 0.5; k_l = 0.5; sc = 0,0001; \\ t_{hs} = 3; t_{ls} = 1.2; k_{li} = 0.05; k_{le} = 0.01; k_{lo} = 0.001 \quad (16)$$

In Figure 4, results for the case where $\gamma=0$ are proposed. α , the variable is given in abscissa and β is deduced from the equality $\alpha + \beta + \gamma = 1$. In Figure 5, $\beta=0$, α is also the variable and γ is deduced. In both cases, optimal temperatures are given (a) and the corresponding power output (b). Corresponding efficiency (c), entropy creations (d), heat expenses (e) and heat rejections (f) for all of the control volumes are presented.

For both cases, when the α weighting increases, temperature differences with the source and sink increase until the limit case where $\alpha=1$ and $\beta=\gamma=0$ which corresponds to the maximization of the power output. Then, heat expenses, heat rejections and entropy creations increase with α weighting. On the other hand, efficiencies decrease.

Physical limitations can be seen in all curves but differ for each quantity observed. The physical limitation for T_H^* differs from T_L^* for example. For both cases, the temperature T_H^* is more restrictive than the temperature T_L^* . While all development trends are the same, the case where β is variable is more constrained than the case where γ is variable.

Limit α value corresponding to zero power of the engine is the same as those linked to the heat consumed and released for the single converter. The limitation on the efficiency is the same whatever the system considered and corresponds to the limitation associated with the power output. The physical

limitation of T_H^* corresponds to the one of the converter heat expense q_h . In symmetry, the physical limitation of T_L^* is the same than the one of the converter heat rejection q_l .

The gap between different systems is due to the heat loss conductance and the internal created entropy choices. A translation of the physical quantities is observed.

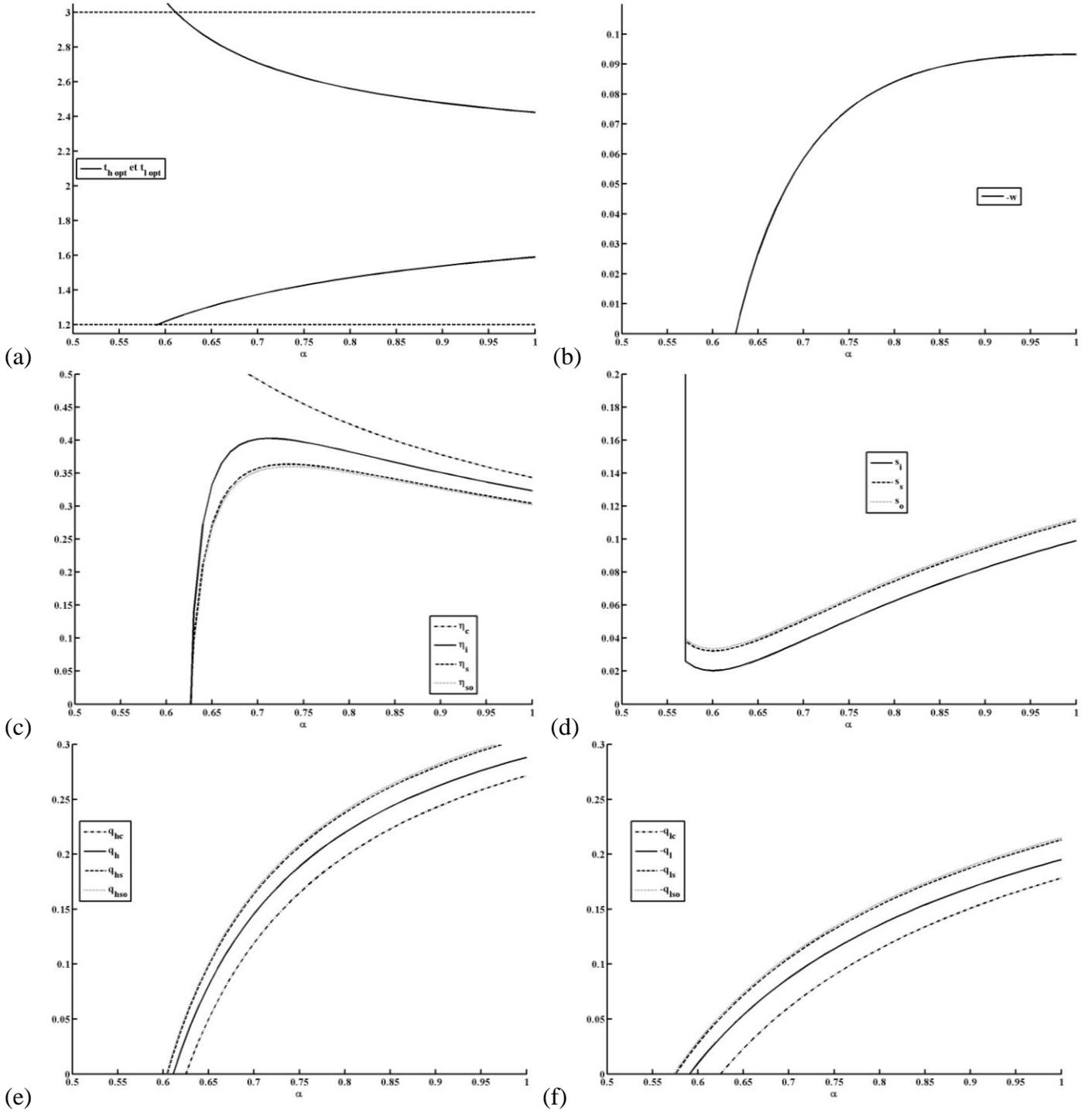


Figure 4 : Results for the case where $\gamma=0$, α and β variables; (a) Optimal temperatures; (b) Power output; (c) Efficiency ; (d) Entropy creations; (e) Heat expenses; (f) heat rejections

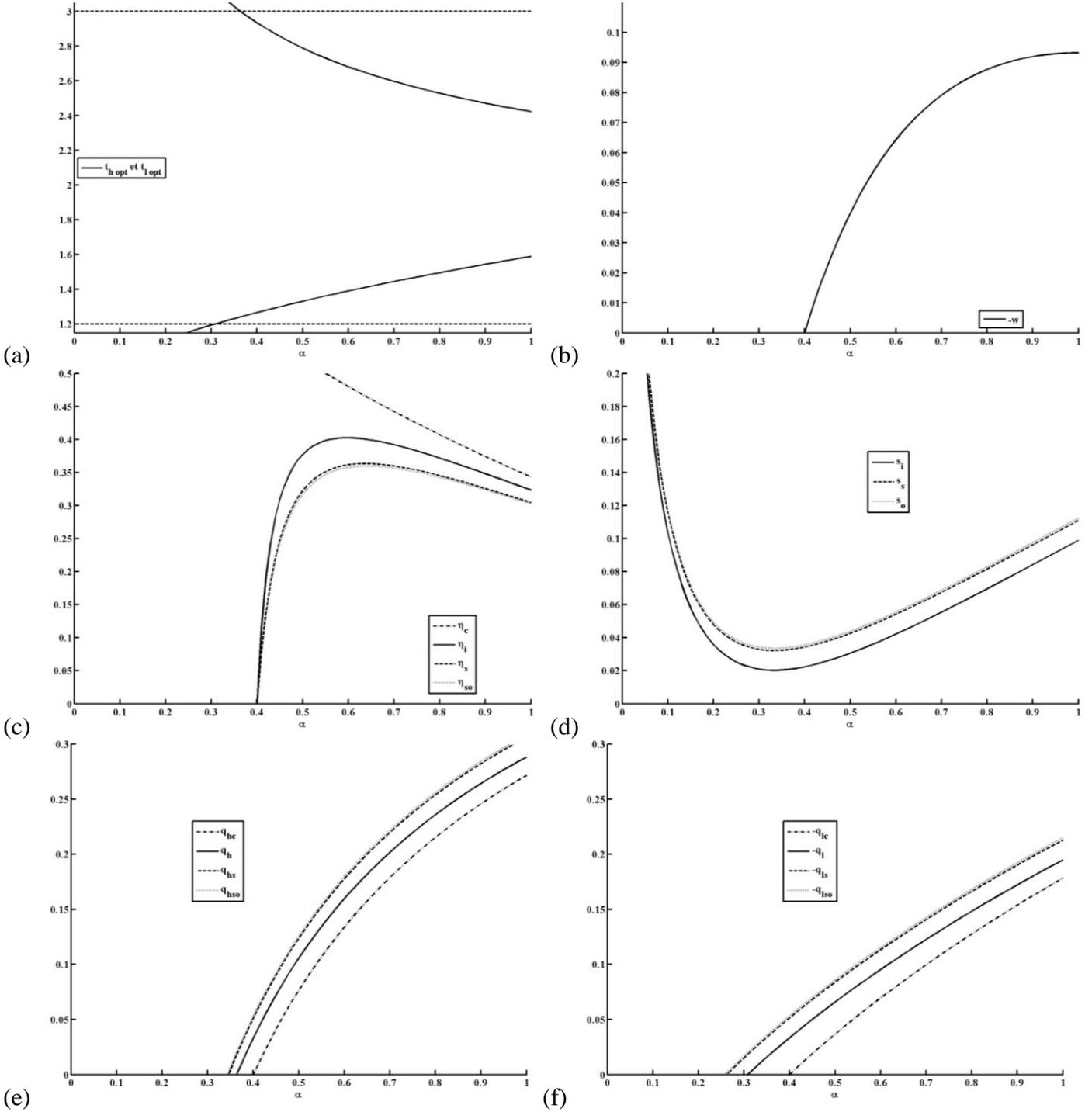


Figure 5 : Results for the case where $\beta=0$, α and γ variables; (a) Optimal temperatures; (b) Power output; (c) Efficiency ; (d) Entropy creations; (e) Heat expenses; (f) heat rejections

4. Discussion and conclusions

Multiobjective optimization appears as an actual goal for energy systems and processes. New appraisal of energy systems and processes is concerned with maximization of useful effect UE, minimization of Energy Expense EE (cost: economical aspect), minimization of rejection R or pollution P (heat, matter: environmental aspect) (Figure 6). This constitutes the fundamental triangle that is proposed here for criterions (Figure 3).

Starting from thermoeconomic approach, we derive a non dimensional weighting method, allowing to combine the three fundamental objectives. The general objective function to maximize OF_G becomes:

$$OF_G = \alpha|UE| - \beta EE - \gamma|R| \quad (17)$$

With the condition $\alpha + \beta + \gamma = 1$

The proposed methodology has been illustrated with the Novikov-Curzon-Ahlborn thermomechanical engine. New results are proposed. Numerous previous results are recovered. A generalized nice radical has been found for the converter.

It appears that this methodology uses the variational calculus instead of heuristic methods, as it becomes more and more frequent now. Moreover we have shown that values of weighting factors are constrained by physical conditions, to satisfy between the objectives. This does not appear in heuristic method.

The proposed methodology is to be applied to various other systems in the near future.

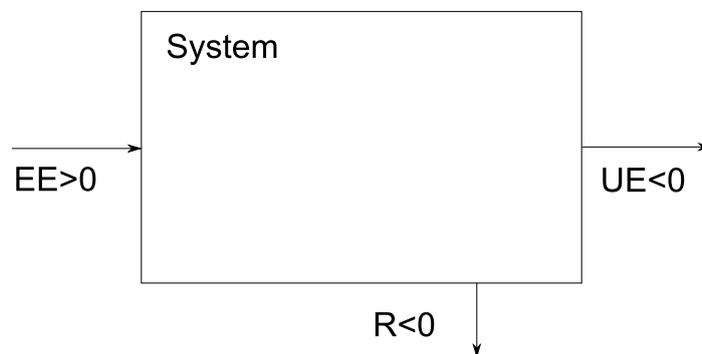


Figure 6 : General energy system scheme

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