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# A hierarchical global-local finite element analysis of variable stiffness composite structures

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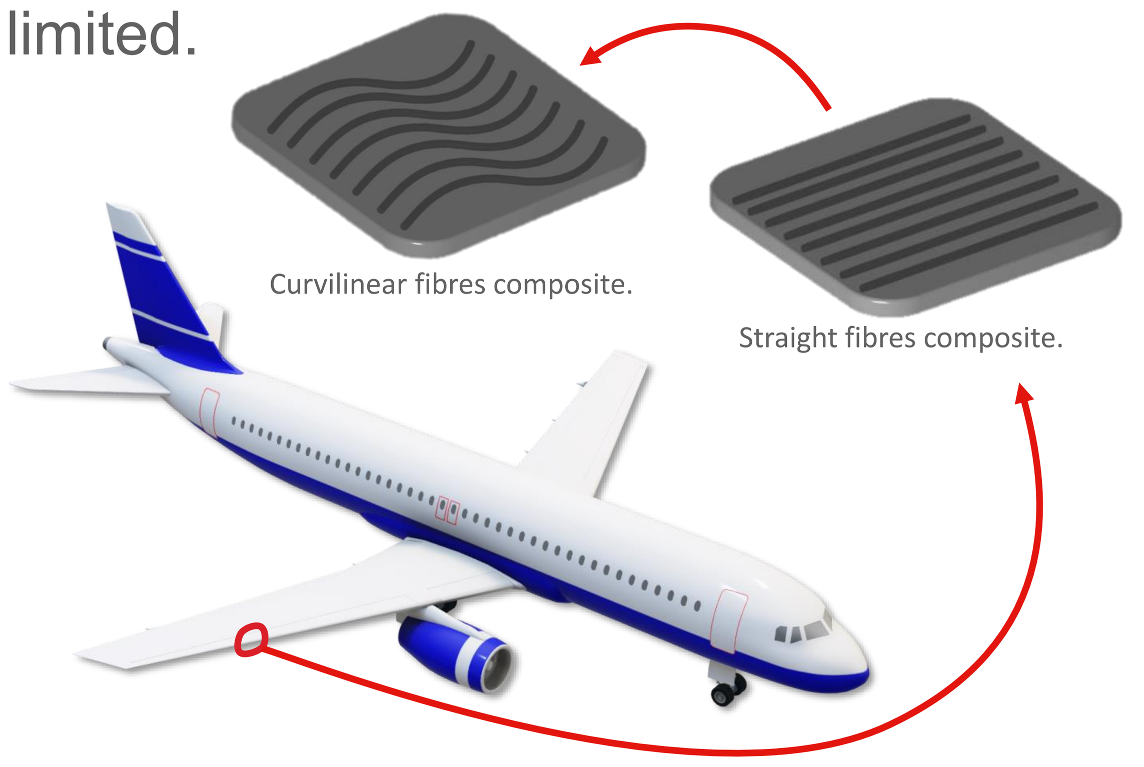
## INTRODUCTION & AIM

Composite materials are usually characterized by a resin matrix where fibres are applied to enhance the mechanical performances of the structure.

**Variable Stiffness Composites (VSCs)** appear to be a great alternative to straight fibres ones, since curvilinear fibres can improve the design and the properties of this kind of materials. Nevertheless, the numeric approaches that can be used for the study VSCs are still limited.

The main **objective** of this work is to develop an innovative framework for the study of curvilinear fibres plate structures.




## METHOD

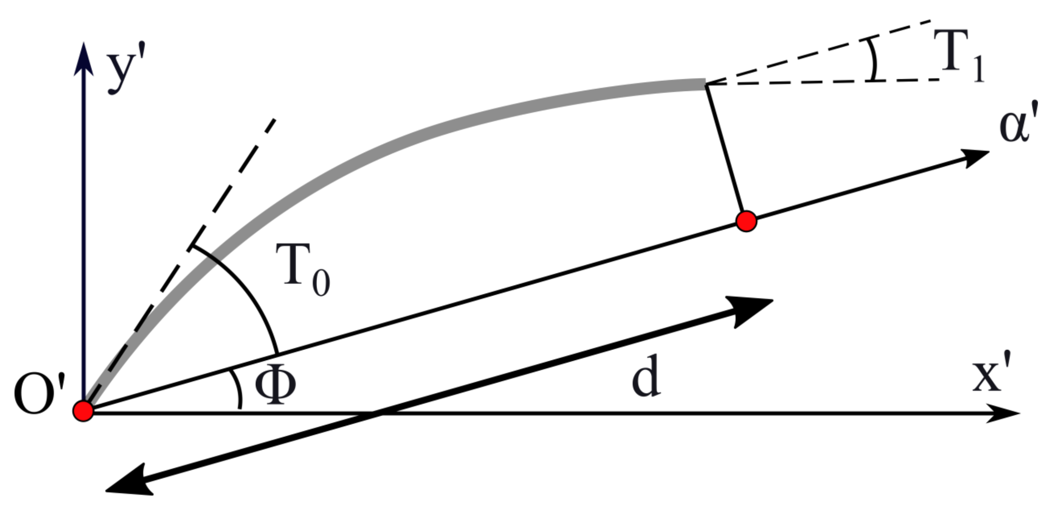
A global/local model using Carrera's Unified Formulation (CUF) within a Finite Element (FE) context is introduced. The proposed method uses CUF plate elements which consider the fibres rotation within the structure middle plane.

**Local orientation of the fibres-path**:

$$\theta(\alpha') = \Phi + T_0 + \frac{T_1 - T_0}{d}|\alpha'|$$

$$\alpha' = x'\cos(\Phi) + y'\sin(\Phi)$$

**Notation**: $\Phi < T_0, T_1 >$

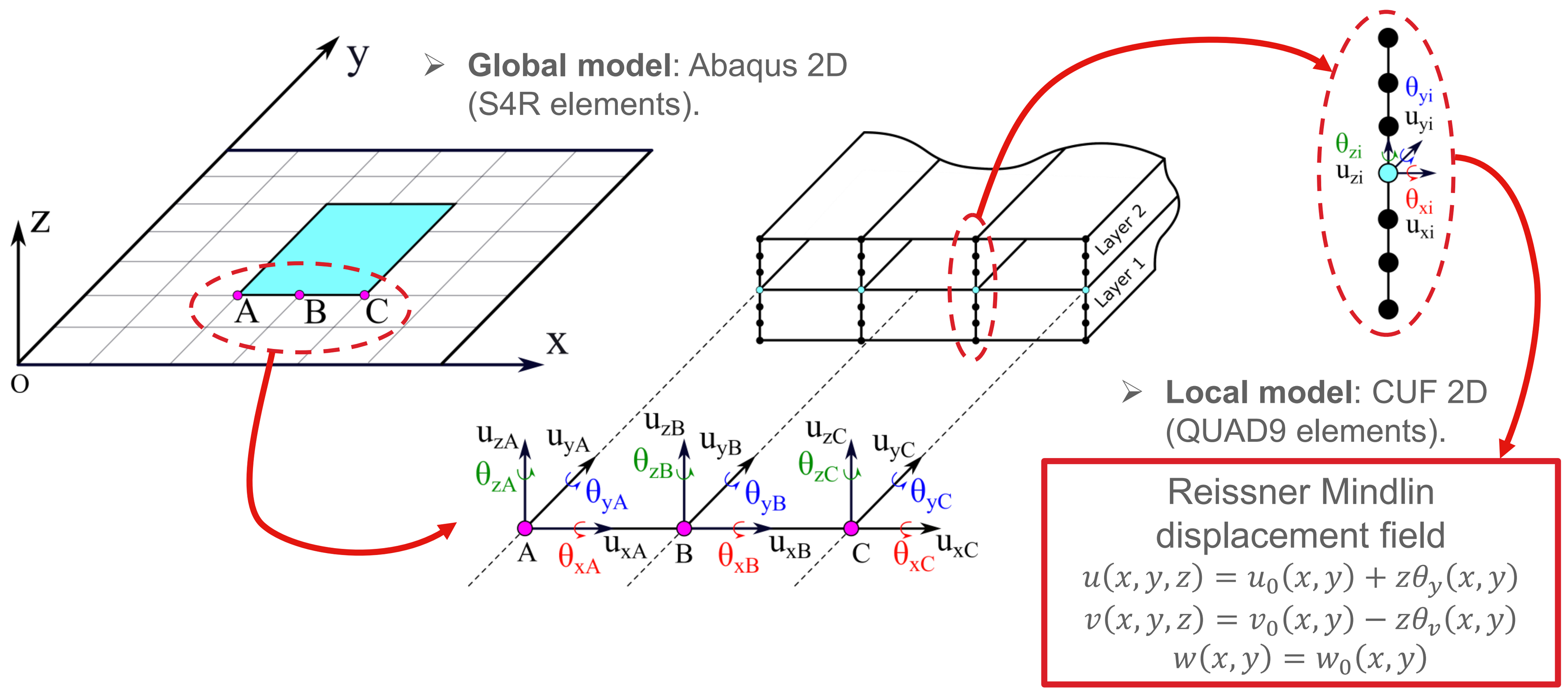



The Principle of Virtual Displacements (PVD) and the Reissner's Mixed Variational Theorem (RMVT) are considered for obtain the problem governing equations.

PVD: $\delta \boldsymbol{q}_{\tau i}^{T} K_{PVD}^{\tau i s j} \boldsymbol{q}_{sj} = \delta L_e - \delta L_{in}$

RMVT: $\delta \boldsymbol{q}_{\tau i}^{T}\left(K_{uu}^{\tau i s j}\boldsymbol{q}_{sj} + K_{u\sigma}^{\tau i s j}\boldsymbol{g}_{sj}\right) = \delta L_e - \delta L_{in}$
$\delta \boldsymbol{g}_{\tau i}^{T}\left(K_{\sigma u}^{\tau i s j}\boldsymbol{q}_{sj} + K_{\sigma\sigma}^{\tau i s j}\boldsymbol{g}_{sj}\right) = 0$

**Acronyms**:
L→Layer-Wise
D→PVD
M→RMVT
Expansion order on thickness → N
L D/M N




## RESULTS & DISCUSSION

Reference solution: Abaqus 3D, C3D20R.

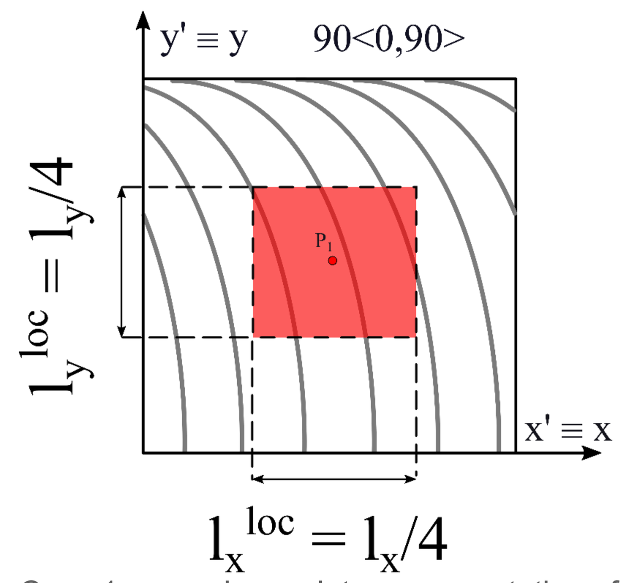
Material:

Cases 1,3

| $E_1$ | $E_2 = E_3$ | $G_{12} = G_{13} = G_{23}$ | $\nu_{12} = \nu_{13} = \nu_{23}$ |
|---|---|---|---|
| $50\ GPa$ | $10\ GPa$ | $5\ GPa$ | 0.25 |

Case 2

| $E_1$ | $E_2 = E_3$ | $G_{23}$ | $G_{12} = G_{13}$ | $\nu_{12} = \nu_{13}$ | $\nu_{23}$ |
|---|---|---|---|---|---|
| $137.9\ GPa$ | $8.96\ GPa$ | $6.21\ GPa$ | $7.1\ GPa$ | 0.3 | 0.49 |



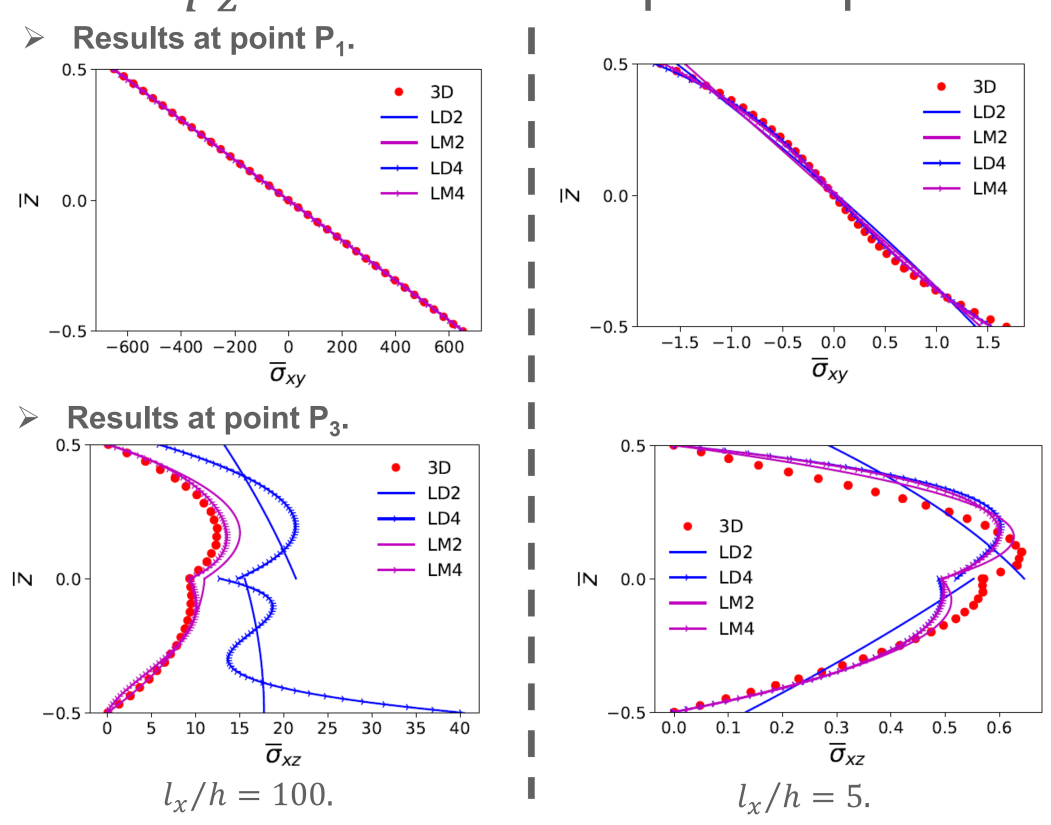

Case 1: mono-layer plate, representation of fibers variational law and local region (red).

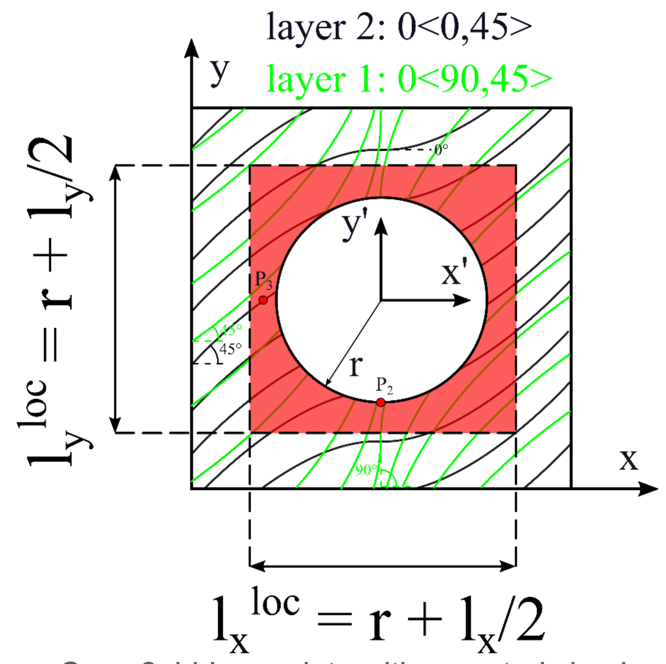
### Bending problems

Geometry: $l_x = l_y = 1m$, $l_x/h = 5, 100$.

Boundary conditions: clamped.

Load: $p_z = -10\ kPa$ at plate top.




Case 2: bi-layer plate with a central circular cut-out, representation of fibers variational law and local region (red).

### Free-edge problem

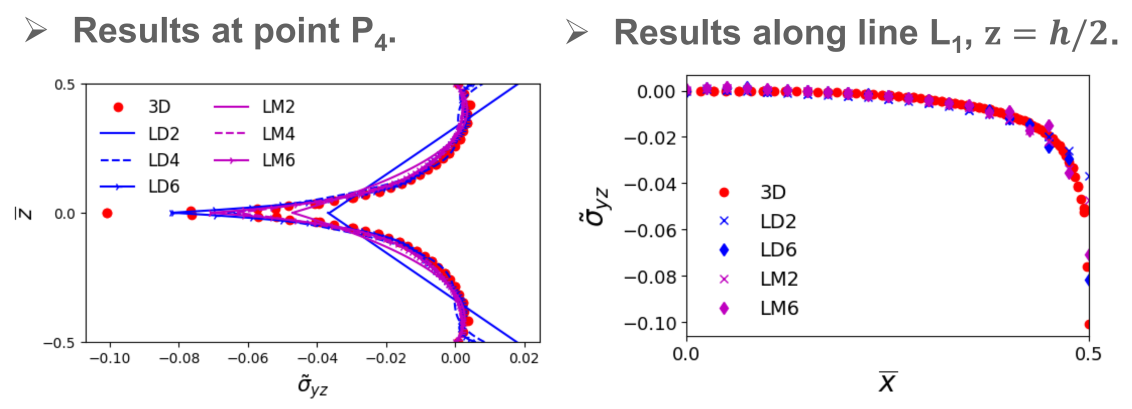
Geometry: $l_x = 1m$, $l_y = 2m$, $h = 0.25m$.

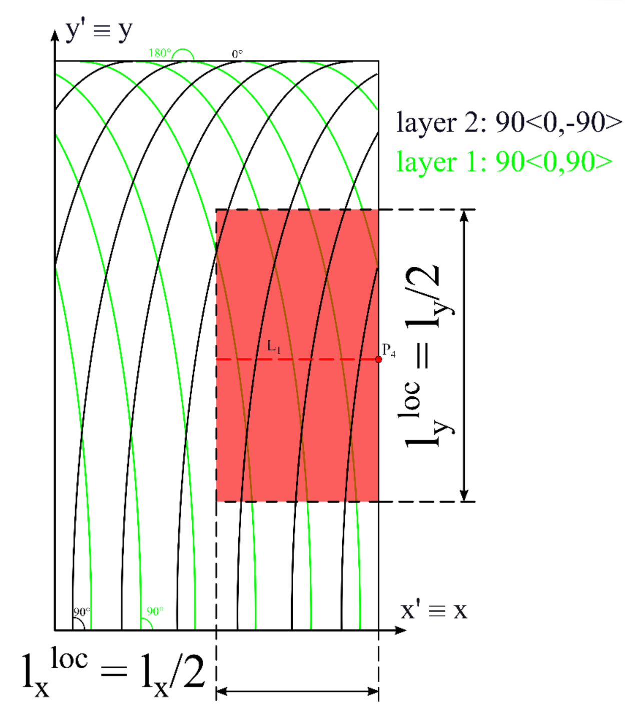
Reference solution: Abaqus 3D, C3D20R.

Boundary conditions:

$y = 0: u_y = -1m; u_x = u_z = 0.$

$y = l_y: u_y = 1m; u_x = u_z = 0.$




Case 3: bi-layer plate, representation of fibers variational law and local region (red).

## CONCLUSIONS

- Excellent agreement with 3D Abaqus benchmark solutions.
- RMVT models better capture transverse stresses.
- Reduced accuracy only near local boundaries.

## FUTURE WORK / REFERENCES

- Extension of the proposed approach to shell structures with constant and variable curvatures.

Iannotta DA, Giunta G, Montemurro M. A mechanical analysis of variable angle-tow composite plates through variable kinematics models based on Carrera's unified formulation. Composite Structures 2024;327:117717.

## ACKNOWLEDGMENTS

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