The 4th International Online Conference on Materials

IOCM 2025 Conference

3–6 November 2025 | Online

Nonlinear buckling analysis of FGM plates based on First-Order Shear Deformation Theory using the Asymptotic Numerical Method.

[1]Achraf Gasbi, [1]Abdessalam Dahmani , [1,2]Hamza Chaabani, [1]Lhoucine Boutahar, [1]Khalid El Bikri

[1]**Modeling of Mechanical Structures and Systems (M2SM), National Higher School of Arts and Crafts of Rabat (ENSAM Rabat), Mohammed V University of Rabat, Rabat 10100, Morocco**

[2] **Laboratory of Mathematics and Engineering Sciences, School of Engineering, International University of Casablanca (UIC), Casablanca , Morocco**

INTRODUCTION

Functionally Graded Materials (FGMs) are increasingly being used in engineering applications where structures are exposed to severe mechanical and/or thermal conditions. Their main advantage lies in the gradual, continuous variation of material properties through the thickness. This tailored compositional gradient effectively mitigates stress concentrations, minimizes thermal mismatch stresses, and significantly improves the overall structural performance and durability. Given these substantial benefits, FGMs are widely adopted in critical fields such as aerospace, energy production, and automotive engineering. One of the main challenges in designing FGM structures is accurately predicting their behavior under complex loading conditions, particularly compressive loads, where nonlinear effects such as buckling and post-buckling come into play. To capture these critical effects with precision, it is essential to employ a robust mechanical model that inherently includes both shear deformation and geometric nonlinearity [1- 3] .

In this study, we adopt the First-Order Shear Deformation Theory (FSDT), also known as the Reissner–Mindlin theory. This theory offers an excellent compromise between computational simplicity and accuracy, particularly for the analysis of moderately thick plates, where the effect of shear is non-negligible. [4] To solve the resulting complex nonlinear problem, we utilize a sophisticated numerical strategy that combines the Finite Element Method (FEM) with the Asymptotic Numerical Method (ANM). While FEM allows for the efficient spatial discretization of the plate structure, the ANM makes it possible to follow the equilibrium path smoothly and systematically. Crucially, this approach enables the accurate detection of bifurcation points (critical buckling loads) without the convergence issues often encountered in traditional iterative methods like Newton–Raphson. This approach was implemented using a custom code developed in MATLAB. This paper aims to provide an efficient and highly accurate numerical tool for analyzing the complex nonlinear buckling and post-buckling behavior of FGM plates. Furthermore, the results obtained with our FSDT-based model are rigorously compared with those from other established models (such as the Von Kármán plate theory [3]) to thoroughly evaluate the performance, reliability, and contribution of our proposed approach.

METHOD

The plate's mechanical behavior is modeled using the First-Order Shear Deformation Theory (FSDT) (Reissner–Mindlin). This model accounts for transverse shear deformation, which is essential for accurate analysis of moderately thick plates. The 3D displacement field at a generic point M(x, y, z) is defined by the mid-surface kinematic variables (u,v,w) and the rotations β_x ,β_y :

$$\begin{cases} U(x,y,z) = u(x,y) + z\beta_x(x,y) \\ V(x,y,z) = v(x,y) + z\beta_y(x,y) \\ W(x,y,z) = w(x,y) \end{cases} \qquad (1)$$

From the displacement field above, the strain is written as follows:

$$\gamma = \frac{1}{2}\left(\nabla \mathrm{u} + {}^t\nabla \mathrm{u}\right) + \frac{1}{2}\nabla \mathrm{u}.{}^t\nabla \mathrm{u} \;= \{\gamma^l\} + \{\gamma^{nl}\} = \{e\} + z\{k\} + \{\chi\} \qquad (2)$$

Where $\{\varepsilon\}$ denotes the membrane strain vector, $\{\kappa\}$ the bending curvature vector, and $\{\chi\}$ is the transverse shear strain vector of the mean surface of the structure ω expressed as:

$$\begin{Bmatrix} e_{xx} \\ e_{yy} \\ 2e_{xy} \end{Bmatrix} = \begin{Bmatrix} u_{,x} + \frac{1}{2}w_{,x}^2 \\ v_{,y} + \frac{1}{2}w_{,y}^2 \\ u_{,y} + v_{,x} + w_{,x}w_{,y} \end{Bmatrix}; \quad \begin{Bmatrix} k_{xx} \\ k_{yy} \\ 2k_{xy} \end{Bmatrix} = \begin{Bmatrix} \beta_{x,x} \\ \beta_{y,y} \\ \beta_{x,y} + \beta_{y,x} \end{Bmatrix}; \quad \begin{Bmatrix} 2\chi_{xz} \\ 2\chi_{yz} \end{Bmatrix} = \begin{Bmatrix} w_{,x} + \beta_x \\ w_{,y} + \beta_y \end{Bmatrix} \qquad (3)$$

A linear elastic constitutive law under the plane stress assumption is employed to describe the stress–strain relationship in the FGM plate. This constitutive relation is expressed as:

$$\{S^{gen}\} = [D]\{\gamma^{gen}\} \qquad (4)$$

The equilibrium equations of the structure are derived by applying the principle of minimum potential energy, which states that the system reaches equilibrium when its total potential energy is stationary. We obtain the following system of equations:

$$\begin{cases} \int\!\!\int_{\omega} \langle\delta\theta\rangle\left({}^t[H] + {}^t[A(\theta)]\right)\{S^{gen}\}\, d\omega = \lambda W_{ext}(\delta u) \\ \{S^{gen}\} = [D]\left([H] + \frac{1}{2}[A(\theta)]\right)\{\theta\} \end{cases} \qquad (5)$$

The Asymptotic Numerical Method (ANM) is an efficient continuation technique for solving the nonlinear system derived via the Finite Element Method (FEM). ANM traces the equilibrium path using a Taylor series expansion around a solution point. This approach is highly robust and cost-effective because it only requires the inversion of the tangent stiffness matrix once per step, ensuring accurate resolution and detection of bifurcation points without convergence issues. [5]

By substituting these approximations into the continuous problem, a global system is obtained and organized according to the powers of the control parameter “a”, as shown below:

Ordre p = 1: $[K_T]\{q_1^e\} = \lambda_1\{F^e\}$; Ordre 1<p ≤ N: $[K_T]\{q_p^e\} = \lambda_p\{F^e\} + \{F_p^{nl}\}$ (6)

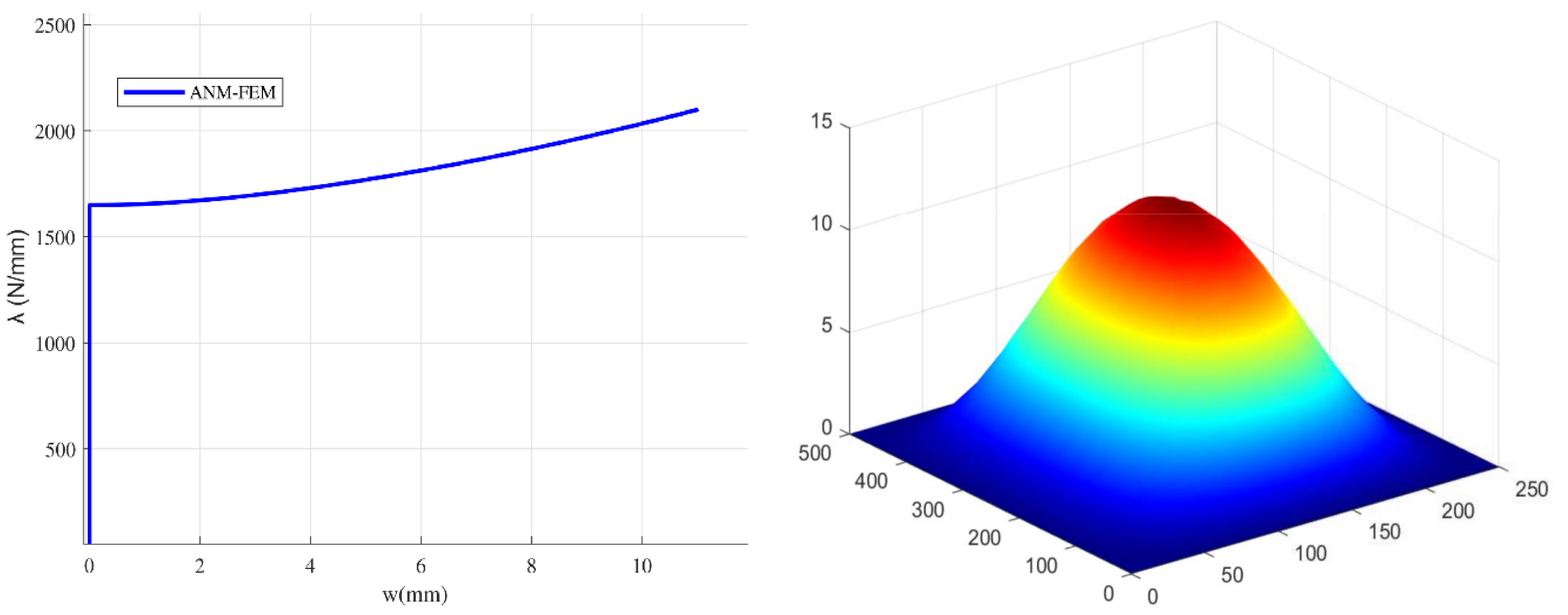
Finally, a continuation procedure is applied to trace the complete solution path. The validity of the Taylor series expansion (6) is limited to an interval [0, a_{max}], where a_{max} is defined as follows:

$$a_{max} = \left(\varepsilon\frac{\|U_1\|}{\|U_N\|}\right)^{\frac{1}{N-1}} \qquad (7)$$

The process is continued by using the last point of the first branch, (u(a_{max}), λ(a_{max})), as the starting point for the next branch. [6]

RESULTS & DISCUSSION

The analysis focuses on the buckling of a thin rectangular FGM plate (Aluminum/Alumina: E_m= 70 GPa, E_c= 380 GPa, ν=0.3) with dimensions a=500 mm, b=250 mm, and h=5 mm. Fully Clamped (CCCC) boundary conditions are imposed. The structure is subjected to an in-plane compressive load P_x. The influence of the biaxial loading ratio R is investigated through the relationshipP_y = R·P_x, covering uniaxial (R=0), biaxial (R=1), and compression-tension (R=−1) scenarios. The solution is obtained using the ANM-FEM approach (Q8 elements, 16 x 10 mesh), with a truncation order N=20, aiming to determine the critical buckling loads and the impact of the ratio R.



(a): Load–deflection curve under uniaxial compression (R=0). (b): Corresponding buckling mode shape of the plate.

Figure 1: Buckling and post-buckling behavior of a CCCC FGM plate (Al/Al_2O_3) under uniaxial compression.

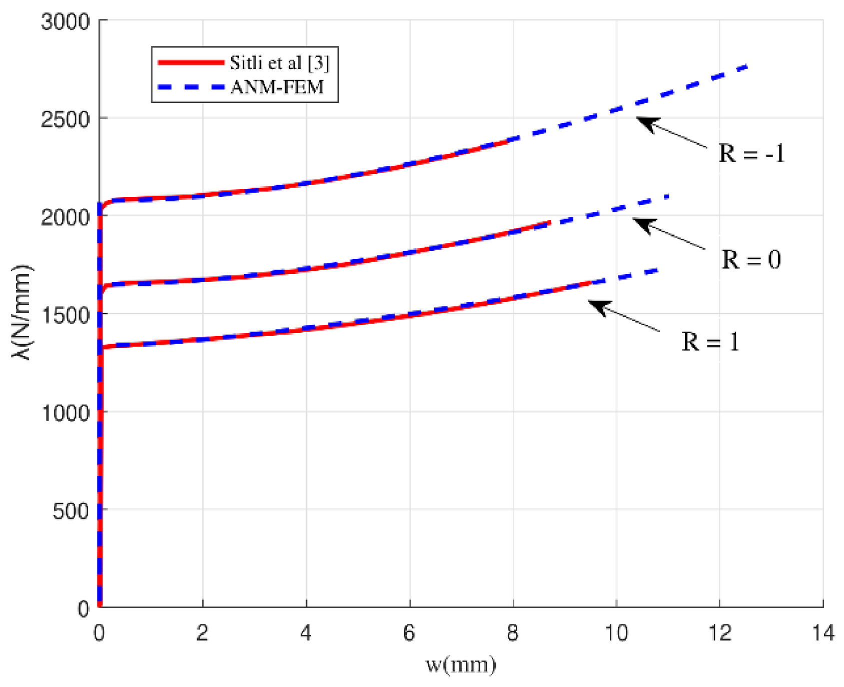


Figure 2: Comparison of Load-Deflection Curves of the FGM plate under different compressions between ANM-FEM and Sitli et al [3].

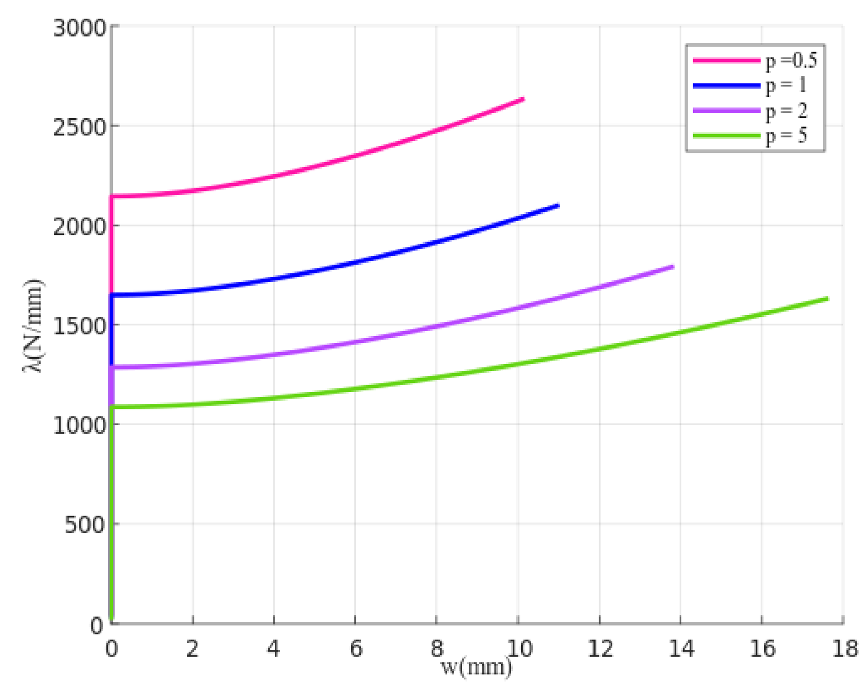


Figure 3: Influence of the Power-Law Index (p) on the Load-Deflection Behavior of the Plate.

The analysis of the base case under uniaxial compression(R=0) identified a stable buckling bifurcation at the critical load P_{cr} = 1648 N/mm (Fig. 1-a), with the plate adopting its first post-critical mode(Fig. 1-b). The ANM-FEM model is validated by the excellent agreement observed with reference literature across the three loading scenarios (Fig. 2). This figure demonstrates the impact of the ratio R on stability: the lowest critical load is found under biaxial compression (R=1) while the highest occurs in compression-tension (R= -1). Finally, the study of the power-law index 'p' (Fig. 3) confirms its role as a critical design parameter, as increasing 'p' (more metallic plate) leads to a drastic reduction in both the critical load and post-buckling stiffness.

CONCLUSION

This study presented a high-performance numerical approach, based on a hybrid ANM-FEM method coupled with FSDT, for the analysis of the nonlinear buckling behavior of FGM plates. The method demonstrated its ability to accurately track post-buckling equilibrium paths and identify critical bifurcation loads by quantifying the influence of key parameters. It was shown that the biaxial load ratio (R) considerably modifies stability, with the tension effect acting as a stiffener, and that the material's power-law index (p) has a crucial impact, as a high value reduces stiffness and buckling resistance.

REFERENCES

[1] H. Chaabani, S. Mesmoudi, L. Boutahar et K. El Bikri, «A high-order finite element continuation for buckling analysis of porous FGM plates,» Engineering Structures, vol. 279, n° 1115597, 2023.

[2] S. Khalil, O. Elmhaia, A. Hamdaoui, B. Braikat, H. Hu, A. Boukamel et N. Damil, «Modeling of Functionally Graded Material Shells,» E3S Web of Conferences, vol. 601, n° 100090, 2024.

[3] Y. Sitli, K. Mhada, O. Bourihane et H. Rhanim, «Buckling and post-buckling analysis of a functionally graded material (FGM) plate by the Asymptotic Numerical Method,» Structures, vol. 31, pp. 1031-1040, 2021

[4] P.V. Jeyakarthikeyan, Siddarth Subramaniam, Vikalp Charuasia et S. Vengatesan, «Richardson extrapolation based integration scheme using FSDT over Quad elements: Application to thick FGM plates,» Composite Structures, vol. 322, n° 1117351, 2023.

[5] H. Mottaqui, B. Braikat et N. Damil, «Discussion about parameterization in the asymptotic numerical method: Application to nonlinear elastic shells,» Computer Methods in Applied Mechanics and Engineering, vol. 199, pp. 1701-1709, 2010.

[6] B. Cochelin, N. Damil et M. Potier-Ferry, «Méthode Asymptotique Numérique un solveur pour les problème non linéaire,» Hermès Lavoisier, 2007.