Entropy and Copula Theory in Quantum Mechanics

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Classical and quantum mechanics density

- In classical mechanics there are individual particles with invariant density in the phase space. In quantum mechanics each particle is sensitive in different ways to all other particles for its position and also for the measure process.

Non-standard entropy vector Sj

$$\begin{cases} S_1 = k\log\rho_1 = \xi_1(x_1,\ldots,x_n \mid \theta_1,\theta_2,\ldots\ldots,\theta_m) \\ S_2 = k\log\rho_2 = \xi_2(x_1,\ldots,x_n \mid \theta_1,\theta_2,\ldots\ldots,\theta_m) \\ \ldots \\ S_N = k\log\rho_N = \xi_N(x_1,\ldots,x_n \mid \theta_1,\theta_2,\ldots\ldots,\theta_m) \end{cases}$$

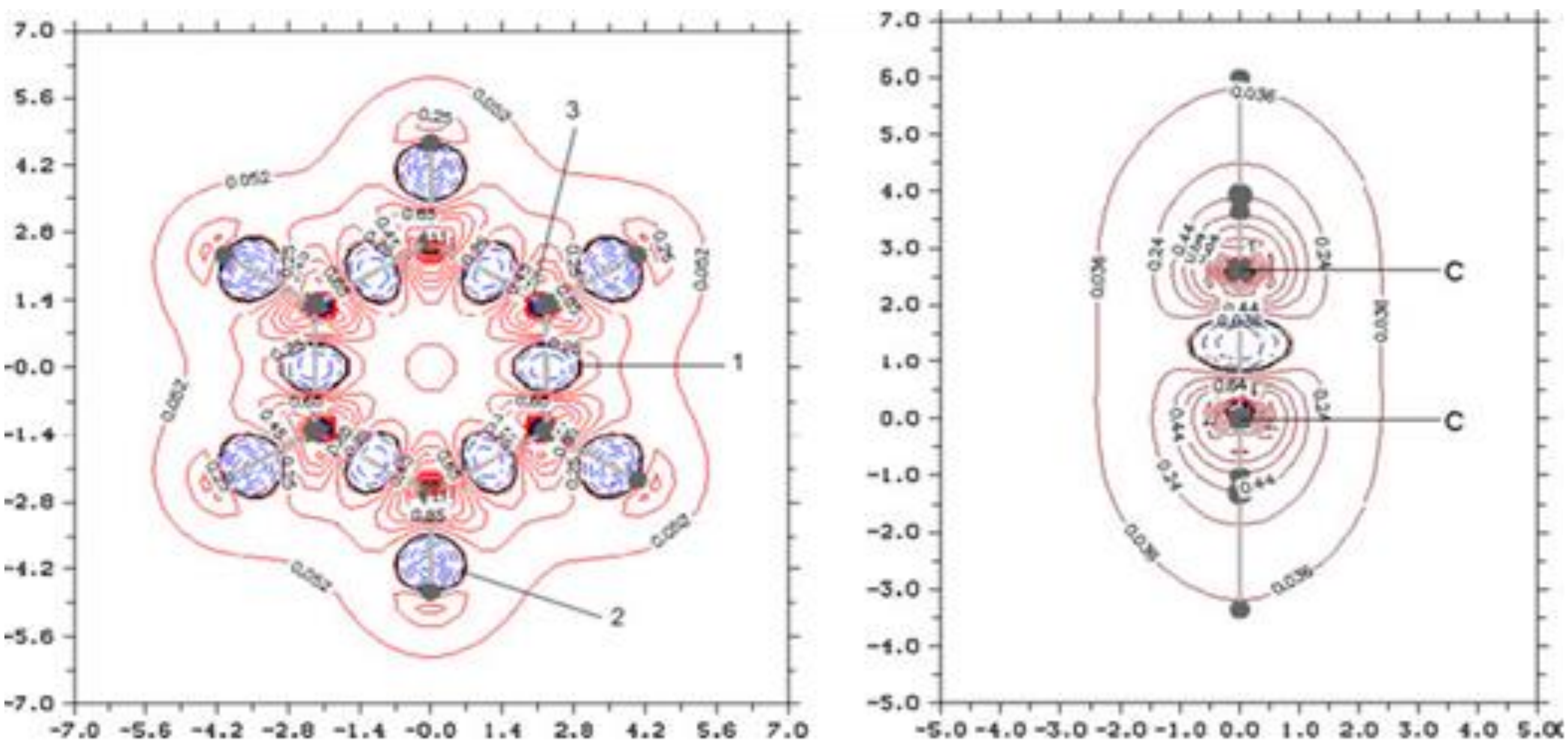
Fisher information as metric for quantum mechanics

$$ds^2 = \sum_{i,k} G_{i,k} \, d\theta^i \, d\theta^k$$

where

$$G_{i,k}(x_1, \ldots x_n) = \sum_j \frac{\partial \xi_j}{\partial \theta_i} \frac{\partial \xi_j}{\partial \theta_k}$$

Fisher information distribution for network of electrons in chemistry



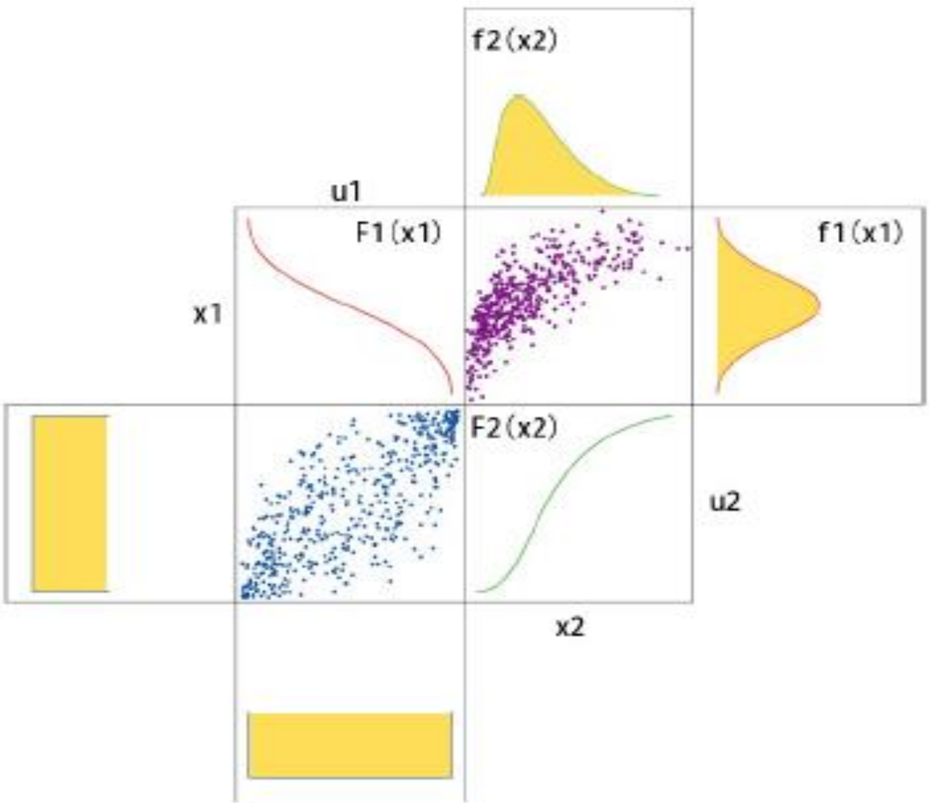
Copula $c(u_1,u_2,...,u_n)$ for joint probability $p(x_1,x_2,...,x_n)$ as entanglement or depedence among variables in quantum mechanics

$$p(x_1,x_2,....,x_n) = c(u_1,u_2,....,u_n)\,p_1(x_1)\,p_2(x_2).....p_n(x_n)$$

Fisher information and copula c

$$\frac{\partial^2 \rho_j}{\partial\theta^k \partial\theta^p} = \frac{\partial}{\partial\theta^k}\left(\frac{\partial\rho_j}{\partial\theta^p}\right) = \frac{\partial}{\partial\theta^k}\left(\frac{\partial c_j}{\partial\theta^p}\rho(\theta_1)...\rho(\theta_N) + c\rho(\theta_1)...\frac{\partial\rho(\theta_p)}{\partial\theta_p}....\rho(\theta_N)\right)$$

Copula as correlation between variables



Covariant derivative for zero quantum field $F_{k,h}$ and commutator

$$D_k = \frac{\partial}{\partial x^k} + \frac{\partial \xi_j}{\partial \theta^k} = \frac{\partial}{\partial x^k} + \frac{\partial \log \rho_j}{\partial \theta^k}$$

$$[D_k, D_h] = F_{k,h} = 0 \qquad (54)$$

Covariant derivative in quantum mechanics

$$\frac{\partial}{\partial x^k} + \frac{\partial^2 \xi_j}{\partial x^k \partial x^p} \frac{\partial x^i}{\partial \xi_j} = \frac{\partial}{\partial x^k} - \frac{\partial \log \rho_j}{\partial x_h} = \frac{\partial}{\partial x^k} + \Lambda_h^j \qquad (57)$$

Quantum potential Q covariant derivative and Lagrangian for quantum mechanics

$\delta S = 0$

For

$$\delta\int \rho(\frac{\partial S}{\partial t} + \frac{1}{2m} p_i p_j + V) dt d^n x = 0 \quad (62)$$

so

$$\frac{\partial S}{\partial t} + \frac{1}{2m} p_i p_j + V + \frac{1}{2m}(\frac{1}{\rho^2}\frac{\partial \rho}{\partial x_i}\frac{\partial \rho}{\partial x_j} - \frac{2}{\rho}\frac{\partial^2 \rho}{\partial x_i \partial x_j}) = \frac{\partial S}{\partial t} + \frac{1}{2m} p_i p_j + V + Q \qquad (63)$$

Covariant derivative and commutator as non zero Casimir field

$$D_k = \frac{\partial}{\partial\theta^k} + \frac{1}{\frac{1}{\rho_j}\frac{\partial\rho_j}{\partial\theta^i}\frac{\partial\rho_j}{\partial\theta^p}}\left(\frac{\partial^2\rho_j}{\partial\theta^k\partial\theta^p} - \frac{1}{\rho_j}\frac{\partial\rho_j}{\partial\theta^k}\frac{\partial\rho_j}{\partial\theta^p}\right)\frac{\partial\log\rho_j}{\partial\theta^p} = \frac{\partial}{\partial\theta^k} + \Gamma^i_{k,p}$$

$$= \frac{\partial}{\partial\theta^k} + K^i_{k,p}(\rho_j)\frac{\partial\log\rho_j}{\partial\theta^p}$$

$$[D_\mu, D_\nu]V_\alpha = -R^\lambda_{\alpha\mu\nu}V_\lambda \qquad (64)$$

Lagrangian for non zero field (Casimir field) in quantum mechanics

$$S=\int\rho[\frac{\partial S}{\partial t}+\frac{1}{2m}(p_i+\Gamma^i_{k,p})(p_j+\Gamma^j_{k,p})+V]dtd^nx=$$

$$S=\int\rho[\frac{\partial S}{\partial t}+\frac{1}{2m}(p_ip_j+p_i\Gamma^j_{k,p}+p_j\Gamma^i_{k,p}+\Gamma^i_{k,p}\Gamma^j_{k,p})+V]dtd^nx=$$

$$S=\int\rho[\frac{\partial S}{\partial t}+\frac{1}{2m}(p_ip_j+K^i_{k,p}(p)\frac{\partial\log\rho}{\partial x_i}\frac{\partial\log\rho}{\partial x_j})+V]dtd^nx=$$

$$\int\rho[\frac{\partial S}{\partial t}+\frac{1}{2m}p_ip_j+V)dtd^nx+\frac{1}{2m}\int\rho K^i_{k,p}(\rho)K^j_{k,p}(\rho)\frac{\partial\log\rho}{\partial x_i}\frac{\partial\log\rho}{\partial x_j})]dtd^nx \qquad (61)$$