Extended Abstract

Two-interval musical scales and binary structures in computer science and biology

Ivan Soshinskiy

Moscow P. I. Tchaikovsky Conservatory, Moscow, 125009, Bolshaya Nikitskaya street 13/6

E-Mail: consoniva@gmail.com

Tel.: +7-903-685-2362

Accepted:

Introduction

From ancient times, understanding the phenomenon of music and building musical structures were associated with mathematics. This report analyses relation of music with binary structures, the example of which is the binary number system, widely used in computer calculations and informatics, noise-immunity coding signals with using dyadic groups and so forth. Principles of binary opposition (or yin-yang principles) permeate living matter at different levels of its organization. Specific examples of this are the complementary pairs of nitrogenous bases of DNA molecules of heredity; division of the alphabet of these nitrogenous bases into pairs of purine-pyrimidine; organization of muscular movement on the base of muscle pairs of flexor-extensor; pairs of male-female, which give life to new generations, etc. In the field of musical culture, the binary principle is realized, in particular, in the existence of two-interval musical scales.

Main part

This report focuses on the analysis of development of two-interval musical scales on the base of the well-known algorithm of Pythagoras. For the author, the initial types of such musical scales were the known two-interval musical scales: the Pythagorean musical scale and so called pentagram scales (or Fibonacci-stage scales) from [1, 2].

Mathematical constructs of such musical scales are based on the Pythagorean algorithm that uses a geometric progression with special coefficients of the progression. For example, in the case of the Pythagorean musical scale, this algorithm uses a quint coefficient of 3/2 for the progression that leads to the construction of the sequence of notes do-re-mi-fa-sol-la-si-do on the interval of frequencies {1, 2} of one octave with basing on the following algorithmic steps:

1. Taking the first seven members of such geometrical progression with the quint factor 3/2, which begins from the inverse value of the quint: $(3/2)^{-1}$, $(3/2)^{0}$, $(3/2)^{1}$, $(3/2)^{2}$, $(3/2)^{3}$, $(3/2)^{4}$, $(3/2)^{5}$;

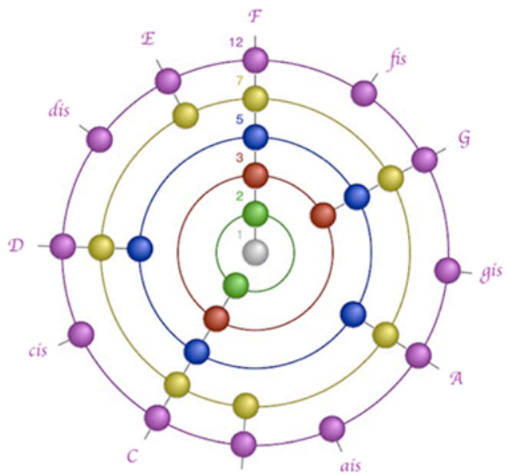
2. Returning into the octave interval {1, 2} for those members of this sequence, values of which overstep the limits of this interval; this returning is made for these values by means of their multiplication or division with the number 2. As a result of this operation, the new sequence is appeared (this sequence can be named "the geometrical progression with the returning into the octave "): $2*(3/2)^{-1}$, $(3/2)^0$, $(3/2)^1$, $(3/2)^2/2$, $(3/2)^3/2$, $(3/2)^4/4$, $(3/2)^5/4$;
3. The permutation of these seven members in accordance with their increasing values from 1 up 2 (the number 2 is included in this sequence as the end of the octave): $(3/2)^0$, $(3/2)^2/2$, $(3/2)^4/4$, $2*(3/2)^{-1}$, $(3/2)^1$, $(3/2)^3/2$, $(3/2)^5/4$, 2.

In this last sequence, a ratio of the greater number to the adjacent smaller number refers to as the interval factor. Two kinds of interval factors exist in this sequence only: 9/8, which is named the tone-interval T, and 256/243, which is named the semitone-interval S. One can check that the sequence of interval factors in this case is T-T-S-T-T-T-S. These five tone-intervals and two semitone-intervals cover the octave precisely: $(9/8)^5 * (256/243)^2 = 2$. If one takes not 7, but 6 or 8 members in the initial quint geometrical progression (see the first step of the algorithm), then the same Pythagorean algorithm does not give a binary sequence of interval factors T and S because three kinds of interval factor arise.

If the coefficient of the progression in the Pythagorean algorithm is equal not to 3/2, but to the square of the golden section (1+5^0.5)/2=1,618... then (with a certain number of members of the initial geometric progression) special two-interval scales are formed, which are called "genetic" by virtue of their relationship with the parameters of molecular genetic system [1, 2].

In this report, the author represents his mathematical theory, which allows to determine for what values of the coefficients of a geometric progression the Pythagorean algorithm generates two-interval scales for certain number of members in the initial progression. The author shows that some well-known in the history of music two-interval musical scales from different historical periods are algorithmically related because they are based on the same Pythagorean algorithm (the difference between them is determined only by differences in the values of their algorithmic parameters). In this theory, a theorem has been proved that only three kinds of scales can be created on the base of the algorithm of Pythagoras: one-interval (rare), two-interval (are relatively regularly) and three-interval (mostly).

The author has also analyzed a logarithmic representation of the algorithm of Pythagoras and he has created a convenient graphical method for the analysis of musical scales, generated by the algorithm for different values of its parameters. He has solved the problem of automation and visual analysis such algorithmic processes of creation of two-interval scales. The solution to this problem is based on the writing of a specialized computer program and a visual representation of the family two-interval scales with a different number of their stages in the form of concentric circles, such as the following:



For the algorithm of Pythagoras the inverse problem has been also solved: knowing the sequence of values of the number of stages inside two-interval scales, which are nested each into other, how one can determine the appropriate multiplying factor. In connection with the decision of this problem the author has developed a theory of the "Pascal's fractal": it is a geometric tree of numerical structure with its recursive organization, in which each number is formed as the sum of the two numbers above it (similar to the Pascal's triangle).

Music is widely used in today's global connections among people and nations. Development of methods and means of musical culture through in-depth understanding of the fundamentals of musical scales can contribute positive effects of musical influences on society and its members, including possibilities of music therapy.

References

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2. Darvas G., Koblyakov A., Petoukhov S., Stepanyan I. Symmetries in molecular-genetic systems and musical harmony. *Symmetry: Culture and Science*, 2012, vol. 23, № 3-4, p. 343-375.