

# Self-energy correction to the bound-electron $g$ factor in H-like ions: approximate treatment of the two-potential contribution.

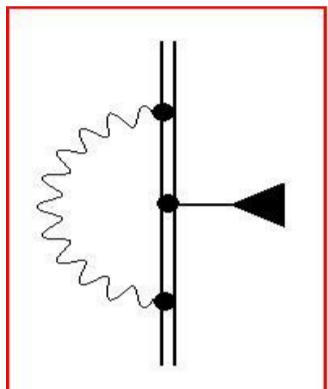
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## Introduction

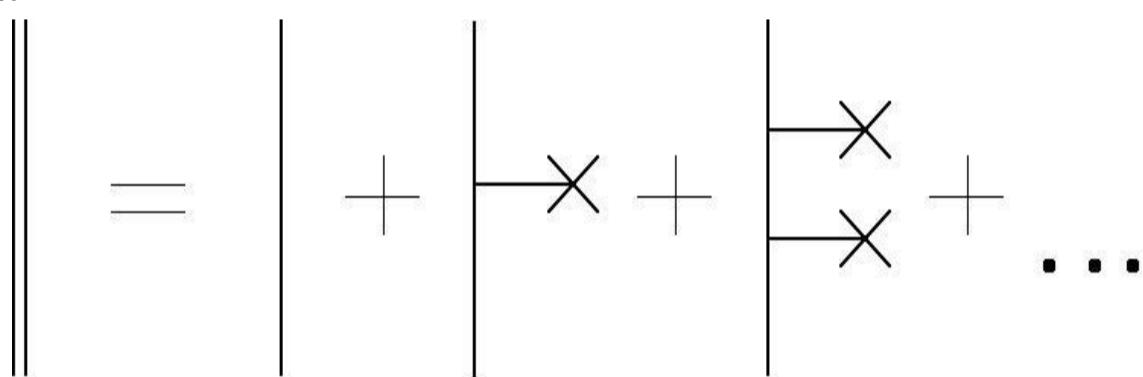
In our work, we consider **highly charged ions**.

Highly charged ions are an atomic system in which  $N_e \ll Z$ , where  $N_e$  is the number of electrons,  $Z$  is the nuclear charge number. In this case, the coupling constant  $\alpha Z \sim 1$ . Therefore, the calculations must be performed nonperturbatively in this parameter.



Feynman diagrams representing the self-energy correction to the bound-electron  $g$  factor

The double line indicates the bound-electron propagator, the wavy line is the photon propagator, and the line ended with a triangle means the interaction with the external magnetic field.

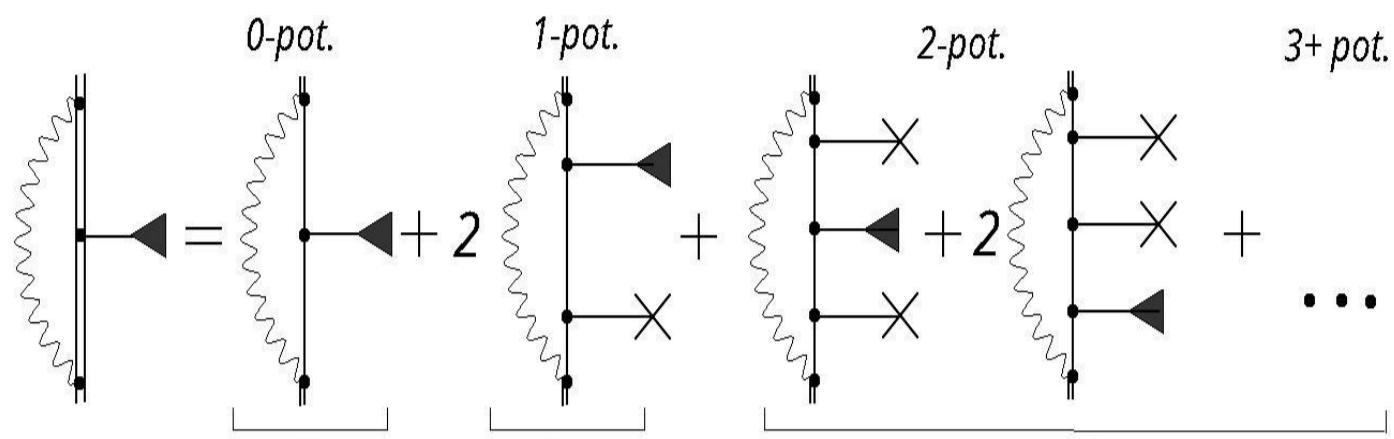


Expansion of the bound-electron propagator in terms of the binding (nuclear) potential  $V$

The single line indicates the free-electron propagator and the line that ends with a cross denotes the interaction with the Coulomb field of the nucleus  $V$ .

## Methods

In Ref. [1], the following method was proposed to consider the vertex part of the SE correction.



The standard method

$P'$  means that this contribution is calculated in momentum space after an ultraviolet renormalization.

$P$  means that this term can be calculated in momentum space, it is not divergent.

$X$  means that the remainder of the series is treated in the coordinate space.

Benefits of the coordinate space:

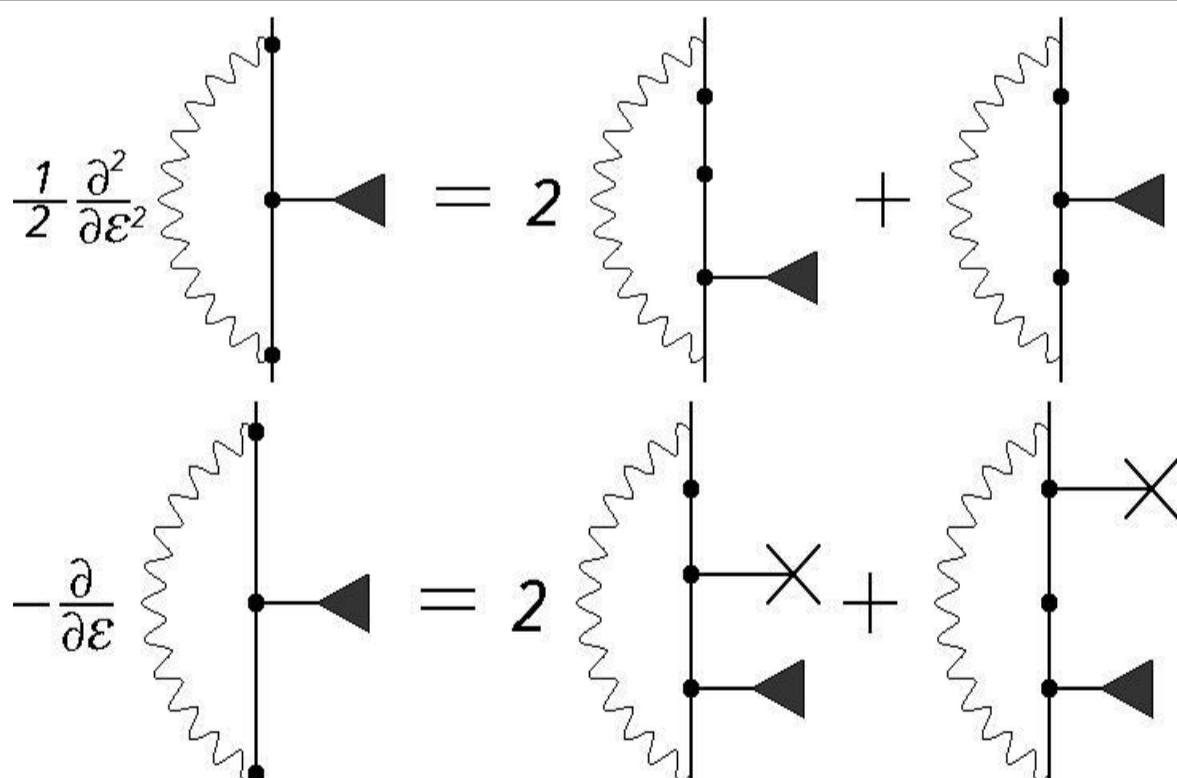
- We can calculate not expanding in  $\alpha Z$  (in single lines)

Drawbacks:

- We have to use the partial-wave expansion, which may converge rather slowly.

The one-potential term was separated out in Ref. [1] to improve the convergence.

## Methods of approximating two-potential contributions



Methods of approx. 2-pot using 0-pot. 1-pot. Contributions.  
 $\epsilon$  is a propagator parameter, and a black dot on the lines corresponds to 1

Our idea is to further improve the convergence of the computational scheme by studying the two-potential term additionally. This term is more complicated, so we consider some approximations to them, according to the ideas suggested in Ref. [2]. The diagrams on the right side look similar to the 2-pot ones. We can not introduce the potential  $V$  manually inside the diagrams, but according to Ref. [2] we can place them into vertices, where the photon line is attached, by modifying the wave functions.

The practical calculations confirm that such transpositions of  $V$  provide a good approximation. In such a way, we obtain the approximations for the 2-pot term, which can be calculated to a high precision and allows one to improve the convergence of the higher-order remainder.

## Results

In this work, we present methods for approximating the 2-pot. contributions to the vertex diagram. These proposed approximations are to improve the convergence of the standard computational scheme. The evaluation of the coordinate-space part is in progress.

## References

1. V. A. Yerokhin et al., Phys. Rev. A 69, 052503 (2004)
2. J. Sapirstein and K. Cheng, Phys. Rev. A 108, 042804 (2023)
3. A. V. Malyshev et al., Phys. Rev. A 109, 062802 (2024)