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Introduction to mutually-inversistic discrete mathematics

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Abstract: *Mutually-inversistic discrete mathematics is constructed by the author, including mutually-inversistic logic, set theory, analytic geometry, calculus, abstract algebra, universal matrix, covering all branches of mathematics, including foundations of mathematics, geometry, analysis, algebra. Unlike conventional discrete mathematics, which put together the branches of mathematics needed by computer science, not interrelated to one another, the branches of mutually-inversistic discrete mathematics are interrelated to one another, and they have many applications in information sciences.*

Keywords: the basis of a new science of information, discrete mathematics, mutual-inversism, mutually-inversistic discrete mathematics

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1. Introduction

Conventional discrete mathematics (Xu, 2009) consists of classical logic, naïve set theory, conventional abstract algebra, graph theory. They are used in computer science. Classical logic and naïve set theory are interrelated to certain extent. The only relationship between naïve set theory and conventional abstract algebra is that the surjection and bijection in naïve set theory become homomorphism and isomorphism in conventional abstract algebra. Graph theory is almost not related with classical logic, naïve set theory, and conventional abstract algebra.

The author finds out that classical logic (Hamilton, 1988) is defective. To overcome its defects, the author constructs mutually-inversistic logic (Zhou, 2000). Based on mutually-inversistic logic, the author builds mutually-inversistic discrete mathematics (Zhou, 2009). Mutually-inversistic discrete mathematics consists of mutually-inversistic logic, set theory, analytic geometry, calculus, abstract algebra, universal matrix. They cover all branches of mathematics, including foundations of mathematics, algebra, geometry, analysis. They are closely interrelated. Mutually-inversistic set theory is based on mutually-inversistic logic. Mutually-inversistic analytic geometry is based on mutually-inversistic set theory. Double-sided discrete calculus (part of mutually-inversistic calculus) is based on mutually-inversistic analytic geometry. Mutually-inversistic abstract algebra is based on mutually-inversistic logic, mutually-inversistic analytic geometry, double-sided discrete calculus. Mutually-inversistic discrete mathematics is very useful. It is applied to logic programming, expert systems, recursion, iteration, program transformation, planning, scheduling, database, semantic network, ontology, program verification, formal semantics, description logic, natural language processing, parallel programming methodology, automated theorem proving, data mining, data warehouse, reasoning with uncertainty, program refinement, multi-valued computer, modern control theory, digital signal processing, temporal reasoning, hardware verification, and so on.

The rest of this paper is organized as follows: Section 2 introduces mutually-inversistic logic. Section 3 introduces mutually-inversistic set theory. Section 4 introduces mutually-inversistic analytic geometry. Section 5 introduces double-sided discrete calculus. Section 6 introduces mutually-inversistic abstract algebra. Section 7 is concluding remarks.

2. Mutually-inversistic logic

Material implication → in classical logic is defined as Table 1.

Table 1 material implication

A	B	A→B
F	F	T
F	T	T
T	F	F
T	T	T

It is well known that material implication has the defect of material implicational paradox. The author finds out that material implication has a not obvious, but more serious defect: it can not be used to make hypothetical inference. There is a basic principle in philosophy that human cognition can only be from the known to the unknown. There is a basic principle in mathematics that mathematical operation can only be from the arguments to the result. If we want to evaluate from the result to the argument, then we must use inverse operations. In the inverse operations, the argument of the original operation becomes the result, the result of the original operation becomes the argument. The evaluation from the result to the argument in the original operation becomes the evaluation from the argument to the result in the inverse operation, not to violate the basic principle of mathematics. In order to mathematize human cognition, we let the known be the arguments, the unknown be the result, then the human cognition from the known to the unknown becomes the mathematical operation from the arguments to the result.

In Table 1, A and B are the known, the arguments, A→B is the unknown, the result, so, Table 1 can only be used from A and B to establish A → B. The affirmative expression of hypothetical inference is from A→B being true and A being true to infer B being true. The negative expression of hypothetical inference is from A→B being true and B being false to infer A being false. In both hypothetical inference, A→B is the known, the argument, so, Table 1 can not be used to make hypothetical inference. But in the inverse operations of Table 1, A→B is the known, the argument, so, the inverse operations of Table 1 can be used to make hypothetical inference. Following this clue, the author constructs mutually-inversistic logic, in which mutually inverse implication $\leq^{-1}$ is defined as Table 2 through Table 4.

Table 2 Inductive composition for $\leq^{-1}$

A	B	$A\leq^{-1}B$
F	F	T
F	T	n
T	F	F

T	T	|	T

Table 3 Decomposition one for $\leq^{-1}$

$A\leq^{-1}B$	A		B
F	F	|	u
F	T	|	u
T	F	|	u
T	T	|	T

Table 4 Decomposition two for $\leq^{-1}$

$A\leq^{-1}B$	B		A
F	F	|	u
F	T	|	u
T	F	|	F
T	T	|	u

N in Table 2 means "needn't determine". U in Tables 3 and 4 means "unable to determine". Table 2 is similar to Table 1, it is used to establish $A\leq^{-1}B$. After $A\leq^{-1}B$ is established, it becomes known, becomes the argument. Tables 3 and 4 are the inverse operations of Table 2. In Tables 3 and 4, $A\leq^{-1}B$ is the known, the argument. So, after $A\leq^{-1}B$ is established, we can use it as the major premise, use Tables 3 and 4 to make hypothetical inference. If A is also known to be true, then we can use Table 3 to determine B to be true. This is the affirmative expression of hypothetical inference. If B is also known to be false, then we can use Table 4 to determine A to be false. This is the negative expression of hypothetical inference. In Table 2 through Table 4, A and B are the special, $A\leq^{-1}B$ is the general. Table 2 is from the special to the general and is called inductive composition. Tables 3 and 4 are from the general to the special and are called decomposition. Inductive composition and decomposition are mutually inverse human cognitive processes, hence mutually-inversistic logic gets its name.

In mutually-inversistic logic, ¬, ∧ , ∨ , are used to form more complex proposition from simpler propositions, and are called composition operators. $\leq^{-1}$, $/\wedge^{-1}$(mutually inverse conjunction) are used to represent the connection between two propositions, and are called connection operators.

In mutually-inversistic logic, the classification of knowledge is as follows: 0, 1, 2, Aristotle, Russell, are 0th terms. X, y, z are 1st terms. +, — , *, height, weight are functions. A function connecting terms is still a term, e.g.2+3 is still a term. =, < , man, mortal are predicate constants. P, q, r are predicate variables. A predicate connecting terms form a fact proposition. 1< 2, man(Aristotle) are 0th facts. X=y, man(x) are 1st facts. P(x), p(x, y) are 2nd facts. P, Q, R are the abbreviations of p(x), p(x, y), q(z), etc. and are called fact variables. Fact composition operators ¬, ∧ , ∨ connecting facts are still facts, e.g. parent(x, y)∧ ancestor(y, z) is still a 1st fact. Empirical or mathematical connection operators $\leq^{-1}$, $/\wedge^{-1}$connecting two facts form a single empirical or mathematical connection proposition. Man(Aristotle) $\leq^{-1}$ mortal(Aristotle) is a 0th single empirical or mathematical connection proposition. Man(x) $\leq^{-1}$mortal(x) (meaning: all men are mortal, corresponding to ∀x(man(x) →mortal(x), even(x)$/\wedge^{-1}$prime(x) (meaning: some even numbers are prime numbers, corresponding to x(even(x) ∧ prime(x))) are 1st single empirical or mathematical connection propositions. A true 1st single empirical or mathematical connection proposition is a single empirical or mathematical theorem. $P\leq^{-1}Q$ is a 2nd single empirical or mathematical connection proposition. Empirical or mathematical composition operators $\neg$, $\wedge$, $\vee$ connecting single empirical or mathematical connection propositions is still a single empirical or mathematical connection proposition, e.g. $\{P\leq^{-1}Q\}\wedge\{Q\leq^{-1}R\}$ is still a 2nd single empirical or mathematical connection proposition. Logical connection operator $\leq^{-1}$ connecting two single empirical or mathematical connection propositions forms a single logical connection proposition. $\{int(x) \leq^{-1} rat(x)\} \leq^{-1} \{\neg rat(x) \leq^{-1} \neg int(x)\}$ is a 1st single logical connection proposition. $\{P\leq^{-1}Q\}\leq^{-1}\{\neg Q \leq^{-1} \neg P\}$ is a 2nd single logical connection proposition. A true 2nd single logical connection proposition is a single logical theorem.

3. Mutually-inversistic set theory

Mutually-inversistic set theory is isomorphic with mutually-inversistic logic with different emphasis. Corresponding to $\neg$, $\wedge$, $\vee$, $\leq^{-1}$, $/\wedge^{-1}$ in mutually-inversistic logic, there are , $\cap$, $\cup$, $\subseteq^{-1}$ (mutually inverse being contained), $|\cap^{-1}$ (mutually inverse intersection) in mutually-inversistic set theory. Mutually-inversistic logic studies the objective world from both intensional and extensional perspective, while mutually-inversistic set theory studies the objective world from purely extensional perspective. Mutually-inversistic logic studies inductive composition and decomposition, while mutually-inversistic set theory studies the properties of the auxiliary and the main. Function and composition operator are the auxiliary. The auxiliary connecting knowledge forms the same level knowledge. Relation (predicate) and connection operator are the main. The main connecting knowledge forms knowledge one level higher. The auxiliary has the properties of idempotency, commutativity, associativity, binary bijection, distributivity, absorbability, etc.. The main has the properties of reflectivity, anti-reflectivity, symmetry, anti-symmetry, transitivity.

Mutually inverse coordinate systems hierarchy consists of term coordinate system, fact coordinate system, single empirical or mathematical connection coordinate system, single set (logical) connection coordinate system. Here, we show the term coordinate system in Fig. 1 and fact coordinate system in Fig. 2.

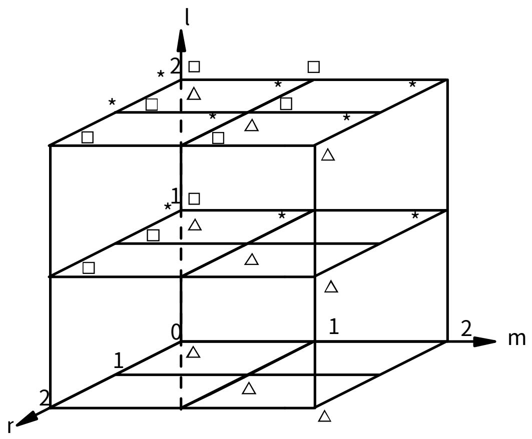

Fig.1 Term coordinate system

The points on the term coordinate axes are terms. The points on fact coordinate axes are facts (relations). The relationship between term coordinate system and fact coordinate system is: a point on the fact coordinate axis, say n=m, is a fact, the type of a relation, containing term variables n and m, varying on the n axis and m axis in Fig. 1; when n=1 and m=1, we obtain a straight line, the value of the relation n=m.

4. Mutually-inversistic analytic geometry

Here, w e use the example of establishing n=m∧m<l≤ $^{-1}$n<l to show mutually-inversistic analytic geometry. N=m is the points marked by Δ in Fig. 1. M<l is the points marked by ÿ in Fig. 1. n=m∧m<l is the points marked by both Δ and ÿ. N<l is the points marked by *. The points marked by both Δ and ÿ is bound to marked by *. Thus, we establish n=m∧m<l≤ $^{-1}$n<l.

5. Double-sided discrete calculus

Double-sided discrete function is defined on term coordinate system. For example, m=－2n is the points marked by Δ in Fig. 3.

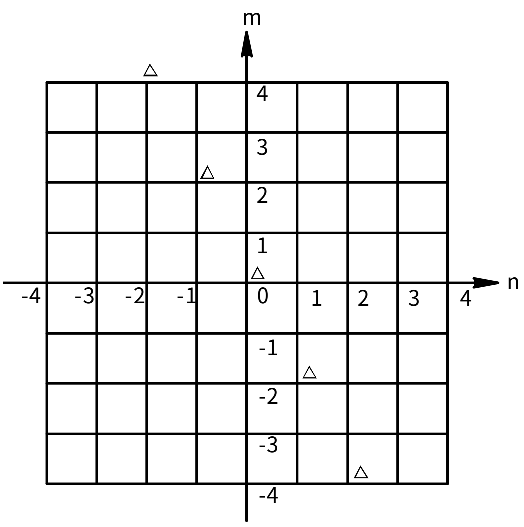

Fig.3 Double-sided discrete function

A straight line has double-sided discrete derivative. The derivative of the function shown in Fig. 3 is $\frac{dm}{dn} = -2$. A constant has double-sided discrete integral. The integral of 3 from n=－4 to n=6 is $\sum_{n=-4}^{6} 3$ =3n$|_{-4}^{6}$=3*(6－(－4))=3*10=30.

6. Mutually-inversistic abstract algebra

Mutually-inversistic abstract algebra is divided into term algebra and fact algebra horizontally, auxiliary algebra and main-auxiliary algebra vertically. Term algebra is established on term coordinate system. Fact algebra is established on fact coordinate system. In auxiliary algebra, only the auxiliaries are the algebraic operators. In main-auxiliary algebra, both the auxiliaries and the main $\subseteq^{-1}$ (partial order) are the algebraic operators.

Suppose S={a, b}, then <ρ(S), ∩, ∪, , Φ, S> is a fact, main-auxiliary algebra (Boolean algebra). Its algebraic operations are represented by the points marked by Δ in the diagrams of ∩ (Fig. 4), ∪ (Fig. 5), (Fig. 6). Its partial ordering is represented by Fig. 7.

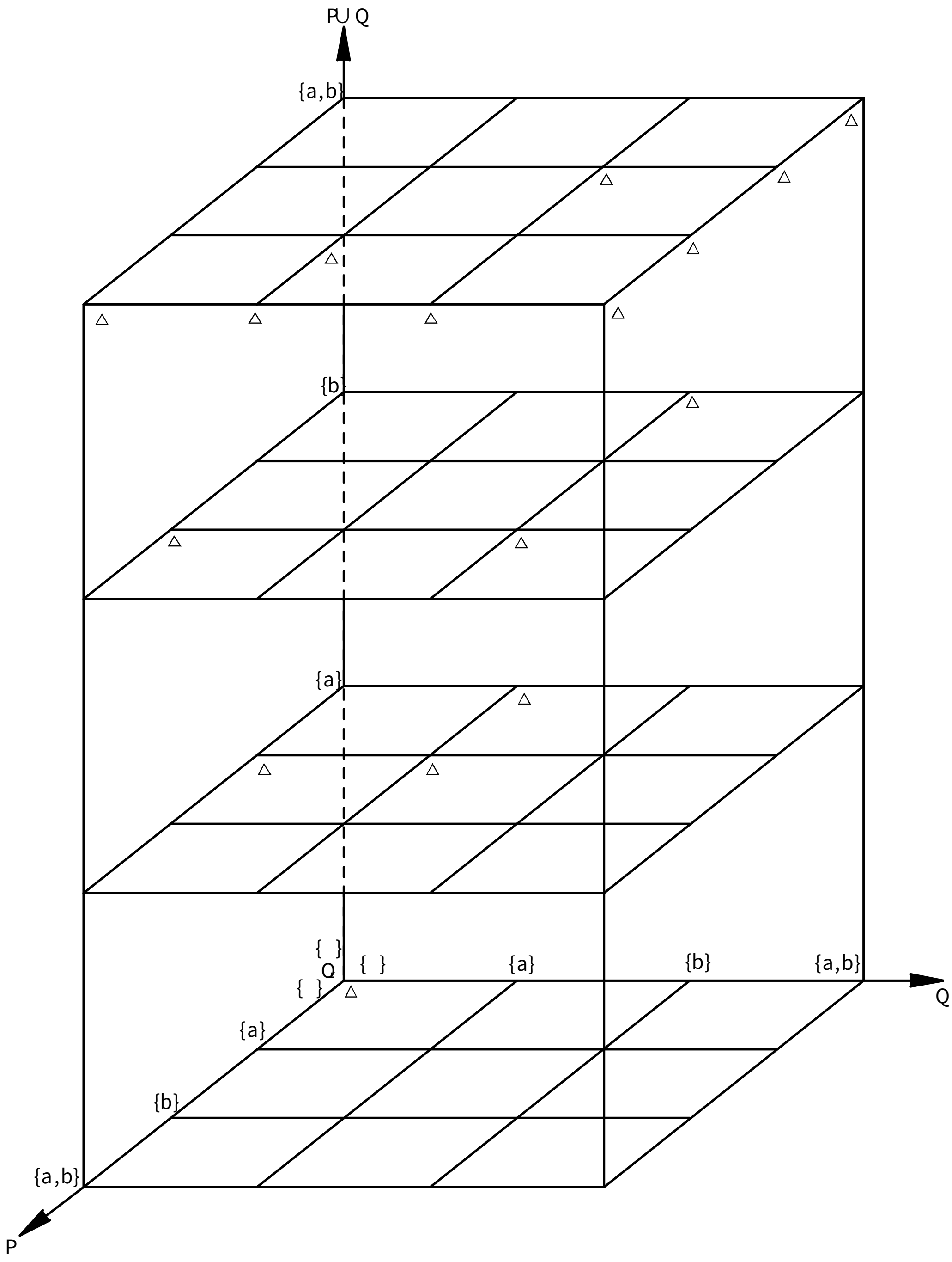

Fig.5 The operation of ∪

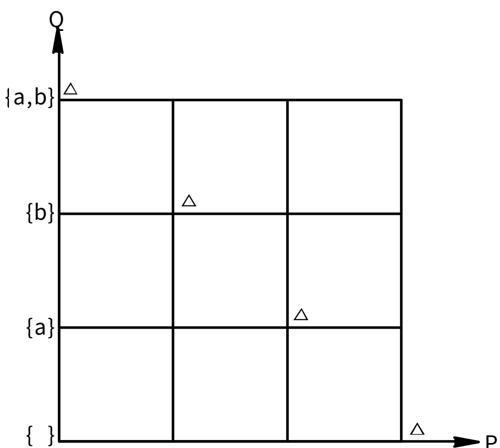

Fig.6 The operation of ∼

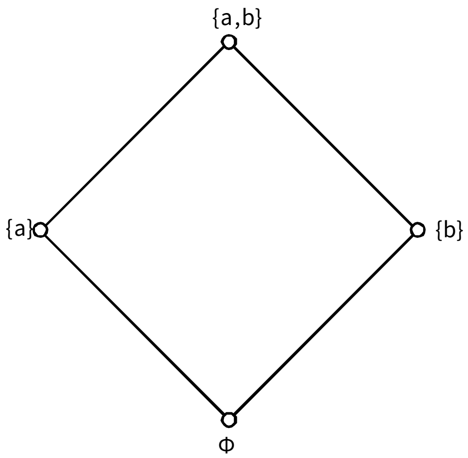

Fig.7 The Hasse diagram of $\subseteq^{-1}$

Each straight line in Fig. 4 through Fig. 6 is a geometric figure, represents a double-sided discrete derivative, reveals a algebraic property.

For example, the straight line marked by A in Fig. 4 represents the partial derivative $\frac{\partial R}{\partial P}|_{Q=\Phi}=0$, revealing the property that when Q=Φ, R keeps constant Φ, whatever the value P is. That is, P∩Φ=Φ, i.e. Q=Φ is the right zero element of ∩.

7. Concluding remarks

The author find out that classical logic is defective. To overcome its defects, the author constructs mutually-inversistic logic. Based on it, the author constructs mutually-inversistic set theory, analytic geometry, calculus, abstract algebra. They cover all branches of mathematics, including foundations of mathematics, geometry, analysis, algebra. They are interrelated to one another.

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About the author

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Professor of the Institute of Information Technology at Beijing Union University. He has been conducting research in mutually-inversistic discrete mathematics and its applications.