A Scalar-Tensor approach to the Dark Universe paradigm: Bayesian Inference and the Hubble tension

Dhiraj Kuniyal

Department of Physics and Astrophysics
University of Delhi, Delhi, India

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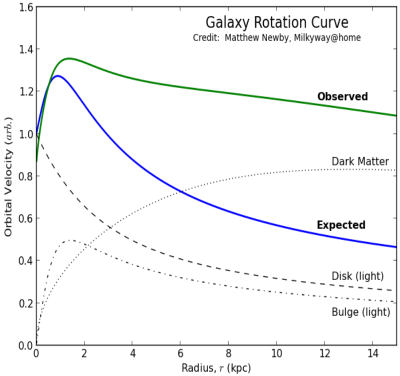
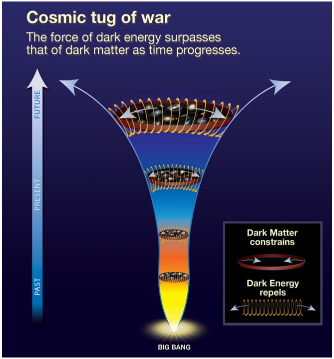
The Cosmic Dark Sector



Courtesy: Matthew Newby, Milkyway@home



Courtesy: https://www.uvic.ca/science/physics/vispa/research/projects/dark

Cosmic dark sector consists of dark matter (DM) and dark energy (DE), which have been conjectured for explaining major observations, viz., the flatness of Galaxy rotation curves and the dimming of Supernovae type Ia, within the realm of Newtonian Gravity/General Relativity (GR).

Standard Cosmology vis-a-vis Modified Gravity

- Conjecture: Cosmic acceleration at low redshifts z, driven by the DE.
- Observations concord to DE-DM density ratio ~ 1 at the present epoch.

Shortcomings of the GR based Standard Cosmology

– Exotic fields/matter sources are required for DE/DM.
– As such, there is no general consensus on the origin of either of them.

Enter the realm of Modified Gravity (MG):

- Can mostly have an equivalent Scalar-Tensor (ST) formulation.
- Makes plausible perception of DE/DM as mere geometric artifacts.
- Effectively interacting DE-DM (IDEM) scenarios may result as well.
- Viable IDEM scenarios have the prospect of alleviating problems such as Hubble tension in parameter estimations using low and high z data.

Cosmology with Interacting Dark Energy-Matter

Consider for brevity, the universe as a system of two dominant components: (Visible+Dark) cosmic matter (denoted by M) and DE (denoted by X).

In a perfect fluid description:

- The (Visible+Dark) cosmic matter is considered as pressure-less (dust).
- Conservation of the system's total energy-momentum does not necessarily imply that for the individual energy-momenta of matter and DE, with the respective energy density and pressure $(\rho_M, 0)$ and (ρ_X, p_X):

$$\boxed{\begin{aligned} \dot{\rho}_M + 3H\rho_M &= Q \\ \dot{\rho}_X + 3H(\rho_X + p_X) &= -Q \end{aligned}},$$

where $H = \frac{\dot{a}}{a}$, $a(t)$: FRW scale factor, overhead dot $(\cdot)$ denotes time derivative,

Q is a scalar which gives the measure of the DE-matter interaction.

IDEM in Scalar-Tensor Cosmology

Generic Scalar-Tensor (ST) Gravity: Features

– Non-minimal metric-scalar coupling in the (original) Jordan frame (JF).

– As such, Newton's constant $G_N \to G$, which evolves with the scalar field.

– Matter fields are, however, coupled minimally to the curvature scalar R.

– Hence, the matter energy-momentum tensor is conserved.

– For a suitable choice of the non-minimal coupling, the action reduces to:

Brans-Dicke (BD) form

$$S = \int d^4x \sqrt{-g} \left[\frac{\phi R}{2} - \frac{\mathfrak{w}}{\phi} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) + L_m(g_{\alpha\beta}, \psi_m) \right]$$

where $g = \det(g_{\mu\nu})$ with $g_{\mu\nu}$ being the metric tensor, ϕ is the BD scalar with potential $V(\phi)$, $\mathfrak{w}$ is the BD parameter, and L_m is the matter (ψ_m) Lagrangian.

– Such a choice of coupling, and hence the BD action, is reminiscent of a bulk of MG theories including $f(R)$ gravity and a plethora of its variants.

IDEM in Scalar-Tensor Cosmology

A conformal transformation $\boxed{g_{\mu\nu} \to \widehat{g}_{\mu\nu} = \phi g_{\mu\nu}}$,

and a field redefinition $\boxed{\phi = \dfrac{e^{2\kappa n \varphi}}{\kappa^2}}$, with $n = \dfrac{1}{\sqrt{6+\mathfrak{w}}}$ and $\kappa = \dfrac{1}{\sqrt{8\pi G_N}}$,

lead to

Einstein Frame action

$$\boxed{\hat{S} = \int d^4x \sqrt{-\widehat{g}} \left[\frac{\widehat{R}}{2\kappa^2} - \frac{1}{2} \widehat{g}^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} - U(\varphi) + \widehat{L}_m(\widehat{g}_{\mu\nu}, \varphi, \psi_m) \right]}$$

where $\widehat{g}_{\mu\nu}$, $\widehat{R}$ are the corresponding metric and curvature scalar, $\widehat{g} \equiv \det(\widehat{g}_{\mu\nu})$, and

$$\boxed{\begin{aligned} & U(\varphi) = e^{-4\kappa n \varphi} V(\phi(\varphi)) \,, \\ & \widehat{L}_m(g_{\alpha\beta}, \psi_m) = e^{-4\kappa n \varphi} L_m(g_{\alpha\beta}(\widehat{g}_{\mu\nu}, \varphi), \psi_m) \,. \end{aligned}}$$

– Non-minimal metric-scalar coupling is lifted.
– In turn, matter Lagrangian $\widehat{L}_m$ depends on φ, explicitly and implicitly.

Einstein Frame Cosmological Setup

- The φ-dependence of $\widehat{L}_m$ implies DE-matter coupling, once the DE is taken to be sourced by the field φ in the Einstein frame (EF).

Friedmann Equations (considering Spatially Flat FRW space-time)

$$\boxed{\begin{aligned} H^2 &= \frac{\kappa^2}{3}\left[\rho_M + \rho_X\right] \\ \dot{H} &= -\frac{\kappa^2}{2}\left[\rho_M + \rho_X + p_X\right] \end{aligned}}, \quad \text{where} \quad \boxed{\begin{aligned} \rho_X &= \frac{\dot{\varphi}^2}{2} + U(\varphi), \\ p_X &= \frac{\dot{\varphi}^2}{2} - U(\varphi) \end{aligned}}.$$

- The (non-)conservation equations for matter (M) and DE (X), shown before, now have an interaction $\boxed{Q = -\kappa n \dot{\varphi} \rho_M}$ arising naturally in EF.
- Matter (non-)conservation gives: $\boxed{\rho_M = \frac{\rho_{M0}}{a^3} e^{-\kappa n \varphi}}$, where $\rho_{M0} = \rho_M|_{a=1}$.
- A BD potential $\boxed{V(\phi) \sim \phi} \Leftrightarrow \boxed{U(\varphi) = \Lambda e^{-2\kappa n \varphi}}$ in EF, where $\Lambda =$constant.
- Such a potential is well-known in various ST-equivalent MG theories, e.g., Metric-Scalar-Torsion theory.

– Sourav Sur, Arshdeep S. Bhatia, *JCAP* 07 (2017) 039.

Cosmological Solution in Einstein Frame

- For the above $U(\varphi)$, the cosmological equations admit an exact solution:

$$\boxed{\varphi = \frac{2nN}{\kappa}}, \quad \text{where} \quad N = \ln a : \text{ Number of e-folds,}$$

whence

$$\boxed{H^2(a) = H_0^2 \left[\frac{\Omega_{M0}}{a^{3+2n^2}} + \frac{\Omega_{X0}}{a^{4n^2}}\right]},$$

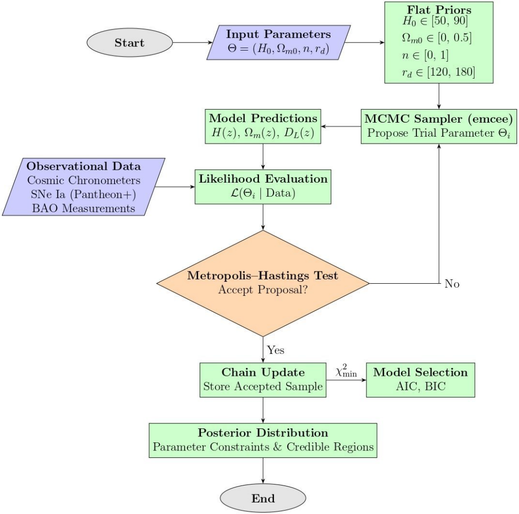
where $H_0 \equiv H|_{a=1}$ is the Hubble's constant,

and $\Omega_{M0} = \dfrac{\kappa^2 \rho_{M0}}{3H_0^2 Y^2}$, $\Omega_{X0} = \dfrac{\kappa^2 \Lambda}{3H_0^2 Y^2}$, with $Y = 1 - \dfrac{2n^2}{3}$.

- Note the following:
  - $\Omega_{X0} = 1 - \Omega_{M0}$. So the free parameters are (H_0, n, Ω_{M0}).
  - The limit $n \to 0$ corresponds to the concordant ΛCDM model.

— Sourav Sur, Arshdeep S. Bhatia, *JCAP* 07 (2017) 039.

Parameter Estimation Methodology



Observational Data

- **Cosmic Chronometers (CC):**
  - Directly measure $H(z) = -\frac{\dot{z}}{1+z}$ from differential aging of galaxies.
  - Data span $z \in (0.1, 2.0)$, with 51 measurements and uncertainties 5 to 15%.
- **Type Ia Supernovae (Pantheon$^+$):**
  - Constrain luminosity distance via the distance modulus,

$$\mu(z) = 5\log_{10}\left[(1+z)\int_0^z \frac{dz'}{H(z')}\right] + 25 .$$

  - Data encompasses 1701 SNe Ia over $z \in (0.1, 2.3)$.
- **Baryon Acoustic Oscillations (BAO):**
  - Provide a standard ruler set by the sound horizon at drag epoch, r_d.
  - Probes: SDSS-IV, DESI-VI, which use r_d^{-1} times the distance measures

$$D_H(z) = \frac{1}{H(z)}, \quad D_M(z) = \int \frac{dz'}{H(z')}, \quad D_V(z) = \left[z\, D_M^2(z)\, D_H(z)\right]^{1/3} .$$

IDEM Parameter Constraints and Diagnostics

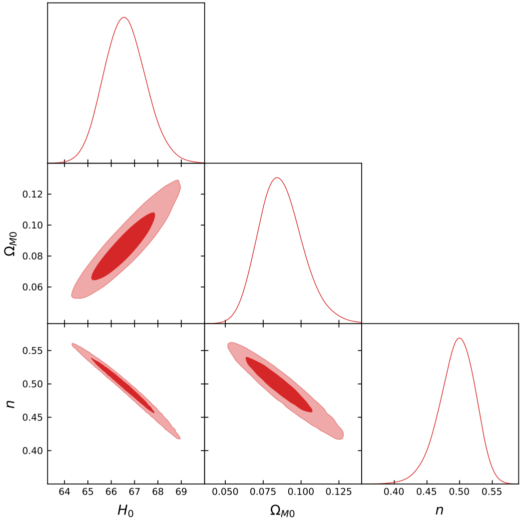
Cosmological parameter estimates and IDEM vs ΛCDM diagnostics

Parameters	**Pantheon$^+$+CC**		**Pantheon$^+$+CC+BAO**	
	ΛCDM	IDEM	ΛCDM	IDEM
H_0 [km/s/Mpc]	$73.901^{+0.160}_{-0.160}$	$66.599^{+0.960}_{-0.895}$	$73.678^{+0.154}_{-0.146}$	$68.141^{+1.146}_{-0.980}$
Ω_{M0}	$0.254^{+0.008}_{-0.008}$	$0.086^{+0.016}_{-0.014}$	$0.271^{+0.007}_{-0.007}$	$0.137^{+0.022}_{-0.017}$
n	–	$0.497^{+0.026}_{-0.031}$	–	$0.436^{+0.034}_{-0.044}$
r_d [Mpc]	–	–	$139.862^{+0.860}_{-0.857}$	$138.872^{+0.901}_{-0.915}$
Diagnostics				
χ^2_{min}	1835.591	1797.917	1873.923	1856.819
χ^2_ν	1.049	1.028	1.062	1.053
AIC	1839.591	1803.917	1879.922	1864.819
BIC	1850.528	1820.323	1896.353	1886.727

— Dhiraj Kuniyal and Sourav Sur, in progress.

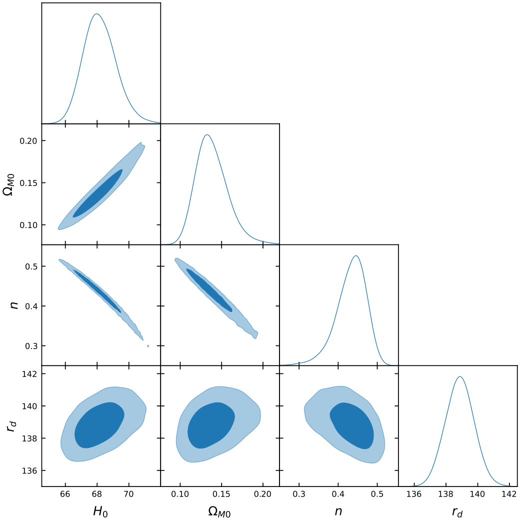
IDEM Posterior Plots

2D Posterior distribution for IDEM, with Pantheon$^+$+CC data



IDEM Posterior Plots

2D Posterior distribution for IDEM, with Pantheon$^+$+CC+BAO data



Summary of Results

- **Hubble Constant**
  - Hubble's constant $H_0 = 66.6$ to $68.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (best fit).
  - Estimate however increases with the inclusion of the BAO data.
  - The Hubble Tension, that affects ΛCDM, is nonetheless alleviated.
- **Evidence for IDEM**
  - Coupling parameter n is significant, ~ 0.436 to 0.497 (best fit).
  - Effective matter density dilution $\sim 1/a^{3+2n^2}$ (steeper than $\sim 1/a^3$).
- **Comparison with ΛCDM**
  - Minimized χ^2, AIC, BIC for IDEM are lower than those for ΛCDM.
  - IDEM is thus favored in spite of having an extra parameter n.

THANK YOU