Study of the effect of the area and curvature terms in the quarks and gluons density of states on the transition temperature

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INTRODUCTION & AIM

In the hadronic phase, quarks and gluons, the fundamental constituents of matter, are confined within hadrons due to the strong interaction [1, 2]. However, at high temperature and/or density, a deconfinig phase transition occurs, leading to the formation of the Quark-Gluon Plasma (QGP). In this state, quarks and gluons are no longer confined and can move freely [3]. In this work, we study the impact of adding the area and curvature terms in the quarks and gluons density of states, on the deconfinement phase transition from a hadronic gas (HG) phase, to a QGP phase, by examining the effective transition temperature $T_c(V)$ in a finite volume. We work within the Bag model [4], and we introduce the color-singletness condition by means of the projection method we use to calculate the partition function of the QGP projected onto the SU(3) color-singlet representation [5]. The quarks and gluons density of states, given by the Multiple Reflection Expansion (MRE) approximation [6], is considered with the contribution of the volume, area and curvature terms. We analyze the variations of the pressure of both HG and QGP phases, with temperature T, for various system sizes V, in different cases of contributing terms in the quarks and gluons density of states, in order to determine the effective transition temperature in a finite volume, $T_c(V)$.

The Partition Function of the HG and QGP phases

As a first step, we need to compute the partition function for both Hadronic Gas and QGP phases. For the HG phase that consists of massive pions, its partition function, in a volume V, at temperature T, is written as:

$$Z_{HG}(T,V_{HG}) = \exp\left(\frac{d_\pi V_{HG}}{2\pi^2 T}\int_0^{+\infty}\frac{k^4 dk}{\sqrt{k^2+m_\pi^2}\left(e^{\beta\sqrt{k^2+m_\pi^2}}-1\right)}\right) \quad (1)$$

For the QGP phase which contains gluons, massless up and down quarks and massive strange quarks, with their antiquarks, the partition function projected on the color-singlet SU(3) representation, in a volume V, at temperature T, and μ= 0, is given by:

$$Z_{QGP}(T,V_{QGP},\mu=0) = \frac{4}{9\pi^2}e^{-\frac{BV_{QGP}}{T}}\int_{-\pi}^{+\pi}\int_{-\pi}^{+\pi}d\varphi d\psi M(\varphi,\psi)e^{(Y_1+Y_2+Y_3+Y_4)} \quad (2)$$

where $M(\varphi,\psi)$ is the weight function or the Haar measure, given by:

$$M(\varphi,\psi) = \left(\mathrm{Sin}\left[\frac{1}{2}\left(\psi+\frac{\varphi}{2}\right)\right]\mathrm{Sin}\left[\frac{\varphi}{2}\right]\mathrm{Sin}\left[\frac{1}{2}\left(\psi-\frac{\varphi}{2}\right)\right]\right)^2 \quad (3)$$

Y_1, Y_2, Y_3 and Y_4correspond, resepectively, to the volume, area, curvature and gluons contributions in the density of states of quarks and gluons given by MRE approximation [6]:

$$Y_1 = d_Q\, V_{QGP}\sum_{q=r,b,g}\left\{\frac{\pi^2}{6}T^3\left[\frac{7}{30}-\left(\frac{\theta_q}{\pi}\right)^2+\frac{1}{2}\left(\frac{\theta_q}{\pi}\right)^4\right]+\frac{(m_S\,T)^{\frac{3}{2}}}{\sqrt{2}\pi^{\frac{3}{2}}}\left(1-\frac{\theta_q^2}{2!}+\frac{\theta_q^4}{4!}\right)e^{\left(-\frac{m_S}{T}\right)}\right\} \quad (4)$$

$$Y_2 = d_Q\left(\frac{3V_{QGP}}{4\pi}\right)^{\frac{2}{3}}\left(\sqrt{\frac{2m_S}{\pi}}T^{\frac{3}{2}}-m_S\,T\right)\sum_{q=r,b,g}\left(1-\frac{\theta_q^2}{2!}+\frac{\theta_q^4}{4!}\right)e^{\left(-\frac{m_S}{T}\right)} \quad (5)$$

$$Y_3 = d_Q\left(\frac{3V_{QGP}}{4\pi}\right)^{\frac{1}{3}}\sum_{q=r,b,g}\left\{\left[\frac{\pi}{3}T\left(-\frac{1}{3}+\left(\frac{\theta_q}{\pi}\right)^2\right)\right]+\left(\frac{2}{3}\sqrt{\frac{2T}{\pi m_S}}-T+\sqrt{\frac{2}{\pi m_S}}T^{\frac{3}{2}}\right)\left(1-\frac{\theta_q^2}{2!}+\frac{\theta_q^4}{4!}\right)e^{\left(-\frac{m_S}{T}\right)}\right\} \quad (6)$$

$$Y_4 = d_G\, V_{QGP}T^3\sum_{G=1}^{4}\frac{\pi^2}{12}\left[-\frac{7}{30}+\left(\frac{\theta_G-\pi}{\pi}\right)^2-\frac{1}{2}\left(\frac{\theta_G-\pi}{\pi}\right)^4\right]+d_G\left(\frac{3V_{QGP}}{4\pi}\right)^{\frac{1}{3}}\sum_{G=1}^{4}\frac{2\pi}{3}T\left[\frac{1}{3}-\left(\frac{\theta_G-\pi}{\pi}\right)^2\right] \quad (7)$$

To achieve our goal, we need to examine the behavior of the HG and QGP pressure, defined by:

$$P_i(T,V) = T\frac{\partial lnZ_i}{\partial T}, \qquad i = HG, QGP \quad (8)$$

Influence of the area and curvature terms on T_c value

After using a numerical integration method, we obtain the HG and QGP pressures, at μ = 0 and with a bag constant $B^{1/4}$ = 225MeV. First, we examine the variations of the pressure for HG and QGP phases with temperature T, illustrated in Figure 1 for a large volume: V=10000fm³, which effectively simulates an infinite system. The Figure reveals an intersection between the pressure curve of the HG phase (black solid line) and the pressure of the QGP phase, which is represented by four curves corresponding to different contributions to the density of states: the contribution of the volume term only (red dashed line), volume and area terms (blue dotted line), volume and curvature terms (magenta dashed dotted line), and volume, area, and curvature terms (green dashed dotted dotted line). The intersection point of the HG and QGP pressure curves defines the infinite-volume transition temperature $T_c(\infty)$, which varies with the contributing terms to the density of states. We notice that $T_c(\infty)$ takes its maximum value with the contribution of all three terms and its minimum value with the volume term only contributing to the density of states, as shown Table 1.

From Figure 2, one can clearly observe that, for each system volume, the curves corresponding to the different contributions to the density of states exhibit very similar behavior. In addition, it is evident that variations in the system size have a significant impact on the transition temperature $T_c(V)$ in a finite volume. Specifically, for smaller volumes, $T_c(V)$ shifts toward higher values, reflecting finite-size effects that become increasingly important as the system size V decreases. Conversely, for larger volumes, it moves to lower values. This demonstrates that the phase transition characteristics are highly sensitive to finite-volume effects. It is clearly noted that, under the color-singletness constraint, $T_c(V)$ is systematically higher than the infinite-volume transition temperature, $T_c(\infty)$, as summarized in Table 2. Furthermore, $T_c(V)$ decreases with increasing system size V. For instance, for the volume contribution only, $T_c^{(Vol)}(V = 2000fm^3)$ = 158,540MeV, is already very close to $T_c^{(Vol)}(\infty)$ = 158,522MeV.

This convergence behavior is consistently observed for all contributions to the density of states. From Tables 1 and 2, its appears that for each case of contributing terms to the density of states, there is a slight difference between the transition temperatures $T_c(V)$ for large volumes, while for small volumes there is a considerable difference. Furthermore, it is clear that the effect of the curvature term on $T_c(V)$ is more considerable than that of the area term.

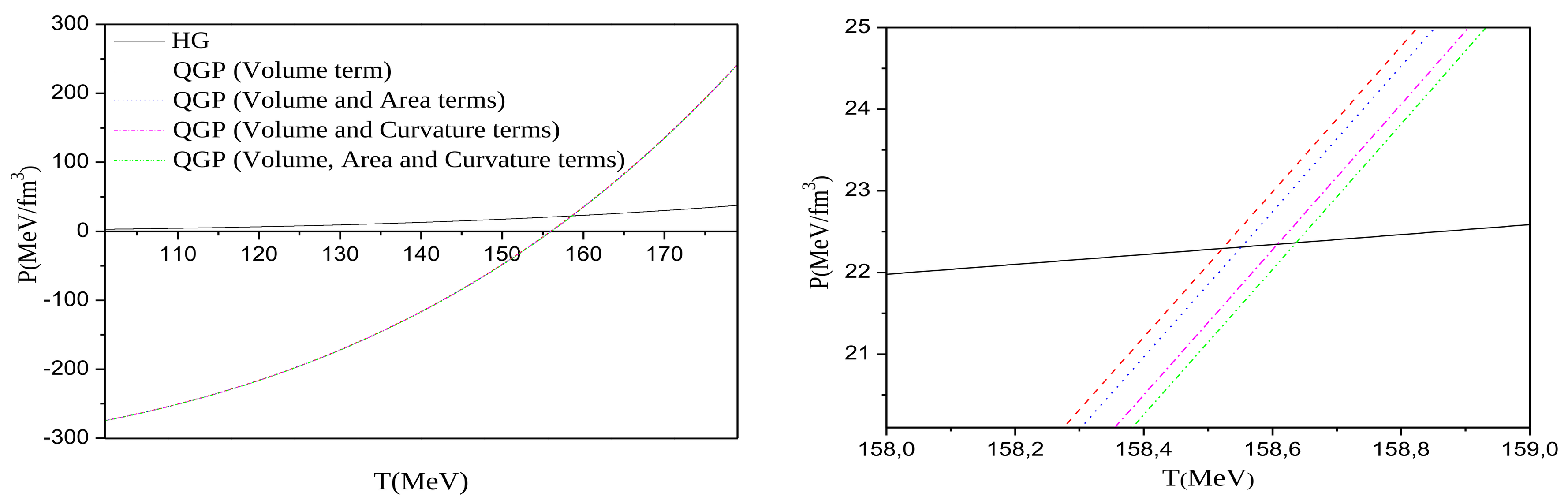

Figure 1. Variations of the pressure as a function of temperature T, for the HG and QGP phases, with the four cases of contributions to the density of states, at μ = 0, and $B^{1/4}$ = 225MeV.

$T_c^{(Vol)}(\infty)$	$T_c^{(Vol+Area)}(\infty)$	$T_c^{(Vol+Curv)}(\infty)$	$T_c^{(Vol+Area+Curv)}(\infty)$
158,522	158,551	158,607	158,636

Table 1. Transition temperature $T_c(\infty)$ values with the various contributions to the density of states.

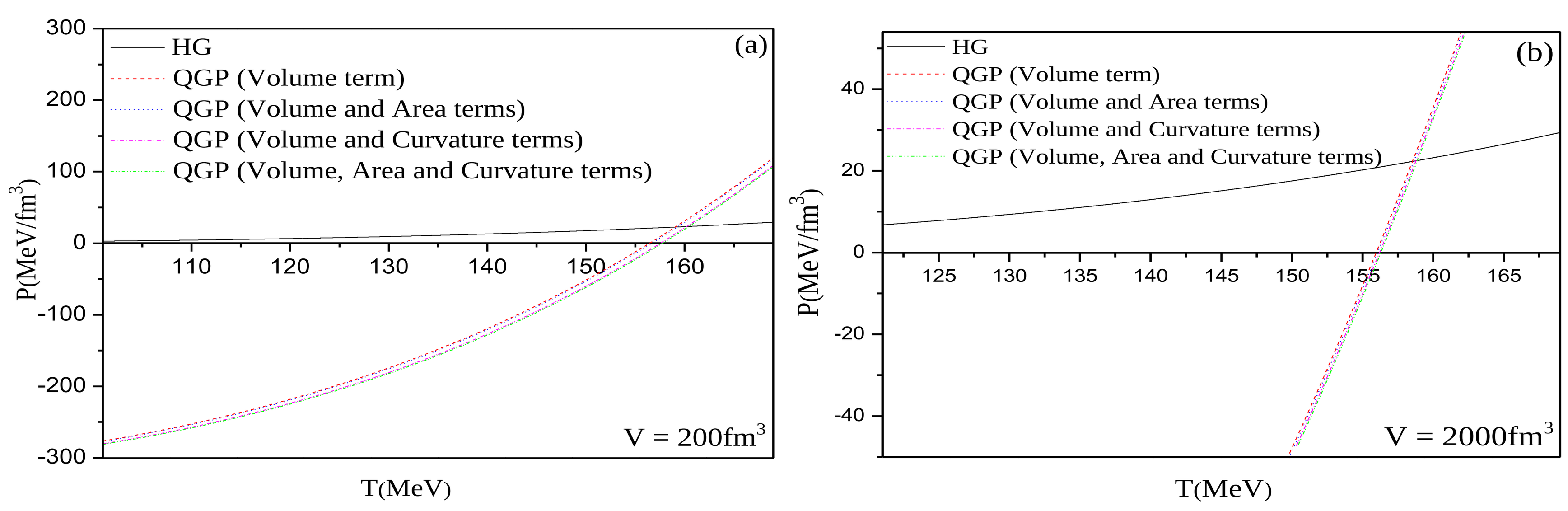

Figure 2. Plots of the pressure of the HG and a color-singlet QGP as a function of temperature T, with the four cases of contributions to the density of states, at μ = 0, and $B^{1/4}$ = 225MeV: (a) for small volume $V = 200fm^3$ (b) for large volume $V = 2000fm^3$.

System Volume (fm^3)	$T_c^{(Vol)}(V)$ (MeV)	$T_c^{(Vol+Area)}(V)$ (MeV)	$T_c^{(Vol+Curv)}(V)$ (MeV)	$T_c^{(Vol+Area+Curv)}(V)$ (MeV)
100	159,240	159,509	161,087	161,345
150	159.103	159,340	160,521	160,758
200	159,050	159,265	160,221	160,434
300	158.994	159,181	159,889	160,073
1000	158.541	158,666	158,937	159,063
2000	158,540	158,640	158,790	158,889

Table 2. The effective transition temperature $T_c(V)$ values for various volumes, with the four cases of contributing terms to the density of states.

CONCLUSION

This study investigates the influence of area and curvature terms on the deconfinement phase transition. To achieve this, we focused on the transition temperature $\boldsymbol{T_c}$(V), by analyzing its value when adding the area and curvature terms in the density of states. Specifically, we found that their inclusion increases the value of $\boldsymbol{T_c}$(V), especially, in small volumes. Furthermore, we notice that $T_c(V)$ reaches its highest value when all three contributions are included, then it slightly decreases when only the volume and curvature terms contribute, while it decreases more with the volume and area contribution, and $\boldsymbol{T_c}$(V) reaches its lower value when the volume term only contributes.

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