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The Evolution of dark energy in a model of a gas of thin tubes of a massless scalar field

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INTRODUCTION

The null string is a classical cosmic string moving at the speed of light. Two null string models:

- One-dimensional model, for studies where the gravitational influence of the null string on its environment is not significant;
- Thin tube of massless scalar field (TToMSF), in the study of the gravitational influence of a null string on its local environment.

If a closed TToMSF has shape of a thin torus and, when moving, does not change its shape and size, then, in a cylindrical coordinate system, the distribution function of the scalar field φ, which will model such a TToMSF, can be represented as [1]

$$\varphi(q,r) = -\gamma \ln\big(\alpha(q) + \lambda(q) f(r)\big)\,, \tag{1}$$

where $q = t + z$, γ is an evolutionary parameter ($\gamma > 0$), the function $\alpha(q)$ is related to the function $\lambda(q)$ by the equality

$$\alpha(q) = 1 - f_0\lambda(q),\ \ f_0 = const., \tag{2}$$

functions $\lambda(q)$ and $f(r)$ are bounded

$$\lambda(q)|_{q\in(-\infty,-\Delta q)\cup(+\Delta q,+\infty)} \to 0, \quad \lambda(q)|_{q\to 0} \to 1/f_0, \tag{3}$$

$$f(r)|_{r\in(0,R-\Delta r)\cup(R+\Delta r,+\infty)} \to f_0, \quad f(r)|_{r\to R} \to 0, \tag{4}$$

where $R = const, R > 0$, Δq and Δr are small positive constants that determine "thickness" of TToMSF (core radius). In [1], by integrating Einstein's equations for function (1), restrictions on the possible values of the constant γ ($\gamma \in \big(0; \sqrt{2/\chi}\big)$) were found, and three possible solutions were obtained:

$$1)\ \text{for}\ \gamma \in \big(0; \sqrt{1/\chi}\big) \cup \big(\sqrt{1/\chi}; \sqrt{2/\chi}\big), \tag{5}$$

$$2)\ \text{for}\ \gamma = \sqrt{2/\chi}\,, \quad 3)\ \text{for}\ \gamma = \sqrt{1/\chi}\,, \tag{6}$$

where $\chi = 8\pi G$ (in the system of units $c = 1$, where c is the speed of light), G is the gravitational constant.

Motion of a test null string

By the term test null string we mean a one-dimensional massless object whose gravitational influence is neglected.

By integrating the equations of motion of the test null string for case (5) we find relation between the variables t, z, r, and θ, with the parameters τ and σ on the world surface of the null string (see, for example, how this was done in [2], [3]). In particular, if the value of the variable θ for each point of the test null string does not change over time, then relation between the variables t, z, and r, with the parameters τ and σ, is determined by the equalities

$$f_L^i(r) = U_L^i + h_L^i/\lambda(q), \tag{7}$$

$$|\lambda(q)_{,\tau}| \frac{\left(1+h_L^i-\lambda(q)\left(f_0-U_L^i\right)\right)^{2-\sqrt{4-2\chi\gamma^2}}}{(\lambda_0\lambda(q)+\beta_0)^k} = \frac{P_q}{\alpha_0}\,, \tag{8}$$

$$s = s_0 + n_L(f_0 - (\lambda(q))^{-1}), \tag{9}$$

where $s = t - z$, index L takes values I - IV, and denotes the number of region to which the solution corresponds (for I: $q < 0, r > R$; for II: $q < 0, r < R$; for III: $q > 0, r > R$; for IV: $q > 0, r < R$), index i takes the values 0, 1 ($i = 0$ corresponds to the case $r_{,\tau} > 0$; $i = 1$ corresponds to the case $r_{,\tau} < 0$), U_L^i, h_L^i, P_q, n_L, α_0, s_0, λ_0, β_0, κ, are constants.

The evolution of dark energy in a gas of TToMSF

It can be noted that when $q \to \pm\infty$ the right side of Eq. (7) takes on unlimited values (taking into account (3)). At the same time, the left side of Eq. (7) can take values only in a limited interval $(0, f_0)$ (see (4)). In this case, Eq. (7) defines boundaries of change of the variable q and, therefore, limits the possible values of the parameter τ and the variable s (see Eqs. (8), (9)). As a consequence, the motion of the test null string will be determined only within a limited region of space-time. This limited region of space-time is called the "interaction zone."

For (7), the boundaries of the interaction zone are reached when

$$\lambda(q) \to a_0 \frac{P_r}{P_q}\frac{c_0}{\gamma^2}/f_0\left(1 + a_0\frac{P_r}{P_q}\frac{c_0}{\gamma^2}\right), \tag{10}$$

where P_r is a constant. From Eq. (10) it is directly evident that

$$\lim_{\gamma\to 0}\lambda(q) \to 1/f_0. \tag{11}$$

For (5), the earliest possible moment of formation of TToMSF corresponds to the value $\gamma \to \sqrt{2/\chi}$, and the values $\gamma \to 0$ correspond to the latest moments of the evolution of the Universe.

From Eq. (10) it follows that as the value of the constant γ decreases, the "width" (size) of the interaction zone also decreases. Comparing the asymptotic Eqs. (3) and (11), one can notice that for values $\gamma \to 0$ the size of the interaction zone also tends to zero.

The studies conducted in [1]-[4] allow us to identify a number of quantities (processes) that influence the evolution of TToMSF gas:

P1. TToMSF gas density (degree of rarefaction). For a rarefied TToMSF gas the equation of state parameter $w \to -1$, and for a compressed gas $w \to 1$;

P2. Thickness (core radius) of TToMSF. The thinner the tubes that form the gas, the faster, upon expansion, this gas can reach the equation of state with the parameter $w \to -1$;

P3. The moment of formation and evolution of the scalar field (the value of the constant γ in the distribution function (1)). In [1] it is shown that the smaller the value of the constant γ in (1), the faster the TToMSF gas will reach the equation of state with the parameter $w \to -1$;

P4. Features of gravitational interaction in TToMSF gas. This feature is related to the fact that two TToMSFs, having different spatial forms, can exist in a gas only outside the zone of interaction between each other [3]. Depending on the initial conditions, the size of the interaction zone for each TToMSF in gas can be different. At the same time, there are always critical values of initial conditions for which the interaction zone for a separate TToMSF in gas can occupy the entire space. In this case, impermeability and size ("width") of the interaction zones for TToMSFs having different spatial forms may be the reason for implementation of a 'strong' gravitational repulsion in such a gas and, as a consequence, be a source of long-term accelerated expansion of the TToMSF gas [3]. The phase of "strong" gravitational repulsion is realized when the total volume of impenetrable interaction zones significantly exceeds the volume of the region containing the gas.

Taking into account the above, the total value of the equation of state parameter for TToMSF gas can be represented as

$$w = w_{P1;P2;P3} + w_{P4},\ \ w_{P1;P2;P3} > -1,\ \ w_{P4} < 0, \tag{12}$$

where $w_{P1;P2;P3}$ is a parameter of the equation of state of the TToMSF gas taking into account the influence of parameters P1, P2, P3, and w_{P4} is a parameter taking into account the features of gravitational interaction in the TToMSF gas (P4).

DISCUSSION

It can be noted that Eq. (12) provides possibility for realization of the phantom dark energy phase in the TToMSF gas. At the same time, during the evolution, contribution of the parameter w_{P4} to the total value of the parameter of the equation of state (12) can only decrease. Reasons leading to a decrease in w_{P4}:

- When expanding, the volume of the region that contains the gas increases. In this case, the total volume of impermeable interaction zones for TToMSFs forming gas gradually approaches the volume of the region that contains the gas.
- In the process of evolution, the value of the constant γ in function (1) decreases, i.e., the size ("width") of the interaction zones for TToMSFs in gas decreases, which accelerates approach of the moment in time starting from which $w_{P4} = 0$.

CONCLUSION

During the evolution of the TToMSF gas, the phantom component of dark energy in the gas should decrease and, starting from a certain point in time, the total value of the equation of state parameter for the TToMSF gas should satisfy the inequality $w > -1$. The obtained result is consistent with recently published observational data from DESI (Dark Energy Spectroscopic Instrument), according to which the preferred w(z) shows a phase $w > -1$ at low redshifts and a phantom crossing with $w < -1$ above redshifts $z \cong 0.4$ [5].

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