SEMI-CLASSICAL DARK UNIVERSE FROM A BOSE-EINSTEIN CONDENSATE IN A SPATIALLY AVERAGED DOMAIN

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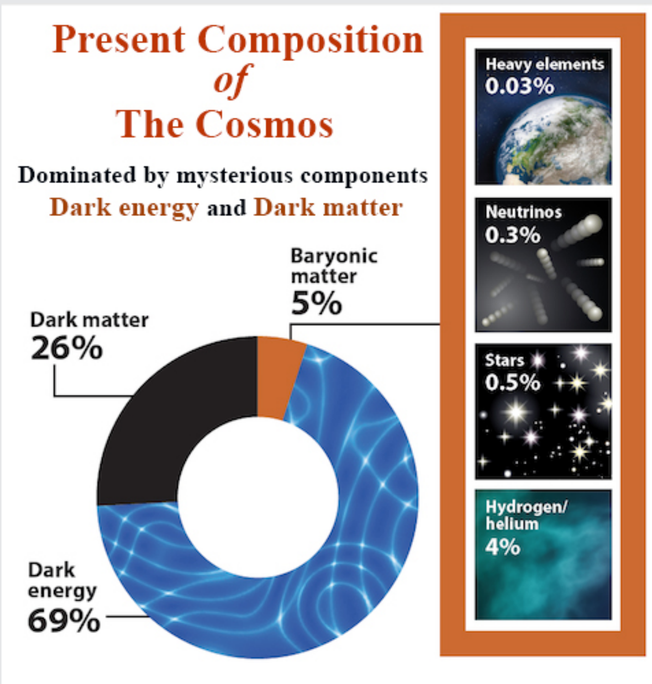
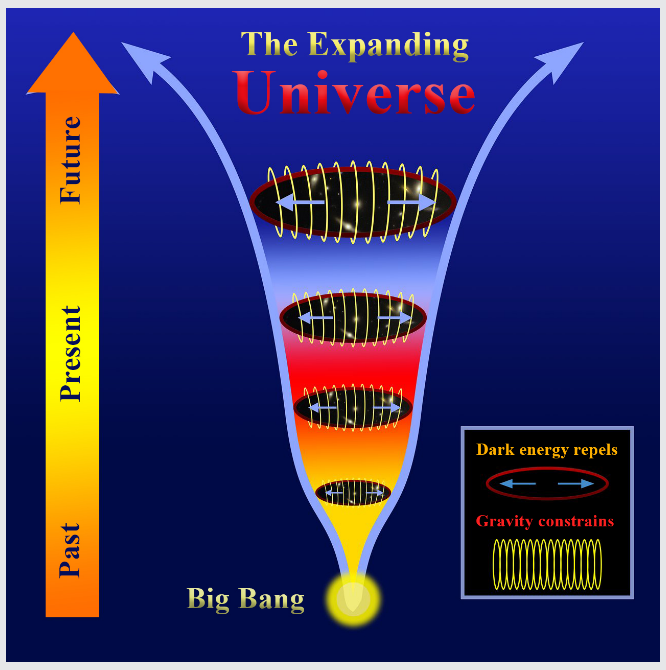
Dark side of the Universe





- Dark matter (DM) and dark energy (DE) at present epoch $(t = t_0)$: $\rho_{DM} \approx \rho_{DE}$
- Lack of a general consensus on the origin and structure of DE and DM, despite numerous proposals.
- A rather attractive proposition: Unified dark sector. Same fundamental origin of DE and DM.
- Makes the redundancy of coincidence obvious.

Quantum Raychaudhury/Friedmann Equation

- **Raychaudhuri Equation**: All geodesics converge in finite proper time (τ) .
- **Friedmann Equations**: Describe geodesics in an expanding universe; can be derived from the Raychaudhuri equation and the energy-momentum conservation relation.
- **Quantum Raychaudhuri Equation (QRE)**: Replacing geodesics with Bohmian/quantal trajectories leads to a quantum correction to Raychaudhuri's expression for $d\theta/d\tau$ where θ : expansion scalar. The correction term is wavefunction-dependent, and it prevents focusing and formation of conjugate points.
- **Quantum Friedmann Equation**: If a constituent fluid has an inherent quantum description, via a wavefunction- $\psi(\vec{x}, t) = \mathcal{R}(\vec{x}, t) \exp iS(\vec{x}, t)$ where, $\mathcal{R}, S \in \mathbb{R}$, the Friedmann acceleration equation $(\ddot{a}/a)$ gets modified by a quantum potential:

  $$U_Q = \frac{\hbar^2}{m^2} h^{\mu\nu} \nabla_\mu \nabla_\nu \left(\frac{\Box \mathcal{R}}{\mathcal{R}}\right) \quad \text{where, } g_{\mu\nu} = h_{\mu\nu} + u_\mu u_\nu$$

- Such a *quantum* cosmic 'fluid' of velocity $u^\mu = (\hbar/m)\, \partial_\mu S$ can ideally be a Bose-Einstein condensate (BEC) of ultra-light bosons of rest mass m, provided m remains within certain limit.

Cosmic dark sector from a BEC

- An important observation: *for a copious supply of bosons of mass $m \lesssim 6$ eV, the corresponding critical temperature T_c exceeds the ambient temperature of the universe at all epochs.* $\Rightarrow$ BEC formation in the early universe.
- A convenient form of the wave function can be $\psi(r, t) = R(t) \exp\left(-\frac{2r^2}{\sigma^2}\right) \exp\left(\frac{-iE_0 t}{\hbar}\right)$ where, E_0, σ:positive constant and $r(t) = xa(t)$
- So, the Quantum potential is

$$U_Q = \frac{3\hbar^2}{m^2}\left[aH^2 \frac{dW}{da} + \frac{4}{\sigma^2}\left(F + \frac{2}{\sigma^2} - \frac{2H}{\sigma^2}\left(aH\frac{dF}{da} + 2H\left(F + \frac{26}{3\sigma^2}\right)\right) x^2 a^2 + \frac{8H^2}{\sigma^4} F a^4 x^4\right)\right]$$

where $W = -\frac{3}{2}(\dot{H} + \frac{3}{2}H^2)$, $F = \dot{H} + 2H^2$

- BEC density: $\rho_B = \rho_{B0} a^{-3} \exp(-2(a^2 - 1)x^2/\sigma^2)$
- Total energy density: $\rho = \rho_B + \rho_b - \frac{2}{\kappa^2} U_Q - 3p$ [ρ_b: baryonic matter density]

Space averaging of the Quantum potential and BEC density

- We used standard techniques of averaging inhomogeneities over a suitably chosen domain $\mathcal{D}$ as:

$$\langle x^2 \rangle = \frac{\int_0^{\mathcal{D}} x^2 d^3x}{\int_0^{\mathcal{D}} d^3x} = \frac{3}{5}\mathcal{D}^2, \quad \langle x^4 \rangle = \frac{3}{7}\mathcal{D}^4$$

- The spatially averaged quantum potential is (Using the above expressions)

$$\langle U_Q \rangle = \frac{3\hbar^2}{m^2}\left[\frac{4}{\sigma^2}\left(\frac{2}{\sigma^2} + \frac{f}{a^2}\right) + \left(\frac{\kappa^2 p'}{4a} - \frac{2sf'}{5a} - \frac{104}{15}\frac{s}{\sigma^2} + \frac{8}{7} f s^2\right)\kappa^2 \rho a^2\right] \quad [f = a^2 F]$$

- The BEC density can also be averaged over the same domain $\mathcal{D}$ as

$$\rho_B = \frac{\int d^3x \rho_B}{\int d^3x} = \rho_{B0} a^{-3}\left[1 - \frac{6\, l(a)}{5} + \frac{6\, l^2(a)}{7} - \cdots\right] \quad [l(a) = (a^2 - 1)s,\ s = \mathcal{D}^2/\sigma^2]$$

Quantum back-reaction and BEC mass bound

- The field equations admit an exact solution (at $s = 0$)

$$U_Q = \kappa^2\left[\Lambda + \epsilon\frac{(\rho_{B0} + \rho_{b0})}{2a^3}\right]$$
$$\rho = \Lambda + \frac{(\rho_{B0} + \rho_{b0})}{a^3}(1 - \epsilon), \quad p = -\Lambda$$

where $\Lambda = \dfrac{24\hbar^2}{\kappa^2 \sigma^2 (m^2\sigma^2 - 8\hbar^2)}$,

$$\epsilon = \frac{4\hbar^2}{4\hbar^2 + m^2\sigma^2} = \frac{H_0^2 \sigma^2 \Omega_0(\Lambda)}{2 + 3H_0^2\sigma^2\Omega_0(\Lambda)}.$$

- The total matter density is $\rho_m = (\rho_{B0} + \rho_{b0})(1 - \epsilon)/a^3$
- An effective ΛCDM universe, albeit with the net *dust*-like cold dark matter content being *back-reacted* by both the BEC and baryon: $\rho_{CDM} = \rho_m - \rho_b = (1 - \epsilon)\rho_{B0}/a^3 - \epsilon\rho_{b0}/a^3$
- Such a Quantum back-reaction implies a unified dark sector in which the effective CDM and DE densities, are not just identified respectively to ρ_B and U_Q, but emerge from a rather involved picture, because of QRE.

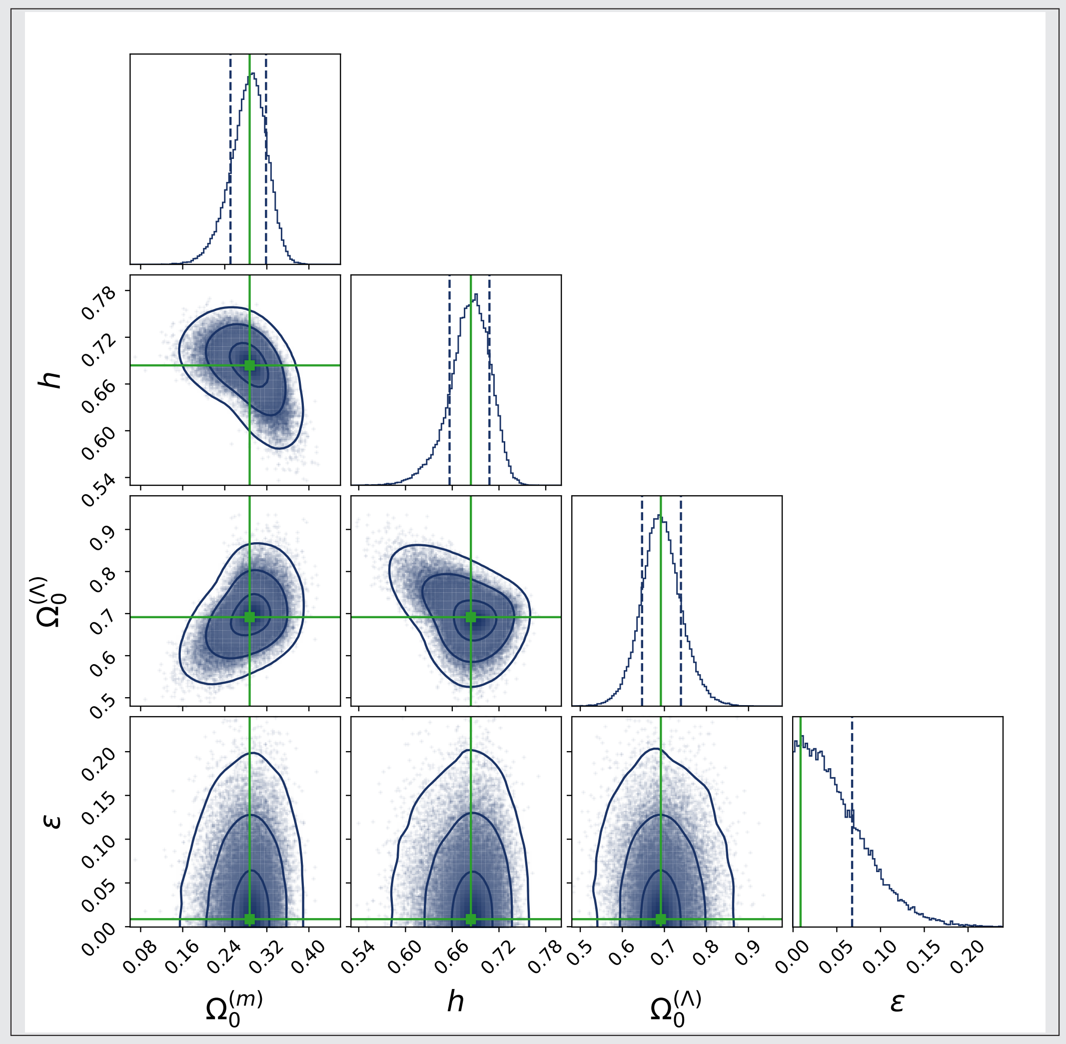
- Back-reaction parameter ϵ determines the mass of BEC : $m \approx \frac{\hbar H_0}{\epsilon}\sqrt{2(1 - \epsilon)(1 - 3\epsilon)\Omega_0(\Lambda)}$
- The requirement of real-valued m imposes $\epsilon < 1/3$, while the condition $\rho_B < \rho_m$ further restricts $\epsilon < \Omega_0(b)/(1 - \Omega_0(\Lambda) + \Omega_0(b)) \rightarrow \epsilon < 0.136$.
- The small ϵ enhances the BEC mass from its Hubble value $m_H = 2\sqrt{2}H_0 \approx 10^{-32}\,\mathrm{eV}$.
- The preferred boson mass in the BEC, that can prevent the formation of small scale structures in CDM via uncertainty principle, is $m \approx 10^{-22}$ eV. Allowed mass range of BEC: $10^{-22} - 10^{-32}$ eV.
- The homogeneous part of the BEC density and the quantum potential gives :

$$U_Q \approx \kappa^2\left[\Lambda + \epsilon\left(\frac{\rho_{B0} + \rho_{b0}}{2a^3} - \frac{3(p_0 + \Lambda)}{a^{2q}}\right)\right]$$
$$p \approx p_0 a^{-2q} - \Lambda(1 - a^{-2q})$$
$$\rho \approx \frac{\rho_{m0}}{a^3} + \Lambda - \frac{3(1 - 2\epsilon)(p_0 + \Lambda)}{a^{2q}}$$

where $q = (1 - 3\epsilon)/(1 - 2\epsilon) \approx 1 - \epsilon$

Results

- Two-dimensional posterior distribution using the SNe Ia Pantheon $+H(z)$ datasets.



- Best fit values and (or) 1σ confidence limits of parameters Ω_0^m, Ω_0^Λ, $h\,(= H_0/100\,\mathrm{km\,s^{-1}\,Mpc^{-1}})$ and ϵ, for estimations using the Pantheon and Pantheon $+H(z)$ datasets.

Observational Datasets	Parametric estimations (Best fit & 1 limits)			
	Ω_0^m	Ω_0^Λ	h	ϵ
Pantheon	$0.2886^{+0.0353}_{-0.0426}$	$0.7018^{+0.0656}_{-0.0531}$	–	< 0.0693
Pantheon$+H(z)$	$0.2873^{+0.0312}_{-0.0364}$	$0.6916^{+0.0478}_{-0.0446}$	$0.6839^{+0.0239}_{-0.0273}$	< 0.0675

- Up to 1σ, the mass enhancement is about 3 orders of magnitude over $m_H \approx 10^{-32}$ eV, which is still fairly in the range expected for a scalar field DE candidate.

General solution (Including space averaging)

- Space averaging of the quantum potential and the BEC density gives a more general solution.

$$U_Q = \kappa^2\left[\Lambda + \epsilon\frac{(\rho_{B0} + \rho_{b0})}{2a^3}\right] + \kappa^2\epsilon s\left[\frac{3\rho_{B0}}{5a^3}(1 - a^2) - \frac{26}{5}\left(\Lambda + \frac{(\rho_{B0} + \rho_{b0})}{a^3}\right)\right] + \mathcal{O}(s^2) + \cdots$$

$$\rho = \Lambda + \frac{\rho_{B0} + \rho_{b0}}{a^3}(1 - \epsilon) - \frac{2s}{5}[13\Lambda\epsilon + \frac{3\rho_{B0}}{a^3}[a^2(1 + \epsilon) - 1] - \frac{\epsilon(23\rho_{B0} + 26\rho_{b0})}{a^3}] + \mathcal{O}(s^2) + \ldots.$$

$$p = -\Lambda + \frac{2\epsilon s}{5}\left(\frac{2\rho_{B0}}{a} + 13\Lambda\right) + \mathcal{O}(s^2) + \cdots$$

Conclusion

- A unified cosmic dark sector can emerge from quantum-corrected Raychaudhuri/Friedmann equations with a BEC of ultra-light bosons, whose mass is enhanced via quantum back-reaction.
- BEC behaves almost as an axion-like scalar field DE rather than DM, based on its mass bound.
- Space averaging shows that, despite corrections to the cosmological solution, the BEC mass bound from quantum back-reaction remains unchanged. In fact the corrections are nominal $(\propto \epsilon)$.

References

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