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Quasi-normal modes of Accelerated-Kerr-Newman-de Sitter black holes via Heun functions

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INTRODUCTION

Motivation:
- Precision and diversity of gravitational-wave (GW) observations enable stringent tests of general relativity (GR).
- Before invoking modified gravity, we must **fully understand all solutions already allowed within GR**.
- The classical no-hair theorem assumes asymptotic flatness.
- Relaxing this assumption leads to a richer class of GR solutions with additional "charges".

What we did:
- **First study** of *uniformly accelerated Kerr–Newman–de Sitter* black holes.
- Asymptotically non-flat due to acceleration and/or cosmological constant.
- **Derived the Teukolsky equation for this generalized metric.**
- **Showed separability into radial and angular sectors.**
- **Showed that both sectors map to Heun's equation.**
- Enables rapid, high-precision computation of quasi-normal mode (QNM) frequencies using Mathematica's Heun framework.

Impact:
1. **Generalizes** previous results for:
   - Kerr–de Sitter black holes [2]
   - Accelerating (C-metric) black holes [1]
2. Broadens the landscape of GR-consistent QNM predictions.
3. Relevant for interpreting future GW observations and robust tests of GR.

TEUKOLSKY EQUATION

Spacetime metric:

$$ds^2 = \frac{1}{\Omega^2}\left(-\frac{Q}{\Sigma}\left[dt - a\sin^2\theta\, d\phi\right]^2 + \frac{\Sigma}{Q}dr^2 + \frac{\Sigma}{P}d\theta^2 + \frac{P}{\Sigma}\sin^2\theta\left[a\,dt - \left(r^2+a^2\right)d\phi\right]^2\right)$$

$$\Omega = 1 - \alpha r\cos\theta \quad ; \quad P(\theta) = 1 - 2\alpha M\cos\theta + \left(\alpha^2 a^2 + \alpha^2 e^2 + \frac{\Lambda a^2}{3}\right)\cos^2\theta$$

$$\Sigma = r^2 + a^2\cos^2\theta\, ; \; Q(r) = \left(r^2 - 2Mr + a^2 + e^2\right)\left(1 - \alpha^2 r^2\right) - \frac{\Lambda}{3}r^2\left(r^2+a^2\right)$$

M :mass, a :spin, e :electric charge, α :acceleration, Λ:cosmological constant

The perturbation equations, formulated in the Newman–Penrose framework, ***are separable for this metric****, yielding* ***decoupled*** ***angular and radial*** *Teukolsky equations.*

Null tetrad used:

$$l^\mu = \left(\frac{\Omega^2\left(r^2+a^2\right)}{Q}, \Omega^2, 0, \Omega^2\frac{a}{Q}\right) \quad ; \quad n^\mu = \left(\frac{r^2+a^2}{2\Sigma}, \frac{-Q}{2\Sigma}, 0, \frac{a}{2\Sigma}\right)$$

$$m^\mu = \frac{\Omega}{\sqrt{2P}(r + ia\cos\theta)}\left(ia\sin\theta, 0, P, \frac{i}{\sin\theta}\right) \; ; \quad \bar{m}^\mu = (m^\mu)^*$$

(* ≡ complex conjugation, also: $l^\mu n_\mu = -1$, $m^\mu \bar{m}_\mu = 1$)

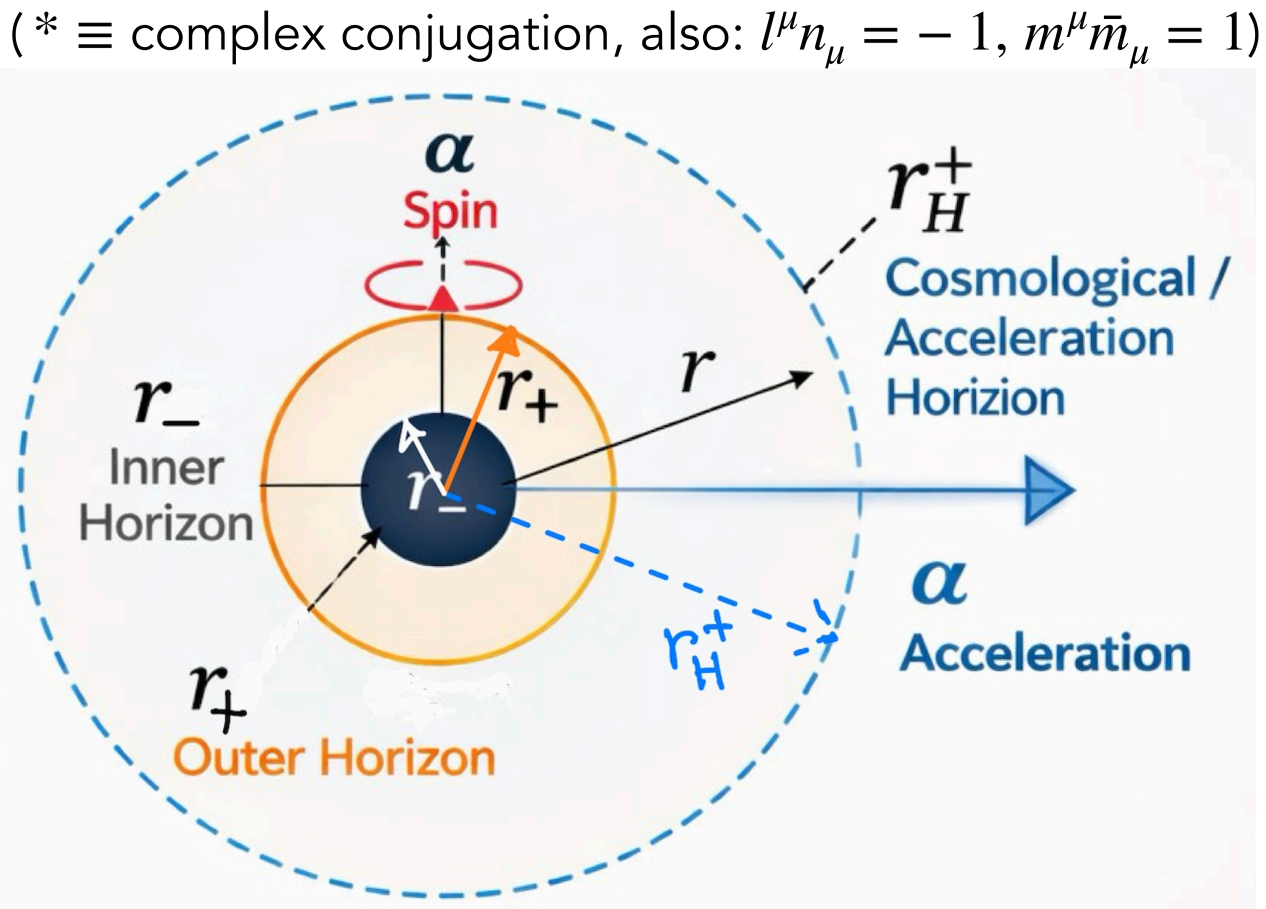

Figure: Cartoon diagram of the system with interpretations of the horizons
Note: r_H^- is unphysical

Weyl scalar separation of variable:

$$\Psi_4 \times \left(-\frac{\Omega}{r - ia\cos\theta}\right)^{2s} = \Omega^{1+2s} e^{-i\omega t} e^{im\phi} R(r)\Theta(\theta)$$

Radial Teukolsky Equation (RTE):

$$Q(r)R''(r) + (1+s)Q'(r)R'(r) +$$
$$\left(-\frac{1}{3} - \frac{2s^2}{3} - \lambda + s(-1 + 4ir\omega) + \frac{K_1^2}{Q(r)} - \frac{isK_1Q'(r)}{Q(r)} + \frac{1}{6}(1+s)(1+2s)Q''(r)\right)R(r) = 0$$

$$K_1 = (a^2 + r^2)\omega - am$$

Similarly we get angular Teukoslky equation.

MAPPING TO HEUN EQUATION

General Heun equation:

$$\frac{d^2 g(z)}{dz^2} + \left(\frac{\gamma}{z} + \frac{\delta}{z-1} + \frac{\epsilon}{z - z_m}\right)\frac{dg(z)}{dz} + \frac{\alpha\beta z - q}{z(z-1)(z-z_m)}g(z) = 0$$

4 **regular** singularities: $0,\ 1,\ z_m,\ \infty$

How to map RTE to general Heun equation?

Let $\quad Q(r) = -\left(\alpha^2 + \frac{\Lambda}{3}\right)\left(r - r_+\right)\left(r - r_-\right)\left(r - r_H^+\right)\left(r - r_H^-\right)$

⇒ 5 singularities of RTE: $r_+, r_-, r_H^+, r_H^-, \infty$

- Map two of the five singularities to 0 and 1
- Get rid of one singularity from the rest of three

Mobiüs transformation:

$$z = \frac{\left(r - r_+\right)\left(r_H^+ - r_-\right)}{\left(r - r_-\right)\left(r_H^+ - r_+\right)}$$

Mapping from $r \to z$: $r_+ \to 0, r_H^+ \to 1, r_- \to \infty, r_H^- \to z_-, \infty \to z_\infty$

$$z_- = \frac{\left(r_H^- - r_+\right)\left(r_H^+ - r_-\right)}{\left(r_H^- - r_-\right)\left(r_H^+ - r_+\right)} \; ; \; z_\infty = \frac{r_H^+ - r_-}{r_H^+ - r_+}$$

Dependent variable transformation:

Let B_1, B_2, B_3 be the indices of $R(r(z))$ for $z = 0, 1, z_-$ (i.e., $z \to 0 \Rightarrow R(r(z)) \sim z^{B_1}$ and so on); then the transformation:

$$R(r(z)) = z^{B_1}(z-1)^{B_2}(z - z_-)^{B_3}\left(z - z_\infty\right)^{2s+1} g(z)$$

factors out the singularity at z_∞. $g(z)$ then satisfies Heun equation.

$\gamma = 2B_1 + s + 1,\ \delta = 2B_2 + s + 1,\ \epsilon = 2B_3 + s + 1$ etc. $\alpha, \beta\, q$ are more complicated

The given set of transformation maps RTE to general Heun equation. The same is true for the angular part of Teukolsky equation.

CONCLUSION AND FUTURE WORK

- We have generalized the results in [1-2] to account for all the parameters simultaneously hence broadened the landscape of GR-consistent QNM predictions.
- Mapping to Heun equation is a **faster alternative** to usual approaches to solve QNMs, Leaver's method for e.g. (computes within seconds).
- Note: at least one among α, Λ needs to be non-zero for this to work. Otherwise, the singularity at $r = \infty$ becomes **irregular**.

REFERENCES

[1] Chen, Cai & Hu, QNM of uniformly accelerated BHs, arXiv:2402.02027 (2024)

[2] Hatsuda, QNM of Kerr–de Sitter BHs, arXiv:2006.08957 (2020)

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