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Bulk Viscosity and Varying Constants

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INTRODUCTION & AIM

- Is the universe a perfect fluid, with no dissipation?
- It may contain bulk viscosity
- In addition, consider variable gravitational and cosmological parameters, $G(t)$ and $\Lambda(t)$, respectively
- There have been several investigations in this direction
- Obviously, the field equations will change with the introduction of bulk viscosity together with variable $G(t)$ and $\Lambda(t)$
- The aim here is to check if this change yields general relativity with viscosity in the appropriate limit of constant G and Λ

METHOD

- Consider a flat (k = 0) Friedmann-Lemaitre-Robertson-Walker (FLRW) model
- Metric:

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \quad (1)$$

- Energy-momentum tensor with viscosity η:

$$T_{ab} = (\rho + \bar{p})u_a u_b + \bar{p} g_{ab}, \bar{p} = p - 3\eta H \quad (2)$$

where $\bar{p}$ is the total pressure and p is the perfect fluid contribution given by:

$$p = (\gamma - 1)\rho, \quad 0 \leq \gamma \leq 2 \quad (3)$$

- Einstein's field equations are:

$$R_{ab} - \frac{1}{2}R g_{ab} = G T_{ab} + \Lambda g_{ab} \quad (4)$$

- In the above equation, allow the gravitational parameter G and the cosmological parameter Λ to vary with time
- Then from equations (1)-(4), we get the following field equations:
- $3H = G\rho + \Lambda, \quad H = \frac{\dot{a}}{a}$ (5)
- $\dot{\rho} + 3(\rho + p)H - 9\eta H^2 + \frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{G} = 0$ (6)
- For a perfect fluid, the pressure is related to the energy density by:
- $p = \omega\rho, \quad \omega = const$ (7)

RESULTS & DISCUSSION

- The coefficient of viscosity η is usually taken to be a power law of the energy density
- $\eta = \eta_0 \rho^n, \quad \eta_0 = const, \quad n = const$ (8)
- How to solve (5)-(7)
- These are 3 equations in the 4 unknowns *a, G*, ρ and Λ
- So to solve, we have to make an assumption
- The generalised energy conservation equation (6) can be split in 2 ways, viz.,

  (a) Take the "usual energy conservation law", viz.,

$$\dot{\rho} + 3H(\rho + p) = 0, \quad (9)$$

  which leads to:

$$-9\eta H^2 + \frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{G} = 0 \quad (10)$$

  (b) The other alternative is to take

$$\dot{\rho} + 3(\rho+p)H - 9\eta H^2 = 0 \quad (11)$$

  which leads to:

$$\frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}}{G} = 0 \quad (12)$$

- The question is which of these alternatives to choose?
- Since we are considering bulk viscosity, we must end up with nonzero viscosity in the appropriate general relativistic limit of no variation in G and Λ
- From (10), then we find by putting G and Λ to be constant that η = 0

CONCLUSION

Hence (11) and (12) yield general relativity with viscosity in the appropriate limit of constant G and Λ

FUTURE WORK / REFERENCES

- Arbab I Arbab, *Astrophysics and Space Science*, **259**(98)371.
- Barrow, J.D., *Nuclear Physics B*, 310(98) 743.