Chiral cosmological models, unifying inflation and PBH formation

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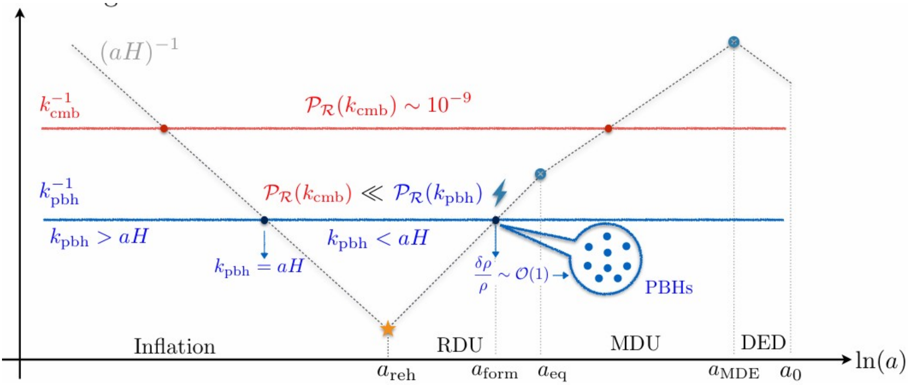
The 3rd International Online Conference on Universe (March 4–6, 2026, online, MDPI)

Introduction

- The idea of particles creation during radiation dominant stage belongs was presented in paper 'Zel'dovich, Ya. B. and Novikov, I. D.,The Hypothesis of Cores Retarded during Expansion and the Hot Cosmological Model, Soviet Astronomy, Vol. 10, p.602
- The Primordial Black hole mass spectrum was obtained from the metric perturbations by Bernard Carr in 1975[1]
- Carr's research was based on the works of S. W. Hawking, Ya. Zeldovich, R.A. Sunyaev and many others (see ref in B.J. Carr 1975).

[1]Bernard J. Carr "The Primordial Black hole mass spectrum" The Astrophysical Journal, **201**: 1-19, 1975 October 1

The modern interpretation of the problem is presented in O. Özsoy and G. Tasinato, "Inflation and Primordial Black Holes," Universe **9** (2023) no.5, 203, doi:10.3390/universe9050203 [arXiv:2301.03600 [astro-ph.CO]].



Chiral cosmological models, unifying inflation and PBH formation

- The modified gravity models with several scalar fields

$$S_F = \int dx^4 \sqrt{-\hat{g}} \left(F(R, \bar{\psi}) - \frac{h_{ij}(\bar{\psi})}{2} \hat{g}^{\mu\nu} \partial_\mu \psi^i \partial_\nu \psi^j - V(\bar{\psi}) \right) \qquad (1)$$

where $\bar{\psi} = \psi^1, \psi^2, ..., \psi^k$ are scalar fields, h_{ij} is a identity matrix.
- For the model (1) one can introduce the scalar field ψ^{k+1} without kinetic term and rewrite S_F as:

$$S_J = \int d^4x \sqrt{-\hat{g}} \left(\frac{\partial F}{\partial \psi^{k+1}} R - \frac{h_{ij}(\bar{\psi})}{2} g^{\mu\nu} \partial_\mu \psi^i \partial_\nu \psi^j - W(\bar{\psi}) \right) \qquad (2)$$

where

$$W = \frac{\partial F}{\partial \psi^{k+1}} (\psi^{k+1}) - F(\bar{\psi}, \psi^{k+1}) + V(\bar{\psi}))$$

Conformal transformation

One can introduce the designation

$$f = f(\bar{\psi}, \psi^{k+1}) = \frac{\partial F(\psi^{k+1}, \bar{\psi})}{\partial \psi^{k+1}} \qquad (3)$$

apply the conformal transformation of the metric:

$$g_{\mu\nu} = \frac{2}{M_{Pl}^2}\, f\, \bar{g}_{\mu\nu} \qquad (4)$$

and get the following action in the Einstein frame:

$$S_E = \int d^4x \sqrt{-g} \left(\frac{M_{Pl}^2}{2} R - \frac{G_{AB}}{2} g^{\mu\nu} \partial_\mu \psi^A \partial_\nu \psi^B - V_E \right) \qquad (5)$$

where

$$G_{AB} = \frac{M_{Pl}^2}{2\,f} \left(h_{AB} + \frac{3\, f_A, f_B}{f} \right), \quad V_E = \frac{M_{Pl}^4}{4} \frac{W}{f^2} \qquad (6)$$

We got $k+1$ field model in GR model with potential. [2]

[2] S. V. Chervon, I. V. Fomin, E. O. Pozdeeva, M. Sami and S. Y. Vernov, Phys. Rev. D **100** (2019) no.6, 063522 doi:10.1103/PhysRevD.100.063522 [arXiv:1904.11264 [gr-qc]]

Working action

Introducing canonical scalar field ϕ instead of ψ^{k+1} as

$$\phi = \sqrt{\frac{3}{2}} M_{Pl} \ln\left(\frac{2}{M_{Pl}^2} f\right) \tag{7}$$

allows one to rewrite the action S_E the standard form:

$$S_E = \int d^4x \sqrt{-g}\left[\frac{M_{Pl}^2}{2} R - \frac{g^{\mu\nu}}{2}\partial_\mu\phi\partial_\nu\phi - \frac{G_{ij}}{2} g^{\mu\nu}\partial_\mu\psi^i\partial_\nu\psi^j - V_E\right] \tag{8}$$

where $i, j = \overline{1, k}$ [3].

[3] V. R. Ivanov, S. V. Ketov, E. O. Pozdeeva and S. Y. Vernov, JCAP **03** (2022) no.03, 058 doi:10.1088/1475-7516/2022/03/058 [arXiv:2111.09058 [gr-qc]]

PBH formation in CCS

- S. V. Ketov, "Multi-Field versus Single-Field in the Supergravity Models of Inflation and Primordial Black Holes," Universe **7** (2021) no.5, 115 doi:10.3390/universe7050115
- M. Braglia, A. Linde, R. Kallosh and F. Finelli, "Hybrid α-attractors, primordial black holes and gravitational wave backgrounds," JCAP **04** (2023), 033 doi:10.1088/1475-7516/2023/04/033 [arXiv:2211.14262 [astro-ph.CO]].
- S. Pi, Y. l. Zhang, Q. G. Huang and M. Sasaki, "Scalaron from R^2-gravity as a heavy field," JCAP **05** (2018), 042 doi:10.1088/1475-7516/2018/05/042 [arXiv:1712.09896 [astro-ph.CO]]
- many others

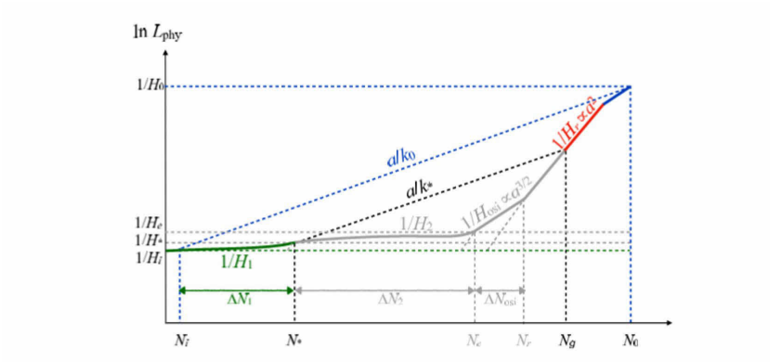


Figure: S. Pi, Y. l. Zhang, Q. G. Huang and M. Sasaki, JCAP **05** (2018), 042 doi:10.1088/1475-7516/2018/05/042 [arXiv:1712.09896 [astro-ph.CO]]

Background metric

• The spatially flat Friedman-Lemaitre-Robertson-Walker (FLRW) universe with the interval

$$ds^2 = -dt^2 + a^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right),$$

• The Hubble parameter $H(t)$ is the logarithmic derivative of the scale factor: $H = \dot{a}/a$. It is suitable to consider the e-folding number $N = \ln(a/a_e)$, where a_e is a constant, as an independent variable during inflation. We use the relation $\frac{d}{dt} = H\,\frac{d}{dN}$.

• The slow-roll parameters in the Einstein frame [4] can be presented as

$$\varepsilon = -\frac{H'}{H} \tag{9}$$

$$\eta = -\frac{1}{2}\frac{\left(H^2\right)''}{\left(H^2\right)'} = \varepsilon - \frac{\varepsilon'}{2\varepsilon} \tag{10}$$

where $' = \frac{d}{dN}$.

At the first stage we use the standard slow-roll formulae to connect the inflationary and slow-roll parameters:

$$n_s = 1 - 6\epsilon + 2\eta, \qquad r = 16\epsilon, \qquad A_s = \frac{2H^2}{\pi^2 M_{\rm Pl}^2 r}. \tag{11}$$

The Starobinsky inflation[5] nicely fits the CMB observations by the Planck/BICEP collaborations

$$n_s = 0.9651 \pm 0.0044\,, \tag{14}$$

but deviates from the latest ACT/DESI data

$$n_s = 0.9743 \pm 0.0034\,. \tag{15}$$

Note that the ACT/DESI data does not significantly change the upper bound on the tensor-to-scalar ratio r and the value of the amplitude of scalar perturbations A_s,

$$A_s = (2.10 \pm 0.03) \times 10^{-9} \qquad \text{and} \qquad r < 0.028\ . \tag{16}$$

[5] The Starobinsky inflationary model is described by the following action,

$$S_{\rm Star.} = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left(R + \frac{1}{6m^2} R^2 \right)\ , \tag{12}$$

with only one parameter $m \sim 10^{-5} M_{\rm Pl}$, which is the inflaton mass. The inflationary parameters n_s and r do not depend on m, but depend on the number of e-foldings during inflation N_i:

$$n_s = 1 - \frac{2}{N_i} + \mathcal{O}(N_i^{-2}), \qquad r = \frac{12}{N_i^2} + \mathcal{O}(N_i^{-3})\ . \tag{13}$$

In particular, $n_s = 0.964$ corresponds to $N_i \approx 55$, whereas $n_s = 0.974$ corresponds to $N_i \approx 77$.

At the ultra-slow-roll regime, we have a nearly-inflection point, at which $V'_E \approx 0$ [6]. To describe this point [7] we use the slow-roll parameter η. If $V'_E \approx 0$, then $\eta \approx 3$. We use the supposition that the transition from the first stage of inflation to the second stage leads to grow of energy density perturbations leading to PBH formation at the movement when perturbations with wavenumber around k_* re-enter the horizon $k_* = a_* H_* = a_{re} H_{re} = k_{re}$ [8]. Modes of perturbations can re-enter the horizon in different stage of the universe evolution. We work in the supposition that it is taking place during radiation dominant stage and e-folding numbers at which PBH formation is possible is very close to the end of second stage of inflation [9].

[6] S. Pi, Y. l. Zhang, Q. G. Huang and M. Sasaki, JCAP **05** (2018), 042 doi:10.1088/1475-7516/2018/05/042 [arXiv:1712.09896 [astro-ph.CO]]

[7] A. Y. Kamenshchik, E. O. Pozdeeva, A. Tribolet, A. Tronconi, G. Venturi and S. Y. Vernov, Phys. Rev. D **110** (2024) no.10, 104011 doi:10.1103/PhysRevD.110.104011 [arXiv:2406.19762 [gr-qc]].

[8] J. Garcia-Bellido, A. D. Linde and D. Wands, Phys. Rev. D **54** (1996), 6040-6058 doi:10.1103/PhysRevD.54.6040 [arXiv:astro-ph/9605094 [astro-ph]].

[9] S. Pi, Y. l. Zhang, Q. G. Huang and M. Sasaki, JCAP **05** (2018), 042 doi:10.1088/1475-7516/2018/05/042 [arXiv:1712.09896 [astro-ph.CO]]

The mass of PBHs depends on the duration of the second stage, $N_e - N_*$, and the value of the Hubble parameter at the end of inflation H_e. To estimate the mass of PBHs we apply the formula [arXiv:1712.09896 [astro-ph.CO] , [arXiv:2207.11878 [astro-ph.CO]] in the form obtained in [arXiv:2407.00999 [gr-qc]]:

$$M_{PBH} \simeq \frac{M_{\mathrm{Pl}}^2}{H_e} \exp\left(2(N_e - N_*)\right), \tag{17}$$

where N_e is the total duration of inflation, N_* is the minimal value of N, at which $\eta(N_*) = 3$.

• We propose a $F(R, \chi)$ inflationary model with the scalar field χ. We compare $F(R)$ gravity inflationary models that have been constructed or developed to fit the ACT data and show that the model proposed in [V. R. Ivanov, arXiv:2508.14250 [gr-qc]] is the suitable for our proposals.
• Using conformal transformation of the metric, we get a CCM model with two scalar fields.
• We analyze the behaviour of scalar fields during inflation by numerical calculations for different values of the model parameters and demonstrate that the constructed inflationary model does not contradict to the recent ACT/DESI observation data and is suitable for PBH formation. The estimation of PBH masses shows that PBHs could be dark matter candidates.

$F(R, \chi)$ model

We consider the modified gravity model with a scalar field χ

$$S_R = \int d^4x\sqrt{-g}\left[F(R,\chi) - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi\right], \tag{18}$$

where $F(R, \chi)$ is a nonlinear double differentiable function:

$$F = \frac{M_{\mathrm{Pl}}^2}{2}\left[(1+X(\chi))\left(1-\frac{1}{3\,\delta}\right)R + \frac{1}{3\delta}\left(R+\frac{m^2}{\delta}\right)\ln\left(1+\frac{\delta\,R}{m^2}\right) - U(\chi)m^2\right], \tag{19}$$

where δ is a dimensionless positive constant, $X(\chi)$ and $U(\chi)$ are dimensionless differentiable functions of the scalar field χ.
We choose the following fourth-order polynomial function $U(\chi)$ and function $X(\chi)$

$$X(\chi) = c\frac{\chi^2}{\chi_0^2}, \qquad U(\chi) = U_0\left[\left(1-\frac{\chi^2}{\chi_0^2}\right)^2 - d\,\frac{\chi}{\chi_0}\right], \tag{20}$$

where c, d, U_0, and $\chi_0 > 0$ are constants. The original $F(R)$ model corresponds to $X(\chi) \equiv 0$ and $U(\chi) \equiv 0$.

The conformal transformation of the metric:

$$\tilde{g}_{\mu\nu} = \frac{2F_{,\sigma}}{M_{\rm Pl}^2} g_{\mu\nu}, \tag{21}$$

gives the following CCM models in the Einstein frame:

$$S_E = \int d^4x \sqrt{-\tilde{g}} \left[\frac{M_{\rm Pl}^2}{2} \tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{y}{2} \tilde{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V_E(\phi, \chi) \right], \tag{22}$$

where

$$y = \frac{M_{\rm Pl}^2}{2F_{,\sigma}} = \mathrm{e}^{-\sqrt{\frac{2}{3}}\,\frac{\phi}{M_{\rm Pl}}}, \qquad V_E(\phi, \chi) = y^2(\phi) V(\sigma(\phi, \chi), \chi). \tag{23}$$

A nice feature of the model (19) is the existence of the potential $V_E(\phi,\chi)$ in the analytic form in the Einstein frame:

$$V_E = \frac{M_{\rm Pl}^2 m^2\, y}{2\delta^2}\left[\frac{y}{3}\,\exp\left(-\frac{(3\,\delta-1)c\chi^2 y+3\,\delta\chi_0^2(y-1)}{\chi_0^2 y}\right)+U_0\,\delta^2 y\frac{\chi^4}{\chi_0^4}\right.$$
$$\left.+\,y\frac{\chi^2}{\chi_0^2}\left(c\delta-\frac{c}{3}-2U_0\,\delta^2\right)-U_0\,d\delta^2 y\frac{\chi}{\chi_0}+\left(U_0\,\delta^2-\frac{1}{3}+\delta\right)y-\delta\right]. \qquad (24)$$

Note that

$$\sigma = \frac{m^2}{\delta}\left(\exp\left(-\frac{(3\,\delta-1)c\chi^2 y+3\,\delta\chi_0^2(y-1)}{\chi_0^2 y}\right)-1\right). \qquad (25)$$

• In our model, we choose N equal to the number of e-folding during only the first stage of inflation, so $35 < N < 40$.
We obtain $3.1 \times 10^{-4} < \delta < 3.7 \times 10^{-4}$ for $N = 35$ and $2.1 \times 10^{-4} < \delta < 2.6 \times 10^{-4}$ for $N = 40$.
• Thus, the suitable interval for the parameter δ is $2.1 \times 10^{-4} < \delta < 3.7 \times 10^{-4}$. It should be noted that this estimation has been obtained for $U_0 = 0$, but it can be used with reasonable accuracy for suitable non-zero values of the parameter U_0.

Results of numerical integration of the evolution equations with the following values of parameters:

$$\delta = 2.5{\cdot}10^{-4},\ U_0 = 0.8,\ \chi_0 = 2\,,\ C = 0.00044,\ d = 0.0005,\ m = 2.3084{\cdot}10^{-5}\, M_{\mathrm{Pl}},$$

The duration of the first stage of inflation, N_*, and the total duration of inflation, N_e, can vary depending on the chosen values of the parameters. The field ϕ_0 is determined by the condition $n_s(\phi_0) = 0.974$. After that, the parameter m is selected such that $A_s(\phi_0) = 2.1 \cdot 10^{-9}$.
The choice of model parameters in formula is not unique. Tables 1 and 2 show that different values of these parameters can lead to viable inflationary scenarios with different inflationary parameters.

δ	$m/M_{\rm Pl}$	$\phi_0/M_{\rm Pl}$	n_s	r	N_*	N_e	$M_{PBH}/M_\odot$
$2.0\cdot10^{-4}$	$2.101\cdot10^{-5}$	5.088	0.974	0.0105	42.2	70.3	$2.14\cdot10^{-9}$
$2.1\cdot10^{-4}$	$2.144\cdot10^{-5}$	5.071	0.974	0.0109	41.5	66.6	$4.25\cdot10^{-12}$
$2.3\cdot10^{-4}$	$2.235\cdot10^{-5}$	5.027	0.974	0.0118	39.9	61.2	$1.53\cdot10^{-15}$
$2.5\cdot10^{-4}$	$2.308\cdot10^{-5}$	5.007	0.974	0.0126	39.1	58.1	$2.11\cdot10^{-17}$
$2.7\cdot10^{-4}$	$2.384\cdot10^{-5}$	4.981	0.974	0.0134	38.1	55.6	$8.28\cdot10^{-19}$
$2.9\cdot10^{-4}$	$2.4595\cdot10^{-5}$	4.952	0.974	0.0143	37.1	53.3	$5.82\cdot10^{-20}$
$3.1\cdot10^{-4}$	$2.532\cdot10^{-5}$	4.927	0.974	0.0152	36.2	51.6	$9.33\cdot10^{-21}$

Table: The dependence of the inflation parameter r, the duration of the first stage of inflation N_*, the total duration of inflation N_e, and the PBH mass on the model parameter δ. The value of the parameter m is fixed by the condition $A_s(\phi_0)=2.1\cdot10^{-9}$. Other model parameters are chosen as follows: $U_0=0.8\,,\ \chi_0=2\,,\ C=0.00044\,,$ $d=0.001\,.$

d	N_e	$N_e - N_*$	$M_{PBH}/M_{\rm Pl}$	$M_{PBH}/M_{\odot}$	M_{PBH}/g	$H_e/M_{\rm Pl}$
0.0012	57.7	18.6	$3.54 \cdot 10^{21}$	$7.72 \cdot 10^{-18}$	$1.56 \cdot 10^{16}$	$4.04 \cdot 10^{-6}$
0.0010	58.1	19.1	$9.70 \cdot 10^{21}$	$2.11 \cdot 10^{-17}$	$4.27 \cdot 10^{16}$	$4.01 \cdot 10^{-6}$
0.0008	58.8	19.7	$3.21 \cdot 10^{22}$	$7.00 \cdot 10^{-17}$	$1.41 \cdot 10^{17}$	$4.02 \cdot 10^{-6}$
0.0005	60.0	20.9	$3.53 \cdot 10^{23}$	$7.70 \cdot 10^{-16}$	$1.55 \cdot 10^{18}$	$4.02 \cdot 10^{-6}$
0.0003	61.4	22.3	$5.85 \cdot 10^{24}$	$1.28 \cdot 10^{-14}$	$2.57 \cdot 10^{19}$	$4.00 \cdot 10^{-6}$
0.0002	62.5	23.5	$6.42 \cdot 10^{25}$	$1.40 \cdot 10^{-13}$	$2.82 \cdot 10^{20}$	$4.02 \cdot 10^{-6}$
0.00015	63.3	24.2	$2.62 \cdot 10^{26}$	$5.71 \cdot 10^{-13}$	$1.15 \cdot 10^{21}$	$4.01 \cdot 10^{-6}$
0.0001	64.4	25.3	$2.36 \cdot 10^{27}$	$5.14 \cdot 10^{-12}$	$1.04 \cdot 10^{22}$	$4.00 \cdot 10^{-6}$
0.00008	64.9	25.9	$7.77 \cdot 10^{27}$	$1.69 \cdot 10^{-11}$	$3.42 \cdot 10^{22}$	$4.04 \cdot 10^{-6}$

Table: The dependence of the duration of inflation N_e and the PBH mass M_{PBH} on the model parameter d. Inflationary parameters, $n_s = 0.974$ and $r = 0.0126$, as well as the duration of the first stage of inflation $N_* \approx 39$ are independent of d. Other model parameters are chosen as follows: $U_0 = 0.8\,,\ \delta = 2.5 \cdot 10^{-4},\ \chi_0 = 2\,,\ C = 0.00044\,,$ $m = 2.3084 \cdot 10^{-5}\, M_{\rm Pl}$.

Induced gravity models and the corresponding CCMs

- The induced gravity model with two scalar fields:

$$S=\int d^4x\sqrt{-\tilde{g}}\left[\frac{\xi}{2}\sigma^2\tilde{R}-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-\tilde{V}(\sigma,\chi)\right], \qquad (26)$$

where ξ is a positive constant, the potential $\tilde{V}$ is a differentiable function, $M_{\rm Pl}$ is the Planck mass.

- The conformal transformation of the metric $g_{\mu\nu}=\frac{\xi\sigma^2}{M_{\rm Pl}^2}\tilde{g}_{\mu\nu}$.
- The action of the two-field CCM:

$$S_E=\int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R-\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-\frac{y}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-V_E\right], \qquad (27)$$

where

$$\phi=M_{\rm Pl}\sqrt{6+\frac{1}{\xi}}\,\ln\left(\frac{\sigma}{M_{\rm Pl}}\right),\quad y=\frac{M_{\rm Pl}^2}{\xi\sigma^2}=\frac{1}{\xi}\exp\left(-2\sqrt{\frac{\xi}{6\xi+1}}\frac{\phi}{M_{\rm Pl}}\right), \qquad (28)$$

and the potential $V_E=y^2(\phi)\tilde{V}(\sigma(\phi),\chi)$.

Inflationary model

We consider the following potential:

$$\tilde{V}(\sigma,\chi) = \lambda\sigma^4 \left(F_1(\chi) + F_2(\chi)\mathrm{e}^{\gamma\,[\ln(\sigma/M_{\rm Pl})]^{2\alpha}}\right), \tag{29}$$

where

$$F_1(\chi) = \left(1 - \frac{\chi^2}{\chi_0^2}\right)^2 - d\,\frac{\chi}{\chi_0}, \qquad F_2(\chi) = \frac{c_2\chi^2}{\chi_0^2} + c_0\,, \tag{30}$$

α, γ, λ, χ_0, c_0, c_2 and d are constants. Note that the potential $\tilde{V}$ is real even if $\ln(\sigma/M_{\rm Pl}) < 0$ and α is not an integer number.
In the Einstein frame, we get

$$V_E(\phi,\chi) = V_0\left(F_1(\chi) + F_2(\chi)\mathrm{e}^{\beta\left(\frac{\phi^2}{M_{\rm Pl}^2}\right)^{\alpha}}\right), \tag{31}$$

where

$$V_0 = \frac{\lambda M_{\rm Pl}^4}{\xi^2}, \qquad \beta = \gamma\left(\frac{\xi}{1+6\,\xi}\right)^{\alpha}. \tag{32}$$

Evolution equations in the Einstein frame

The evolution equations have the following form:

$$H^2 = \frac{2V_E}{6M_{\rm Pl}^2 - {\phi'}^2 - y{\chi'}^2}\,, \qquad (33)$$

$$H' = -\frac{H}{2M_{\rm Pl}^2}\left[{\phi'}^2 + y{\chi'}^2\right]\,, \qquad (34)$$

$$\begin{aligned} \phi'' =& (\epsilon_1 - 3)\phi' + \frac{1}{2}\frac{dy}{d\phi}{\chi'}^2 - \frac{6M_{\rm Pl}^2 - y{\chi'}^2 - {\phi'}^2}{2V_E}\frac{\partial V_E}{\partial \phi}\,, \\ \chi'' =& (\epsilon_1 - 3)\chi' - \frac{1}{y}\frac{dy}{d\phi}\chi'\phi' - \frac{6M_{\rm Pl}^2 - {\phi'}^2 - y{\chi'}^2}{2yV_E}\frac{\partial V_E}{\partial \chi}\,. \end{aligned} \qquad (35)$$

Numerical solutions of the evolution equations

We solve system of the evolution equations numerically to analyze the evolution of scalar fields during inflation and to get values of inflationary parameters. We define the e-folding number N in such a way that $N = 0$ corresponds to the moment at which inflationary parameters are calculated. For the values of models parameters

$$\begin{aligned} &V_0 = 10^{-10} M_{\rm Pl}^4, \quad \alpha = -0.37, \quad \beta = -1.8, \\ &\chi_0 = 3.5\, M_{\rm Pl}, \quad c_0 = 12, \quad c_2 = 147, \quad d = 10^{-3}, \end{aligned} \tag{36}$$

we get inflationary parameters

$$n_s = 0.9622, \quad r = 0.0266, \quad A_s = 2.10 \cdot 10^{-9}\,, \tag{37}$$

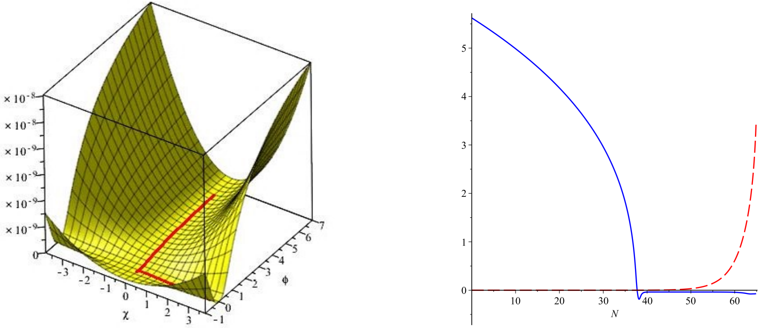


Figure: Potential with trajectory and the fields evolution

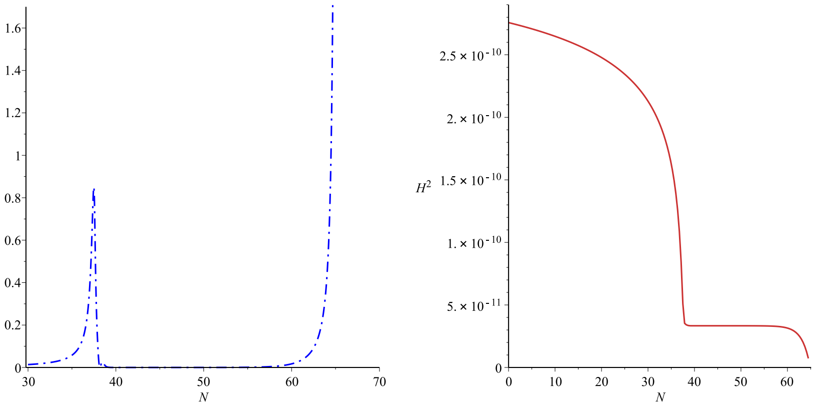


Figure: Slow-roll parameters ϵ_1 and the square of the Hubble parameter

α	β	ϕ_0/M_{Pl}	n_s	r	N_*	N_{tot}
-0.40	-2	5.867	0.962	0.027	38.2	61.9
-0.40	-1.8	5.502	0.959	0.027	35.4	58.9
-0.40	-1.5	4.936	0.954	0.027	31.1	53.5
-0.37	-2	6.008	0.965	0.026	40.8	64.2
-0.37	-1.8	5.623	0.962	0.026	37.6	60.7
-0.37	-1.5	5.018	0.957	0.028	32.8	54.9
-0.35	-2	6.104	0.967	0.025	42.7	65.4
-0.35	-1.8	5.701	0.964	0.026	39.2	61.9
-0.35	-1.5	5.072	0.959	0.027	34.1	56.2

Table: Dependence of inflation parameters, duration of the first stage of inflation N_* and total duration of inflation N_{tot} on the model parameters α and β. Other model parameters are chosen as follows:
$V_0 = 10^{-10} M_{\mathrm{Pl}}^4, \quad \chi_0 = 3.5\, M_{\mathrm{Pl}}, \quad c_0 = 12, \quad c_2 = 147, \quad d = 0.003.$

d	N_{tot}	$N_{tot} - N_*$	M_{PBH}/M_{Pl}	$M_{PBH}/M_{\odot}$	M_{PBH}/g
0.001	64.5	26.9	$8.57 \cdot 10^{28}$	$1.87 \cdot 10^{-10}$	$3.72 \cdot 10^{23}$
0.002	62.1	24.5	$7.07 \cdot 10^{26}$	$1.54 \cdot 10^{-12}$	$3.07 \cdot 10^{21}$
0.003	60.7	23.1	$4.31 \cdot 10^{25}$	$9.40 \cdot 10^{-14}$	$1.87 \cdot 10^{20}$
0.007	57.7	20.1	$1.18 \cdot 10^{23}$	$2.57 \cdot 10^{-16}$	$5.14 \cdot 10^{17}$
0.01	56.5	18.9	$9.92 \cdot 10^{21}$	$2.16 \cdot 10^{-17}$	$4.30 \cdot 10^{16}$

Table: The dependence of duration of inflation N_{tot} and the PBH mass M_{PBH} from the model parameter d. Other model parameters are given by (36). The end of the first stage of inflation is at $N_* = 37.6$ independent on d.

α	λ	n_s	N_*	N_{tot}	$N_{tot} - N_*$	$M_{PBH}/M_{\odot}$
-0.2	$1.98 \cdot 10^{-10}$	0.973	44.41	64.92	20.51	$1.4 \cdot 10^{-16}$
-0.18	$1.95 \cdot 10^{-10}$	0.973	43.12	63.74	20.62	$1.8 \cdot 10^{-16}$
-0.15	$1.75 \cdot 10^{-10}$	0.974	43.40	64.22	20.82	$2.9 \cdot 10^{-16}$
-0.13	$1.62 \cdot 10^{-10}$	0.974	42.53	63.51	20.98	$4.1 \cdot 10^{-16}$

Table: The dependence of duration of inflation N_{tot} and the PBH mass M_{PBH} from the model parameter α at fixed $c_0 = 1.2$, $\xi = 1$, $\beta = -1.3$, $d = 0.005$, $c_2 = 147$, $\chi_0 = 3.5$.

The current estimation of the mass region of PBHs considered as candidates for dark matter is $10^{-17}\,M_\odot \leqslant M_{PBH} \leqslant 10^{-12} M_\odot$, where $M_\odot$ is the Solar mass (Ozsoy (2023) and references therein). As shown in Table 4, the proposed model with $0.002 \leqslant d \leqslant 0.01$ allows us to reproduce the masses of the PBH from this interval.

Conclusion

• We considered two chiral cosmological models with two scalar fields inspired by modified gravity models:

1. $F(R, \chi)$ model
2. and induce gravity model

• In both inflationary models we get ultra-slow-roll regimes corresponding to the nearly-inflection point of the potential and increasing of energy density perturbations which leads to PBH formation in the radiation dominant stage.
• For some values of model parameters, we reproduce appropriate values for inflationary parameters and the PBHs masses from the region of dark matter candidates is $10^{-17}\,M_{\odot} \leqslant M_{PBH} \leqslant 10^{-12} M_{\odot}$.

This study was conducted within the scientific program of the National Center for Physics and Mathematics, section 5 'Particle Physics and Cosmology'.