

Construction of slow-roll approximations in the Jordan frame

Sergey Vernov and Ekaterina Pozdeeva

Skobeltsyn Institute of Nuclear Physics, Lomonosov
Moscow State University

based on

E.O. Pozdeeva, M.A. Skugoreva,

A.V. Toporensky, S.Yu. Vernov,

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The standard method for constructing an inflationary model with nonminimal coupling involves the conformal transformation of the metric and the construction of the corresponding inflationary model in the Einstein frame.

This method can sometimes be complicated, as the potential in the Einstein frame can only be obtained in parametric form.

In 2025, models of inflation have been revisited in light of the recent observations of cosmic microwave background radiation by Atacama Cosmology Telescope (ACT) [T. Louis *et al.* [ACT], JCAP **11** (2025) 062]

and DESI Collaboration [A. G. Adame *et al.* [DESI], JCAP **02** (2025) 021].

These observations have shown that inflation models are significantly constrained by demanding a higher (than the Planck Telescope value) cosmological tilt n_s of scalar perturbations and a positive running index α_s favored by ACT.

In particular, the ACT observations have merely $\sim 2\sigma$ tension with the predictions of the Starobinsky inflationary model [A.A. Starobinsky, *Phys. Lett. B* **91** (1980) 99].

In this report, we present a new slow-roll approximation and the way of inflationary model construction in the Jordan frame only.

This approximation has been proposed in the paper by [E.O. Pozdeeva, M.A. Skugoreva, A.V. Toporensky, S.Yu. Vernov, *JCAP* **05** \(2025\) 081.](#)

Using this slow-roll approximation, we provide a new modification to the Starobinsky inflation model within the framework of $F(R)$ gravity.

The model constructed satisfies all observational constraints.

This part of the presentation is based on the paper by [S.V. Ketov, E.O. Pozdeeva, S.Yu. Vernov, *JCAP* **12** \(2025\) 040.](#)

MODELS WITH A MINIMALLY COUPLED SCALAR FIELD

A single-field model with a minimally coupled scalar field is described by

$$S = \frac{1}{2} \int d^4x \sqrt{-g} [M_{\text{Pl}}^2 R - g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 2V_E(\varphi)].$$

In the spatially flat Friedmann-Lemaître-Robertson-Walker metric with

$$ds^2 = -dt^2 + a^2(t) (dx^2 + dy^2 + dz^2),$$

the evolution equations are

$$3 M_{\text{Pl}}^2 H_E^2 = \frac{1}{2} \left(\frac{d\varphi}{dt_E} \right)^2 + V_E, \quad (1)$$

$$\frac{dH_E}{dt_E} = -\frac{1}{2M_{\text{Pl}}^2} \left(\frac{d\varphi}{dt_E} \right)^2, \quad (2)$$

$$\frac{d^2\varphi}{dt_E^2} + 3H_E \frac{d\varphi}{dt_E} + V_{E,\varphi} = 0, \quad (3)$$

where $V_{E,\varphi} = \frac{dV_E(\varphi)}{d\varphi}$.

The slow-roll approximation in the Einstein frame

One can see that $H_E(t_E)$ is a monotonically decreasing function.
The slow-roll parameters in the Einstein frame are defined by

$$\varepsilon^{(E)} = -\frac{1}{H_E^2} \frac{dH_E}{dt_E} = \frac{3 \left(\frac{d\varphi}{dt_E} \right)^2}{\left(\frac{d\varphi}{dt_E} \right)^2 + 2V_E}, \quad (4)$$

$$\eta^{(E)} = \varepsilon^{(E)} - \frac{1}{2\varepsilon^{(E)} H_E} \frac{d\varepsilon^{(E)}}{dt_E} = -\frac{1}{2 H_E} \frac{\frac{d^2 H_E}{dt_E^2}}{\frac{dH_E}{dt_E}}. \quad (5)$$

If $\varepsilon^{(E)} \ll 1$ and $|\eta^{(E)}| \ll 1$, then Eqs. (1) and (3) can be simplified:

$$H_E^2 \approx \frac{V_E}{3 M_{\text{Pl}}^2}, \quad (6)$$

$$\frac{d\varphi}{dt_E} \approx -\frac{V_{E,\varphi}}{3H_E}. \quad (7)$$

Equation (6) gives $H_E(\varphi)$.

Note that differentiating Eq. (6) and using Eq. (2), one gets Eq. (7).

The inflationary parameters

The CMB observables are given by the amplitude of scalar perturbations A_s , the scalar spectral index n_s , the tensor-to-scalar ratio r , and their running, α_s and α_t , respectively.

In the *leading* approximation with respect to the SR parameters, we get

$$\begin{aligned} n_s &\approx 1 - 2\epsilon^{(E)} + \frac{d \ln(\epsilon^{(E)})}{dN_E} = 1 - 4\epsilon^{(E)} + 2\eta^{(E)}, & \alpha_s &\approx -\frac{d n_s}{dN_E}, \\ r &\approx 16\epsilon^{(E)}, & A_s &\approx \frac{2H_E^2}{\pi^2 M_{\text{Pl}}^2 r^{(E)}}, & \alpha_t &\approx \frac{1}{8} \frac{d r}{dN_E}, \end{aligned} \quad (8)$$

the e-folds number $N_E = -\ln(a_E/a_{E_e})$.

MODELS WITH NONMINIMAL COUPLING

A generic model with a nonminimally coupled scalar field σ can be described by the following action:

$$S = \int d^4x \sqrt{-g} \left[U(\sigma)R - \frac{c}{2} g^{\mu\nu} \sigma_{,\mu} \sigma_{,\nu} - V(\sigma) \right], \quad (9)$$

where $c = 0$ or $c = 1$.

The case of $c = 0$ corresponds to $F(R)$ gravity, because the action

$$S_R = \int d^4x \sqrt{-g} F(R) \quad (10)$$

is equivalent to the action of the following scalar-tensor gravity model

$$S_J = \int d^4x \sqrt{-g} [F_{,\sigma}(R - \sigma) + F], \quad (11)$$

where σ is a scalar field without a kinetic term.

In the spatially flat Friedmann-Lemaître-Robertson-Walker metric with

$$ds^2 = - dt^2 + a^2(t) (dx^2 + dy^2 + dz^2) , \quad (12)$$

the evolution equations are

$$6UH^2 + 6\dot{U}H = \frac{c}{2}\dot{\sigma}^2 + V, \quad (13)$$

$$2U(2\dot{H} + 3H^2) + 4\dot{U}H + 2\ddot{U} + \frac{c}{2}\ddot{\sigma}^2 - V = 0, \quad (14)$$

and

$$c(\ddot{\sigma} + 3H\dot{\sigma}) + V_{,\sigma} = 6(\dot{H} + 2H^2) U_{,\sigma}, \quad (15)$$

where dots denote the time derivatives and $A_{,\sigma} = \frac{dA(\sigma)}{d\sigma}$ for any A .

We introduce the function

$$Y(t) = \frac{M_{\text{Pl}}}{\sqrt{2U}} \left(H + \frac{\dot{U}}{2U} \right). \quad (16)$$

and rewrite Eqs. (13)–(15) in the following equivalent form¹:

$$3M_{\text{Pl}}^2 Y^2 = \frac{A}{2} \dot{\sigma}^2 + V_{\text{eff}}, \quad (17)$$

$$\dot{Y} = - \frac{A\sqrt{U}}{\sqrt{2}M_{\text{Pl}}^3} \dot{\sigma}^2, \quad (18)$$

$$\ddot{\sigma} = -3\sqrt{\frac{2U}{M_{\text{Pl}}^2}} Y \dot{\sigma} - \frac{A_{,\sigma}}{2A} \dot{\sigma}^2 - \frac{V_{\text{eff},\sigma}}{A}, \quad (19)$$

where

$$V_{\text{eff}} = \frac{M_{\text{Pl}}^4 V}{4U^2}, \quad A(\sigma) = \frac{M_{\text{Pl}}^4}{4U^2} \left(c + \frac{3U_{,\sigma}^2}{U} \right). \quad (20)$$

¹M.A. Skugoreva, A.V. Toporensky and S.Yu. Vernov, Phys. Rev. D **90** (2014) 064044 [arXiv:1404.6226],

A.Yu. Kamenshchik, E.O. Pozdeeva, A. Tribolet, A. Tronconi, G. Venturi and S.Yu. Vernov, Phys. Rev. D **110** (2024) 104011 [arXiv:2406.19762].

THE EINSTEIN FRAME

Let us demonstrate that the function $Y(t)$ is the Hubble parameter in the Einstein frame as a function of the cosmic time in the Jordan frame. The conformal transformation

$$g_{\mu\nu} = \Omega^2 g_{\mu\nu}^E \quad (21)$$

with $\Omega^2 = \frac{M_{\text{Pl}}^2}{2U}$ gives the GR action with a minimally coupled scalar field. We express the Einstein frame Hubble parameter H_E through Jordan frame variables,

$$H_E = \frac{d \ln(a_E)}{dt_E} = \Omega^{-1} \left(H + \frac{d \ln \Omega}{dt} \right) = \frac{M_{\text{Pl}}}{\sqrt{2U}} \left(H + \frac{\dot{U}}{2U} \right) = Y. \quad (22)$$

The last expression is nothing else, but the variable Y introduced in Eq. (16) without any connection to the Einstein frame.

SLOW-ROLL PARAMETERS

Using the Einstein frame slow-roll parameters and the conformal transformation, we can define slow-roll parameters in the Jordan frame. The slow-roll parameter

$$\varepsilon^{(E)} \equiv -\frac{1}{H_E^2} \frac{dH_E}{dt_E} = \varepsilon_1 + \frac{\zeta_1(1-\varepsilon_1)}{2+\zeta_1} - \frac{2\zeta_1\zeta_2}{(2+\zeta_1)^2}, \quad (23)$$

where the slow-roll parameters in the Jordan frame are

$$\varepsilon_1 = -\frac{\dot{H}}{H^2} = \frac{1}{2} \frac{d \ln(H^2)}{dN}, \quad \varepsilon_n = \frac{\dot{\varepsilon}_{n-1}}{H\varepsilon_{n-1}} = -\frac{d \ln(\varepsilon_{n-1})}{dN},$$

$$\zeta_1 = \frac{\dot{F}_{,R}}{HF_{,R}} = \frac{F_{,RR}\dot{R}}{HF_{,R}} = -\frac{d \ln(F_{,R})}{dN}, \quad \zeta_n = \frac{\dot{\zeta}_{n-1}}{H\zeta_{n-1}} = -\frac{d \ln(\zeta_{n-1})}{dN}.$$

So, there are two sets of the slow-roll parameters in the Jordan frame. To describe cosmic evolution during inflation, we use the e-folds number $N = -\ln(a/a_e)$, where a_e is a constant.

So, we connect slow-roll parameters in the Einstein and Jordan frames:

$$\varepsilon^{(E)} \approx \varepsilon_1 + \frac{1}{2}\zeta_1 - \frac{\zeta_1}{4}(\zeta_1 + 2\zeta_2 + 2\varepsilon_1) = \frac{1}{4}\zeta_1(\zeta_1 - 4\varepsilon_1) \approx 3\varepsilon_1^2 + \mathcal{O}(\varepsilon_1^3), \quad (24)$$

$$\eta^{(E)} = \varepsilon^{(E)} + \frac{1}{2} \frac{d \ln(\varepsilon^{(E)})}{dN_E} \approx -\varepsilon_2 \frac{dN}{dN_E} + 3\varepsilon_1^2 + \mathcal{O}(\varepsilon_1^3). \quad (25)$$

The relation

$$\frac{dN}{dN_E} = \frac{2}{2 + \zeta_1} \approx 1 + \varepsilon_1 \quad (26)$$

gives

$$\eta^{(E)} \approx -\varepsilon_2 - \varepsilon_1 \varepsilon_2 + 3\varepsilon_1^2 + \mathcal{O}(\varepsilon_1^3). \quad (27)$$

The known slow-roll approximation in the Jordan frame² includes the equation

$$H^2 \approx \frac{V}{6U}.$$

We propose to use another slow-roll equation:

$$Y^2 \approx \frac{V_{\text{eff}}}{3M_{\text{Pl}}^2}. \quad (28)$$

After differentiating this equation with respect to t and using Eq. (18), we obtain the second slow-roll equation

$$\dot{\sigma} \approx - \frac{M_{\text{Pl}} V_{\text{eff},R}}{3YA\sqrt{2F_{,R}}}. \quad (29)$$

These equations allow us to get H^2 and all slow-roll and inflationary parameters as functions of σ .

²D.I. Kaiser, Phys. Rev. D **52** (1995) 4295.

$F(R)$ GRAVITY MODELS

We consider a generic metric $F(R)$ gravity model (10) and assume $F_{,R}(R) > 0$.

In terms of the function $F(R)$ and its derivatives, we obtain

$$Y = \frac{M_{\text{Pl}}}{\sqrt{2F_{,R}}} \left(H + \frac{F_{,RR}\dot{R}}{2F_{,R}} \right), \quad (30)$$

$$A = \frac{3M_{\text{Pl}}^4 F_{,RR}^2}{4F_{,R}^3}, \quad V_{\text{eff}} = \frac{M_{\text{Pl}}^4}{4F_{,R}^2} (RF_{,R} - F). \quad (31)$$

Equations (17) and (18) take the following form:

$$3M_{\text{Pl}}^2 Y^2 = \frac{A}{2} \dot{R}^2 + V_{\text{eff}}, \quad (32)$$

$$\dot{Y} = - \frac{A\sqrt{2F_{,R}}}{2M_{\text{Pl}}^3} \dot{R}^2. \quad (33)$$

In the proposed slow-roll approximation, we get for $F(R)$ gravity:

$$Y^2 \approx \frac{V_{\text{eff}}(R)}{3M_{\text{Pl}}^2}, \quad (34)$$

$$R' \approx -\frac{2F_{,R}(RF_{,R} - 2F)}{F_{,RR}(2RF_{,R} - F)}. \quad (35)$$

From these equations, we get

$$\zeta_1(R) \approx \frac{2(RF_{,R} - 2F)}{2RF_{,R} - F}, \quad (36)$$

$$H^2(R) \approx \frac{(2RF_{,R} - F)^2}{54F_R(RF_R - F)}, \quad (37)$$

and

$$\varepsilon_1 = 2 - \frac{R}{6H^2} \approx 2 - \frac{9F_{,R}R(RF_{,R} - F)}{(2RF_{,R} - F)^2}. \quad (38)$$

In this approximation, we obtain

$$\varepsilon_1 = -\frac{1}{2}\zeta_1 + \frac{1}{4}\zeta_1^2. \quad (39)$$

INFLATIONARY PARAMETERS

Using Eqs. (8), (24), and (39), we connect the inflationary parameters and the slow-roll parameters in the Jordan frame:

$$r \approx 16\varepsilon^{(E)} \approx 4|\zeta_1(\zeta_1 - 4\varepsilon_1)| \approx 48\varepsilon_1^2, \quad (40)$$

$$n_s \approx 1 - 4\varepsilon^{(E)} + 2\eta^{(E)} \approx 1 - 2\varepsilon_2 - 6\varepsilon_1^2 - 2\varepsilon_1\varepsilon_2 \approx 1 - 2\varepsilon_2, \quad (41)$$

$$\alpha_s \approx -2\varepsilon_2\varepsilon_3, \quad (42)$$

$$\alpha_t \approx 2 \frac{d\varepsilon^{(E)}}{dN_E} \approx -12\varepsilon_1^2\varepsilon_2, \quad (43)$$

$$A_s = \frac{H_E^2}{8\pi^2 M_{\text{Pl}}^2 \varepsilon^{(E)}} \approx \frac{H^2}{48\pi^2 F_{,R} \varepsilon_1^2} \approx \frac{H^2}{\pi^2 r F_{,R}}.$$

In these formulae, we have kept only the leading contributions.

OBSERVATION DATA

Recent CMB observations due to Atacama Cosmology Telescope
T. Louis *et al.* [ACT], JCAP **11** (2025) 062 [arXiv:2503.14452].
combined with the Dark Energy Spectroscopic Instrument (DESI) data
A. G. Adame *et al.* [DESI], JCAP **02** (2025) 021 [arXiv:2404.03002]
gives values of inflationary parameters

$$n_s = 0.974 \pm 0.003, \quad \alpha_s = 0.0062 \pm 0.0052. \quad (44)$$

These values are different from the Planck/BICEP data

$$n_s = 0.9651 \pm 0.0044, \quad \alpha_s = -0.0069 \pm 0.0069, \quad (45)$$

The ACT/DESI data does not significantly change the upper bound on r
and the value of A_s ,

$$A_s = (2.10 \pm 0.03) \times 10^{-9} \quad \text{and} \quad r < 0.028. \quad (46)$$

Thus, the ACT/DESI data favours a *higher* scalar spectral index n_s with
small *positive* running α_s .

To check the propose approximation we apply it to the well-known Starobinsky inflationary model³ with

$$F(R) = \frac{M_{\text{Pl}}^2}{2} \left(R + \frac{R^2}{6 m^2} \right). \quad (47)$$

We find in the SR approximation that

$$H^2 \approx \frac{(2m^2 + R)^2}{12(3m^2 + R)} \approx \frac{R}{12} \quad (48)$$

$$R' \approx \frac{4m^2(3m^2 + R)}{2m^2 + R} \approx 4m^2, \quad (49)$$

when $R \gg m^2$.

After solving Eq. (49), we get the running e-folds as

$$N = \frac{R}{4m^2} - \frac{1}{4} \ln \frac{3m^2 + R}{R_0}, \quad (50)$$

where R_0 is an integration constant.

³A.A. Starobinsky, *Phys. Lett. B* **91** (1980) 99.

The SR parameters of the Starobinsky model in the Jordan frame can be expanded during inflation with respect to the small variable

$$z = \frac{m^2}{R}. \quad (51)$$

Using $z \ll 1$, we find the leading terms as follows:

$$\zeta_1(z) = -4 \frac{z}{2z+1} \approx -4z, \quad (52)$$

$$\zeta_2(z) = 4 \frac{z(3z+1)}{(2z+1)^2} \approx 4z, \quad (53)$$

$$\zeta_3(z) = 4 \frac{z(4z+1)}{(2z+1)^2} \approx 4z, \quad (54)$$

$$\varepsilon_1(z) = 2 \frac{z(4z+1)}{(2z+1)^2} \approx 2z, \quad (55)$$

$$\varepsilon_2(z) = 4 \frac{z(6z+1)(3z+1)}{(2z+1)^2(4z+1)} \approx 4z, \quad (56)$$

$$\varepsilon_3(z) = 4 \frac{z(108z^3 + 74z^2 + 16z + 1)}{(2z+1)^2(4z+1)(6z+1)} \approx 4z. \quad (57)$$

We also obtain

$$n_s \approx 1 - 2\zeta_2 = 1 - 8z \frac{(3z + 1)}{(2z + 1)^2} \quad (58)$$

and

$$r \approx 12\zeta_1^2 = \frac{192z^2}{(2z + 1)^2} . \quad (59)$$

The end of inflation corresponds to $\varepsilon_1(R_{\text{end}}) = 1$, which yields

$$R_{\text{end}} \approx (\sqrt{5} - 1)m^2 \approx 1.2m^2 . \quad (60)$$

The initial value R_{in} of the spacetime scalar curvature at the beginning of inflation is related to the duration of inflation measured by e-folds $N_* = N_{\text{in}} - N_{\text{end}}$. Our results for the Starobinsky model are summarized in Table 1, while they agree with those derived in the Einstein frame,

Table: The values of n_s , r , R_{in} , and N_* in the Starobinsky model.

n_s	0.961	0.964	0.969	0.971	0.974
r	0.0043	0.0037	0.0027	0.0024	0.0019
$R_{\text{in}}/m^2 = 1/z_{\text{in}}$	209.6	226.7	262.6	280.4	312.2
N_*	51	55	64	69	77

In the leading order of the SR approximation, we find that

$$N_* \approx \frac{1}{4z} \approx \frac{1}{2\varepsilon_1} . \quad (61)$$

So, we reproduces the well-known results for the Starobinsky model,

$$n_s = 1 - \frac{2}{N_*} + \mathcal{O}(N_*^{-2}), \quad r = \frac{12}{N_*^2} + \mathcal{O}(N_*^{-3}) , \quad (62)$$

and

$$\alpha_s = -\frac{2}{N_*^2} + \mathcal{O}(N_*^{-3}), \quad \alpha_t = -\frac{3}{N_*^3} + \mathcal{O}(N_*^{-4}) . \quad (63)$$

As is clear from Table 1, it is possible to increase the value of n_s in the Starobinsky model by increasing the duration of inflation measured by the e-folds beyond 70 that is too high to be acceptable. It is impossible to get a positive running α_s without a modification of the model.

Since the ACT observations have merely $\sim 2\sigma$ tension with the predictions of the Starobinsky model, it is reasonable to study small deformations of the model in the context of $F(R)$ gravity for the limited range of the spacetime scalar curvature R relevant to inflation. For example, inflationary models defined by

$$F_3(R) = \frac{M_{\text{Pl}}^2}{2} \left(R + \frac{R^2}{6m^2} + \frac{\delta_3 R^3}{36m^4} \right) \quad (64)$$

and

$$F_4(R) = \frac{M_{\text{Pl}}^2}{2} \left(R + \frac{R^2}{6m^2} + \frac{\delta_4 R^4}{48m^6} \right) \quad (65)$$

with the dimensionless parameters δ_i , can give a suitable values of n_s , but $\delta_i < 0$, so $F_{,R}(R) < 0$ at large R .

A. L. Berkin and K.i. Maeda, Phys. Lett. B **245** (1990) 348

T. Saidov and A. Zhuk, Phys. Rev. D **81** (2010) 124002

V.R. Ivanov, S.V. Ketov, E.O. Pozdeeva and S.Yu. Vernov, JCAP **03** (2022) 058

A. Addazi, Y. Aldabergenov and S.V. Ketov, Phys. Lett. B **869** (2025) 139883

Models with

$$F(R) = \frac{M_{\text{Pl}}^2}{2} (R + \alpha R^\beta), \quad (66)$$

where $1 < \beta < 2$, can describe new observation values of n_s , but have problems at $R = 0$.

H. Motohashi, Phys. Rev. D **91** (2015) 064016,

S.D. Odintsov and V.K. Oikonomou, Phys. Lett. B **870** (2025) 139907.

New model without the above-mentioned problems has been proposed in V.R. Ivanov, arXiv:2508.14250. It has

$$F(R) = \frac{M_{\text{Pl}}^2}{2} \left[\left[1 - \frac{1}{3\delta} \right] R + \frac{1}{3\delta} \left[R + \frac{m^2}{\delta} \right] \ln \left(1 + \frac{\delta}{m^2} R \right) \right], \quad (67)$$

where δ is a dimensionless positive constant.

Note that $\alpha_s < 0$ for all known models.

THE PROPOSED $F(R)$ MODEL

Let us consider a new model by adding an R^5 -term as well, with

$$F_5(R) = \frac{M_{\text{Pl}}^2}{2} \left(R + \frac{1}{6m^2} R^2 + \frac{c_3}{m^4} R^3 + \frac{c_4}{m^6} R^4 + \frac{c_5}{m^8} R^5 \right), \quad (68)$$

where c_3 , c_4 and c_5 are the dimensionless coupling constants. In this model, we find ($z = \frac{m^2}{R}$):

$$\zeta_1 = - \frac{4(z^4 - c_3 z^2 - 2c_4 z - 3c_5)}{2z^4 + z^3 + 10c_3 z^2 + 14c_4 z + 18c_5}. \quad (69)$$

The observational bound $r < 0.028$ also implies

$$|\zeta_{1in}| < 0.048. \quad (70)$$

Hence, any term except the R^2 in Eq. (68) should not dominate, all the parameters c_i should be small, $|c_i| \ll 1$, whereas a small positive value of c_5 can be responsible for changing the sign of ζ_1 .

The stability conditions $F_{,R}(R) > 0$ and $F_{,RR}(R) > 0$ are satisfied for all positive R .

The effective potential V_{eff} has a maximum at $R > 150 m^2$, so that $R(t)$ is a monotonically decreasing function during inflation.

The values of n_s , r , and α_s are independent upon the value of m that is fixed by the observed value of A_s .

Table: The values of R_{in} (in units of m^2), N_* and r for $n_s = 0.974$ and $\alpha_s = 0.0062$.

c_3	c_4	c_5	R_{in}	N_*	r
3.874×10^{-4}	-2.583×10^{-6}	6.084×10^{-9}	130	64.2	0.0030
3.412×10^{-4}	-2.282×10^{-6}	5.355×10^{-9}	130	56.1	0.0040
3.073×10^{-4}	-2.068×10^{-6}	4.845×10^{-9}	130	51.1	0.0050
2.805×10^{-4}	-1.903×10^{-6}	4.459×10^{-9}	130	47.6	0.0060
2.377×10^{-4}	-1.529×10^{-6}	3.361×10^{-9}	140	49.0	0.0060

Using the SR approximation formulae, the values of c_i can be chosen to meet the ACT favored values of n_s and α_s .

Our results are summarized in Table 2.

Those results are robust against small changes in the values of c_i , while R_{in} can be adjusted to keep N_* within the desired range.

- We propose a new slow-roll approximation for $F(R)$ gravity inflationary models. New slow-roll approximation are based on the use of the function $H(R)$. Similar approximation for models with nonminimally coupled scalar fields has been proposed in [E.O. Pozdeeva, M.A. Skugoreva, A.V. Toporensky, S.Yu. Vernov, *JCAP* **05** \(2025\) 081, *arXiv:2502.13008*](#).
- The recent ACT/DESI observational data disfavors the Starobinsky inflation model by 2σ and thus motivates a search for its modifications. The simplest modifications including only one additional term proportional to R^3 or R^4 can provide the desired increased value of the cosmological tilt n_s of scalar perturbations but the running index α_s remains negative.

- We proposed and studied a new inflation model that agrees with the ACT constraints to the CMB observables for some values of the model parameters.
- To the best of our knowledge, the proposed model is the first $F(R)$ -gravity-based model of inflation with a positive running index α_s and 1σ -agreement with the ACT/DESI data. The other inflationary parameters are in agreement with the observations as well.

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