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Finite-Size Effects on the Density-Driven Deconfinement Phase Transition in QCD

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INTRODUCTION & AIM

The phase transition from confined hadronic matter to a deconfined quark-gluon plasma (QGP) is a fundamental prediction of Quantum Chromodynamics (QCD) [1]. Such a transition is expected to occur under extreme conditions of high temperature and/or large quark chemical potential. While temperature-driven deconfinement transitions have been extensively investigated [2–4], density-driven transitions in finite systems remain less explored [5], despite their relevance to ultra-relativistic heavy-ion collision (URHIC) experiments, where the produced matter is finite in size and short-lived.

In this context, the phase coexistence model (PCM) provides an effective framework for describing the mixed phase of hadronic matter and QGP [6]. The QGP sector is modeled using a modified MIT bag model that explicitly incorporates the color-singletness constraint [6, 7], ensuring global color neutrality and yielding a more realistic description of the confinement–deconfinement transition in finite volumes.

In the present work, we investigate the behavior of the order parameter, defined as the mean value of the volume fraction of the hadronic phase, together with its chemical susceptibilities. In particular, we analyze the first-order chemical susceptibility and the corresponding second-order derivative with respect to the quark chemical potential (μ). Extending earlier studies [3–5], we systematically examine finite-volume effects on the density-driven deconfinement transition. By analyzing the order parameter and its chemical derivatives within a phase coexistence model, we can measure the effect of the system size on the transition region width and the location of the effective transition point.

METHOD

We use in the following our result obtained in [5] for the partition function (PF) of the mixed HG-QGP system, in the state h defined in the phase coexistence model. The total PF is given by:

$$Z(h,T,\mu,V) = \frac{4}{9\pi^2} exp\left(\frac{\pi^2}{30}VT^3\right)\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} d\varphi d\psi\, M(\varphi,\psi)\, exp[(1-h)\mathcal{R}(T,\mu,V;\varphi,\psi)], \quad (1)$$

where $M(\varphi,\psi)$ represents the Haar measur which has the following expression

$$M(\varphi,\psi) = \left[sin\left(\frac{1}{2}\left(\psi+\frac{\varphi}{2}\right)\right) sin\left(\frac{\varphi}{2}\right) sin\left(\frac{1}{2}\left(\psi-\frac{\varphi}{2}\right)\right)\right]^2, \quad (2)$$

and

$$\mathcal{R}(T,\mu,V;\varphi,\psi) = VT^3\left(g\left(\varphi,\psi,\frac{\mu}{T}\right) - \frac{\pi^2}{30} - \frac{B}{T^4}\right), \quad (3)$$

where $g\left(\varphi,\psi,\frac{\mu}{T}\right) = g_0(\varphi,\psi) + g_1\left(\varphi,\psi,\frac{\mu}{T}\right)$, with the function $g_0(\varphi,\psi)$ given by

$$g_0(\varphi,\psi) = \frac{\pi^2}{12}\left(\frac{21}{30}d_Q + \frac{16}{15}d_G\right) + \frac{\pi^2}{12}\frac{d_Q}{2}\sum_{q=r,b,g}\left\{\left[\left(\frac{\theta_q}{\pi}\right)^2 - 1\right]^2 - 1\right\} - \frac{\pi^2}{12}\frac{d_G}{2}\sum_{G=1}^{4}\left[\left(\frac{\theta_G-\pi}{\pi}\right)^2 - 1\right]^2, \quad (4)$$

here, d_Q and d_G being the degeneracy factors of quarks and gluons respectively. The angles $\theta_q(q = r;\ b;\ g)$ are given by:

$$\theta_r = \frac{\varphi}{2} + \frac{\psi}{3}, \theta_b = \frac{-\varphi}{2} + \frac{\psi}{3}, \theta_g = \frac{2\psi}{3}, \quad (5)$$

and $\theta_G(G = 1, \dots, 4)$ are expressed as follows:

$$\theta_1 = \theta_r - \theta_g, \theta_2 = \theta_g - \theta_b, \theta_3 = \theta_b - \theta_r, \theta_4 = 0, \quad (6)$$

the function $g_1(\varphi,\psi)$ is given by

$$g_1\left(\varphi,\psi,\frac{\mu}{T}\right) = \left(1 - \frac{\varphi^2}{2\pi^2} - \frac{2\psi^2}{3\pi^2}\right)\left(\frac{\mu}{T}\right)^2 + \frac{1}{2\pi^2}\left(\frac{\mu}{T}\right)^4. \quad (7)$$

The main quantities of interest in this study are the order parameter and its first and second derivatives with respect to chemical potential. First, we need to derive the order parameter $\langle h(T,\mu,V)\rangle$ which represents here the mean value of the HG volume fraction, as defined in the Phase coexistence model, which may be written as

$$\langle h(T,\mu,V)\rangle = \frac{\int_0^1 h(T,\mu,V)Z(h,T,\mu,V)dh}{\int_0^1 Z(h,T,\mu,V)dh}. \quad (8)$$

The first and second order chemical susceptibilities are defined as the first and second derivatives of the order parameter $\langle h(T,\mu,V)\rangle$, with respect to the chemical potential:

$$\chi_\mu(T,\mu,V) = \left.\frac{\partial\langle h(T,\mu,V)\rangle}{\partial\mu}\right|_{T,V}, \quad (9)$$

$$\chi'_\mu(T,\mu,V) = \left.\frac{\partial^2\langle h(T,\mu,V)\rangle}{\partial\mu^2}\right|_{T,V}. \quad (10)$$

RESULTS & DISCUSSION

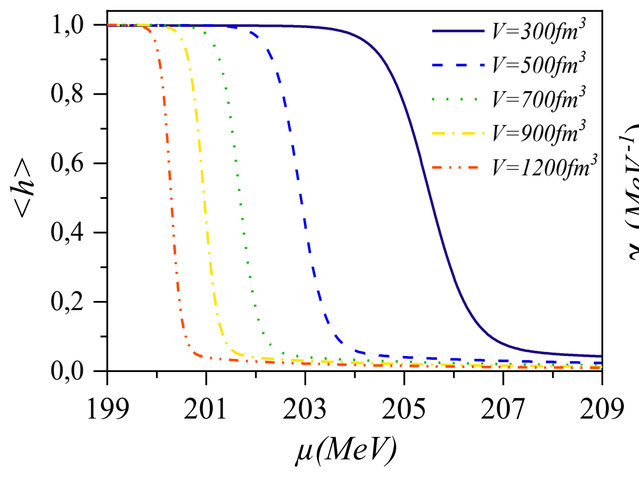

Fig 1. Variation of the order parameter $\langle h\rangle$ as function of chemical potential at $T = 125MeV$, for various volume selections with $B^{1/4} = 200MeV$.

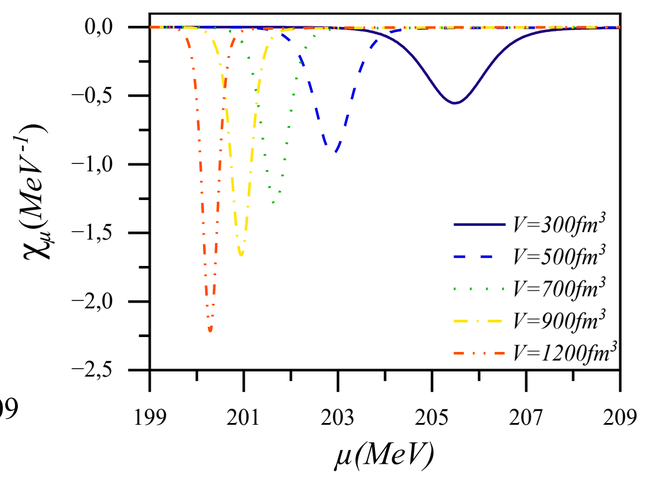

Fig 2. Variation of the chemical susceptibility χ_μ as function of chemical potential at $T = 125MeV$, for various volume selections with $B^{1/4} = 200MeV$.

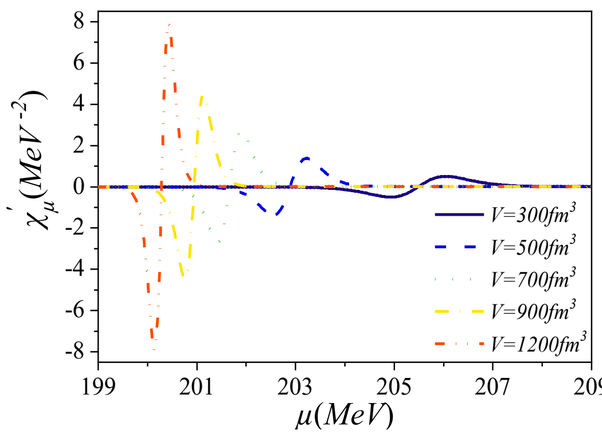

Fig 3. Variation of the second derivative of order parameter χ'_μ as function of chemical potential at $T = 125MeV$, for various volume selections with $B^{1/4} = 200MeV$.

The behavior of the order parameter $\langle h\rangle$ within the chemical potential interval surrounding the effective-transition point is illustrated in Fig 1, for the selected finite volumes. As the system volume decreases, the transition becomes progressively smoother, reflecting the suppression of critical fluctuations in finite systems. Correspondingly, in Fig, 2, the first-order chemical susceptibility, noted χ_μ exhibits pronounced peaks whose heights are reduced and whose positions are shifted with decreasing volume, signaling a modification of the effective critical point due to finite-size effects.

The behavior of the second-order chemical susceptibility, noted χ'_μ is illustrated in Fig. 3, which shows that in the transition region, the second-order chemical susceptibility exhibits sharp peaks reflecting critical fluctuations, which become more pronounced in large volumes. As the volume decreases, these peaks smooth out, reduce in amplitude, and shift position, signaling finite-size effects, color-singletness constraint, leading to an effective transition point shifted away from the true transition point at the thermodynamic limit.

Thus, as shown in Fig. 3, this second-order chemical susceptibility provides a sensitive probe of the deconfinement transition in finite systems under the influence of the color-singletness constraint.

CONCLUSION

Our results indicate that decreasing the system volume leads to a progressive smoothing of the order parameter across the transition region. In parallel, the first derivative exhibits pronounced peaks, while the second derivative shows rapid variations whose amplitudes and locations are strongly volume dependent. These features signal a modification of the effective transition point due to finite-size effects and highlight the role of exact conservation constraints in small systems. The behavior of the order parameter and its derivatives with chemical potential thus provides a reliable framework for characterizing the deconfinement transition in finite systems relevant to URHIC.

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