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1 BARYONIC TULLY-FISHER RELATION (a farewell to DM)

Flat rotation curves of galaxies suggest extra-gravity at galactic scales, which is usually explained by dark matter (DM). However, astrophysicists (P. Kroupa et al.) point to facts and data that contradict the DM hypothesis. Instead, the MOND model is advocated.

The baryonic Tully–Fisher relation $\log M_{bar} = c + b \log V_{flat}$ links the baryonic mass of spiral galaxies with their asymptotic velocity. MOND requires $b = 4$. M_{bar} includes stars, atomic and molecular gas (i.e. HI/HeI/H_2; for dwarfs, HII component is important because intergalactic radiation ionizes HI, as N.Gnedin notes). Conducting calculations is hard and requires assumptions and/or modeling. In [1] slope $b \approx 3$ was obtained for 32 spirals ($V_{flat} \approx 45...280$ km/s). Slope $b = 2$ appears in modeling [2;App.G] (except for dwarfs). And this slope prefers $F \sim M_{bar}/R$ gravity; some people called it MOGA, MOdified Gravity Attraction, [3]. MOGA appears also in one 5D-variant of Teleparallelism with 4th-order gravity and expanding brane-universe [4]: Newton's law transforms to $F/m = GM/(LR)$ when $R > L$ (L is brane's co-moving thickness along the extra-dimension).

[1] Ponomareva A.A. et al. From light to baryonic mass: the effect of the stellar mass-to-light ratio on the Baryonic Tully-Fisher relation // MNRAS. 2018. V.474. 4366. hal-02117129v1 .

[2] Valageas P., Schaeffer R. The mass and luminosity functions of galaxies and their evolution // Astron. Astrophys. 1999. V. 345, 329; arXiv: astro-ph/9812213.

[3] Toxvaerd S. Approximations and modifications of celestial dynamics tested on the three-body system. arXiv:2512.03823 [gr-qc] (see also arXiv:2403.02848v2).

[4] Zhogin I.L. Longitudinal waves in 5D Absolute Parallelism and relativistically expanding Brane cosmology. Space,Time and Fund. Inter. #2 (2025) 40.
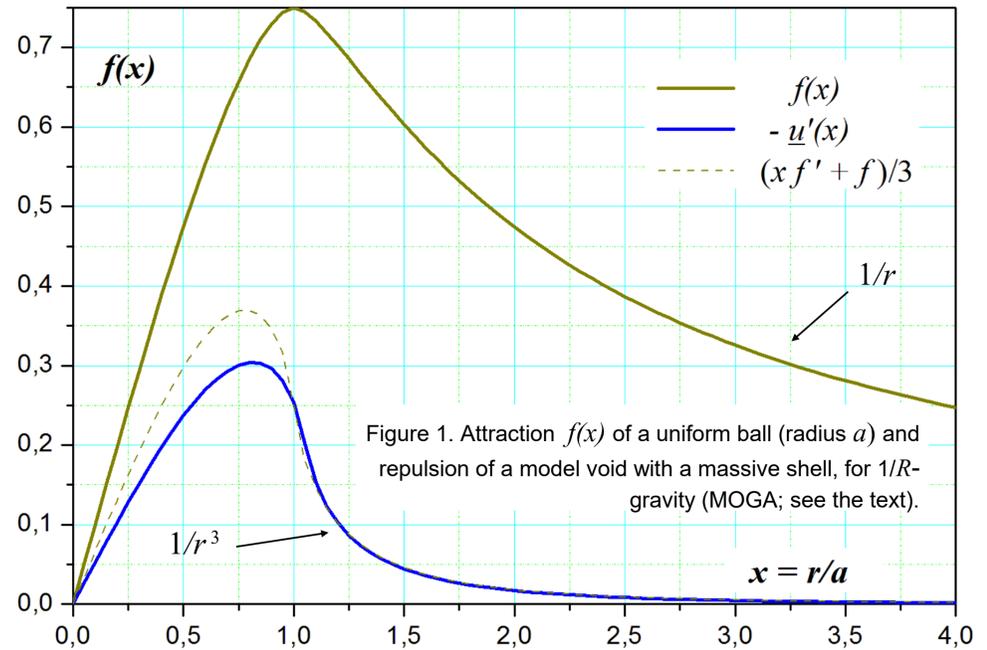
Figure 1. Attraction $f(x)$ of a uniform ball (radius a) and repulsion of a model void with a massive shell, for $1/R$-gravity (MOGA; see the text).

2 MOND vs MOGA (two variants of MOGA)

«Simple» MOGA suggests this correction of Newton's law: $F = GmM/R^2 (1 + R/L)$, see [3b].

AP-MOGA [4] (and references therein) gives smaller corrections at «small» distances ($R < L$):

$F = GmM/R^2(1+ \kappa R^4/L^4 + …)$ – Rindler's term can vanish; it could be better for the near observations, in the Solar system (κ depends on the mass distribution along the extra dimension).

MOGA, unlike MOND, preserves the momentum and angular momentum, as well as stability of three-body systems [3].

Note the interesting systems for comparing MOND and MOGAs (with $R \ll L$; $L \sim 5$ kpc):

☼ α Centauri – three nearest stars (and their planets; e.g., see Phys.org);

☼ wide binaries (some stars have extra velicities; see arXiv:2602.17884 [gr-qc] and references);

☼ ω Centauri star cluster (few stars are too fast; they involve a BH, Nature, V.631 (2024) 285).

Now we consider the large scales, $R > L$. For thin disks (SO_2-symmetry; simple $1/R$-model), the force in the disk plane at radius R depends on the mass M_R inside R: $F(R) = GM_R m / (LR)$; on the axis Z (uniform discs, radius a; 'test-mass' m): $F(Z)/m = GMZ/(La^2) \ln(1+a^2/Z^2)$.

For uniform spherical balls (bulges), one finds: $F(R) = Gmm/(La) f(R/a)$, where (see Figure 1)

$$f(x) = 3/8\{x + 1/x − 0.5(x−1/x)^2 \ln[(x+1)/|x−1|]\} = x\{1−x^2/5−...−3x^{2n}/[(2n−1)(2n+1)(2n+3)]−...\}. \quad (1)$$

One can further integrate (in parts) this equation and find the potential:

$$u(x) = \int f(x)\, dx = 0.5 \ln|x^2 − 1| + x^2/8 − (x^4 − 6x^2 − 3)/(16x) \ln[(x+1)/|x−1|] (+ C) . \quad (2)$$

Logarithmic potential growth stops when «underdensities» appear, and voids should be objects with negative masses. Our MW galaxy resides in the KBC void (one among the largest – redshift z is small!) with a shift from its center. Defocussing caused by the KBC-void provides some extra-dipoles (redshift-dependent).

3 SPHERICAL VOIDS WITH A MASSIVE SHELL

Let us consider one simple model of a void with a massive shell, which involve two co-centeered uniform balls with positive and negative densities, ρ_b and $−\rho_a$, of radius b and a .

The total mass of the shell is $M_s = \rho_b V_b − \rho_a V_a = \rho_0 V_b$, where $\rho_0 \sim 0.2 \rho_c$ is the mean baryonic density of the Universe. Let, for simplicity, $\rho_b = \rho_a$ (totaly empty void), and $b = \varepsilon a$, $\varepsilon = 1.1$.
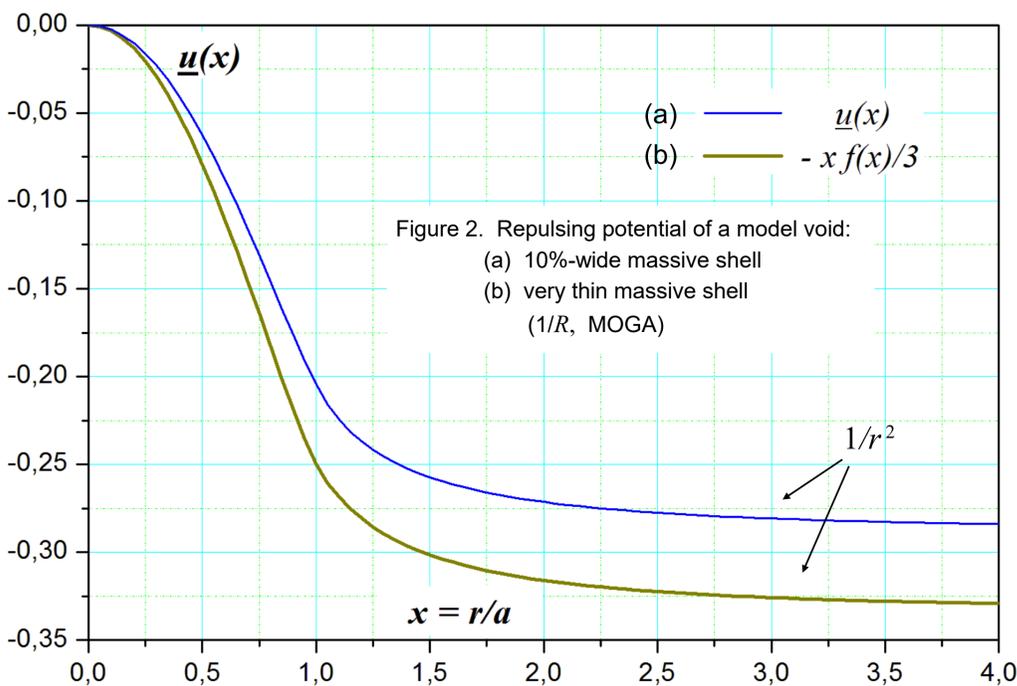

Figure 2. Repulsing potential of a model void:
(a) 10%-wide massive shell
(b) very thin massive shell
(1/R, MOGA)

One can can find the following expressi1on for the void potential [again, $x = r/a$; see (2)]

$$U(x) = GM_s /L\, \underline{u}(x) \text{ where } \underline{u}(x) = [u(x/\varepsilon) − u(x)]/(\varepsilon^3 − 1). \quad (3)$$

We took into account that the uniform density ρ_0 gives no gravity, no potential (the case $a \to 0$).

In the case of very thin massive shell when $a \to b$ and $\varepsilon \to 1$ we obtain (see Figures 2 and 1):

$$\underline{u}(x) \to − x f(x) / 3 . \quad (4)$$