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Cosmological Constant or Cosmological Curvature Parameter? A Conformal Reinterpretation of Λ

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INTRODUCTION & AIM

1. Background

Observations across multiple cosmological probes indicate that the Universe possesses a non-zero cosmological constant. However, the large discrepancy between observationally inferred and theoretically predicted vacuum energy densities, together with recent analyses in which combined SNe Ia, H(z), BAO, LSS, BBN, and CMB datasets mildly prefer models with a time-dependent cosmological parameter [1].

This study re-examines the assumptions underlying Λ using interaction field equations derived from an action incorporating bulk influence, which reduce to the Einstein field equations in the flat-background limit.

METHOD

2. Extended Action (The Bulk – Cloud Interaction)

In the modern view, General Relativity (GR) is the leading term in the low-energy effective field theory of gravity [2]. As GR couples spacetime geometry to all stress–energy sources, including vacuum contributions, vacuum effects generically induce an effective cosmological-constant term, sharpening the cosmological constant problem.

Our bimetric framework, inspired by the theory of Elasticity, instead promotes vacuum energy to a dynamical background bulk with its own intrinsic curvature $\mathcal{R}$ and associated Lagrangian density $\mathcal{L}$. The vacuum's resistance to deformation is encoded by a scalar stiffness modulus $E_D \propto \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$ representing vacuum energy density. The extended action is:

$$S = E_D \int_C \left[\frac{R}{\mathcal{R}} + \frac{L}{\mathcal{L}}\right]\sqrt{-g}\, d^4x \qquad (1)$$

The interaction between the vacuum bulk and the active geometry is

$$S = \int_B k\left[-\frac{1}{4}\mathcal{F}_{\mu\nu}\tilde{g}^{\beta\nu}\mathcal{F}_{\alpha\beta}\tilde{g}^{\alpha\mu}\right]\sqrt{-\tilde{g}}\int_C\left[\frac{R_{\mu\nu}g^{\mu\nu}}{\mathcal{R}_{\mu\nu}\tilde{g}^{\mu\nu}} + \frac{L_{\mu\nu}g^{\mu\nu}}{\mathcal{L}_{\mu\nu}\tilde{g}^{\mu\nu}}\right]\sqrt{-g}\, d^4x\, d^4\xi \qquad (2)$$

The first integral represents the vacuum field strength tensor $\mathcal{F}_{\mu\nu}$ coupled to the background bulk metric $\tilde{g}_{\mu\nu}$. The second integral operates over the Cloud (C) coordinates x, normalizing the physical curvature $R_{\mu\nu}$ and Lagrangian density $L_{\mu\nu}$ to their background counterparts $\mathcal{R}_{\mu\nu}, \mathcal{L}_{\mu\nu}$.

2.1 The Cloud Classical Field Equations (CCFEs)

Varying the dual action in Ref. [3] yields the interaction field equations:

$$\underbrace{G_{\mu\nu}}_{\text{Einstein Tensor}} - \underbrace{\frac{R}{\mathcal{R}}\mathcal{R}_{\mu\nu}}_{\text{Global Tensor}} + \underbrace{\mathcal{K}\left[\left(K_{\mu\nu} - K\hat{p}_{\mu\nu}\right) - \frac{R}{\mathcal{R}}\left(\mathcal{K}_{\mu\nu} - \mathcal{K}\hat{q}_{\mu\nu}\right)\right]}_{\text{Geometric Slip}} = \underbrace{\frac{\mathcal{R}}{\mathcal{T}}}_{\text{Coupling}\left(\frac{\text{Bulk Curvature}}{\text{Vacuum Density}}\right)} \underbrace{\left(T_{\mu\nu} + \mathcal{K}\tau_{\mu\nu}\right)}_{\text{Matter and Vacuum Stresses}} \qquad (3)$$

K is the extrinsic curvature and $\hat{p}_{\mu\nu}$ is the induced metric on boundary.

2.2 Global Tensor, Scalar Mode Dynamics & Vainshtein Unlocking

In a maximally symmetric background, $\mathcal{R}_{\mu\nu}/\mathcal{R} = \sigma\tilde{g}_{\mu\nu}$, with σ a geometric factor. The deviation of the physical metric $g_{\mu\nu}$ from the bulk's metric $\tilde{g}_{\mu\nu}$, can be investigated using the scalar degree of freedom $\pi(x)$ inherent in the conformal mismatch between these two metrics: $\tilde{g}_{\mu\nu} = e^{2\pi(x)}\, g_{\mu\nu}$. Substituting this relation into the geometric action (Eq.1) and isolate the kinetic sector for the scalar mode π:

$$S_\pi \propto \frac{E_D}{\mathcal{R}}\int d^4x\sqrt{-\tilde{g}}\,(\partial\pi)^2 \Rightarrow Z(\beta, \partial^2\pi)\,\Box\pi \approx T/M_{pl} \qquad (4)$$

Varying the action with respect to π gives a non-linear equation of motion of the Galileon type (Eq.4), where Z is the effective kinetic stiffness of the scalar field, governed by β. In high density regions, $\beta \gg 1 \Rightarrow \Box\pi \approx 0$, effectively locking the conformal mode yielding Eq. (5), the GR limit. On cosmological scales, $\beta \approx 1$, the conformal relation becomes dynamically active. The stiffness drops leading to the background-softened in Eq. (6).

RESULTS & DISCUSSION

The kinetic stiffness of the conformal mode $\pi(x)$, set by $\beta = R_{\text{local}}/\mathcal{R}_{\text{bulk}}$, governs two distinct physical regimes (Figure 1). Accordingly, the CCFEs can be expressed as effective field equations in a symmetric background:

2.3 The Screened State (High Density − GR Limit, $\beta \gg 1$)

$$R_{\mu\nu} - \frac{1}{2}R\underbrace{\left(g_{\mu\nu} + \frac{1}{2}\tilde{g}_{\mu\nu}\right)}_{\text{Background Stiffening}} + \underbrace{\mathcal{K}S_{\mu\nu}}_{\text{Geometric Slip}} = \underbrace{\frac{16\pi G}{c^4}}_{\text{Effective Coupling}}\left(T_{\mu\nu} + \mathcal{K}\tau_{\mu\nu}\right) \qquad (5)$$

2.4 The Unlocked State (Low Density − Holographic limit, $\beta \approx 1$)

$$R_{\mu\nu} - \frac{1}{2}R\underbrace{\left(g_{\mu\nu} - \frac{1}{4}\tilde{g}_{\mu\nu}\right)}_{\text{Background Softening}} + \mathcal{K}S_{\mu\nu} = \frac{4\pi G}{c^4}\left(T_{\mu\nu} + \mathcal{K}\tau_{\mu\nu}\right) \qquad (6)$$

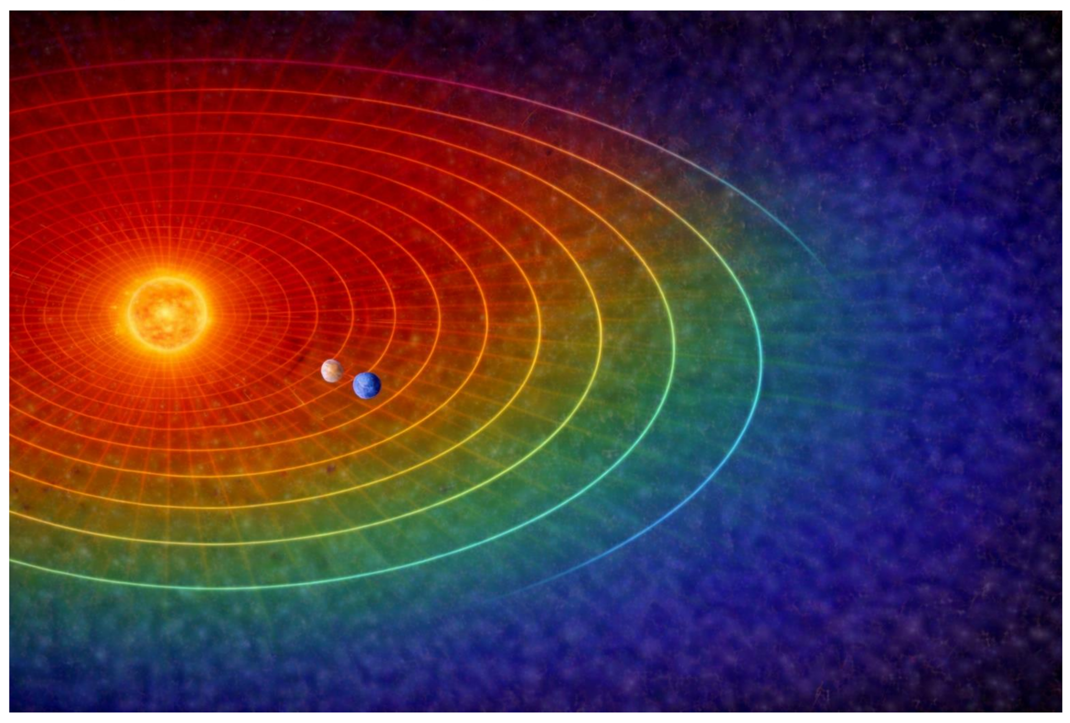

Figure 1:Schematic of local $\beta \gg 1$ to deep-space limit $\beta \approx 1$ (not to scale).

3. The CCFEs-de Sitter Identity and Modified Friedmann Dynamics

We consider the vacuum field equations (6) in the presence of geometric cosmological parameter Λ_g, acting as bulk geometric pressure scaling the effective metric: $R_{\mu\nu} - \frac{1}{2}R\left(g_{\mu\nu} - \frac{1}{4}\tilde{g}_{\mu\nu}\right) + \Lambda_g\left(g_{\mu\nu} - \frac{1}{4}\tilde{g}_{\mu\nu}\right) = 0 \Rightarrow R = 6\Lambda_g$. For flat FLRW, $R = 6/c^2\left(\dot{H} + 2H^2\right)$. Taking the trace of Eq.6 gives:

$$\dot{H} + 2H^2 = c^2\Lambda_g + \frac{4\pi G}{c^4}\left(\rho c^2 - 3P\right) \Rightarrow H^2 = \frac{c^2}{2}\Lambda_g \Rightarrow L_H = \frac{c}{H} = \sqrt{2/\Lambda_g} \qquad (8)$$

4. CCFEs Predictions and Observations

Observation	Empirical Value	ΛCDM (Standard)	CCFES (Derived)	Comparison Status
Hubble Radius ($\bar{L}_H$)	1.37×10^{26} m	1.65×10^{26} m ($L = \sqrt{3/\Lambda}$)	1.35×10^{26} m ($L = \sqrt{2/\Lambda_g}$)	98.5%
Growth Rate ($f\sigma_8$)	0.76 ± 0.04	0.83 (Planck Tension)	$0.75 - 0.77$	Alleviates Tension
BAO Scale Evolution	150 Mpc	150 Mpc (Numerical Fit)	148 Mpc	Compatible
CMB 1st Acoustic Peak	$l \approx 220$ [5]	$l \approx 220$ (Numerical Fit)	$l \approx 215$	Approximate match
Horizon Temp.	$\sim 2.7 \times 10^{-30}$ K	Input Parameter	2.7×10^{-30} K	Theoretical agreement

CONCLUSION & FUTURE WORK

In this framework, Λ_g is interpreted as a normalized bulk curvature scale rather than a vacuum energy density, thereby reframing the Λ problem as a geometric misidentification rather than a fine-tuning issue. Furthermore, the coupling relation in Eq. (3), $\mathcal{R}/\mathcal{T} \equiv 8\pi G/c^4$ implies that higher local curvature induced in the bulk corresponds to a proportionally higher effective local vacuum energy density. Future work will implement the perturbation equations in a Boltzmann solver (e.g., CLASS/CAMB).

REFERENCES

[1] Solà Peracaula, J. (2023). The cosmological constant problem and running vacuum in the expanding universe. Universe, 9(6), 262.
[2] Donoghue, J. F. (1994). General relativity as an effective field theory. Ph. Rev. D, 50, 3874–3888.
[3] Al-Fadhli, M. B. (2023). Gravitational, electromagnetic and quantum interaction: From string to cloud theory. Physical Sciences Forum, 7(1), 55.
[4] Randall, L. (1999). An alternative to compactification. Physical Review Letters, 83, 4690–4693.
[5] N. Aghanim et al. (2020) Planck 2018 results. VI. Astron. Astrophys., vol. 641, p. A6.