The Public Opinion Evolution dynamics on Scale Free Network in the External Field

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Abstract：In the paper considering the influence of node's inertia and a constant external field, we establish an public opinion evolution model in which public opinion evolution and network topological structure interact with each othe. It is found that degree distribution gradually gets away from typical power law distribution to Poisson distribution with time evolution under the infuence of node's inertia and a constant external field. With the time evolution in system, there is obvious convergence effect of the public opinions distribution, which not only relates with node's inertia but also depends on the constant external field. It can be obtained that adjusting the value of node's inertia factor γ and constant external field ϕ_0 can control the opinion value's number, which even controls the rate of change.

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1. Introduction

Different people have different ideas or attitudes to some topics of politics, entertainment, health, and so on. Therefor, People give them different concern. It is defined as public opinion to a topic, which are characterized by the serial number in

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the paaper. The society system is formed from different relationships of people in reality. To study the complicated phenomenon of society system, we consider the society system as a complex network, and shortly named a social network. In the system people will adjust its own opinion to adapt social development through people's contacting and influencing each other, which is named public opinion dynamics.

Recently, the research on opinion evolution dynamics becomes a hot point [1-9]. Actually it is a cross-cutting area between natural science and social science. There are some of well-known public opinion dynamics models: Sznajd model [10], distributioneffuant et al model [11], Krause-Hegselmann model [12].

These public opinion evolution mechanisms can be sorted to two kinds: one is that every individual's public opinion is based on its familiar neighbors， which make up of a small group. Another is that the small group is made up of people having same public opinion, even they are not familiar neighbors each other. But in reality the public opinion evolution mechanism is not one of them but mixture of them. In 2006, Petter holme and M.J.Newman give a new public opinion model based on small world, which is synthesized evolution dynamics[13].

Our society is an open complicated system, and people's social activities are offen effected by external circumstance, such as political ads, national policy and so on. We look these external affect as external fields. The evolutional law of public opinion dynamics is investigated on scale free network in which the topology structure is always changing in this paper. Here the influence of a constant external field ϕ_0 and node's inertia $e^{\gamma k_h}$ on public opinion evolution are considered, and the public opinion evolution parameter is defined as $\phi_0 e^{\gamma k_h}$. It is indicated that public opinion evolution and network topological structure interact with each other, and they are all influenced by the external field ϕ_0 and the node's inertia factor γ.

2. The model

g_h is the serial number representing the *h* node's public opinion(g_h =1,2…,M, M is Integer), which terms different views about some topic. In our paper, M is 50 respecting fifity kinds of views. The initial network is BA Scale-Free network[14]. The sum of node is N=20000, amd the average degree value is $\bar{k} = 8$.

It is defined the strength of external field as a constant ϕ_0, which reflects the external field similar affect on nodes' public opinion in the network. ϕ_0 is a real number varying from 0 to 1. So it is a outside factor for public opinion evolution. Different individual own different ability to gather and deal with info in reality, so node's inertia is defined as $e^{\gamma k_h}$ (*k* is *h* node's degree value), where γ is the adjustment parameter of node's inertia. Therfore, node's inertia reflects node's own trait influence on public opinion evolution, and it is a internal factor and relate with the stucture of network. It can be more obviously seen in the back that the node's inertia restrain the emergence of convergence phenomenon with larger γ when $\gamma > 0$ in general, and when $\gamma < 0$ it accelerate the emergence of convergence phenomenon. This is the reason we call it node's inertia.

At the beginning of public opinion evolution, every node is randomly given public opinion serial number. After that, we randomly choose one node *i* in every step, and randomly select another one *j* which is linked by node *i*. If the two nodes possess the same public opinion serial number, they will do nothing. If the two nodes have the different public opinion serial number, nodes *i* will be given to random evolution order parameter ϕ_i. Compared to the relationship between ϕ_i and $\phi_0\ e^{\gamma k_i}$, the system has two kinds of probable evolution.

(1) If $\phi_i \le \phi_0\ e^{\gamma k_i}$, the line connecting between *i* and *j* will be removed ,and randomly choose another remaining one to connect node *i*. For the remainder, it must has the same public opinion value ϕ_i as node *i* without connecting before, as shown in Fig.1 (a). In this case the network structure will change but not the public opinion.

(2) If $\phi_i > \phi_0\ e^{\gamma k_i}$, the serial number in the node *j* will be given to node *i*, as shown in FIG.1 (b). In this case the public opinion of node *i* will change, and the network structure will not change.

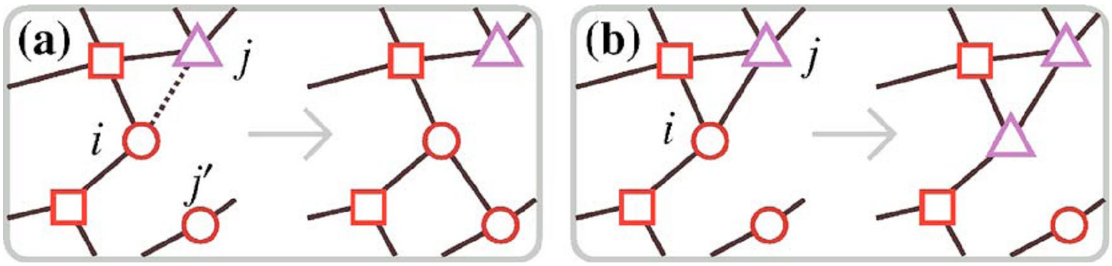


FIG..1 An illustration of the model, with vertex shapes representing different opinions. At each time step the system is updated according to the process illustrated in panel (a) and (b)

3. Results and discussion

In the following simulations cellular automaton method is used and periodic boundary conditions are considered. We have performed extensive numerical simulations for the initial conditions with fifty different public opinion uniformly distributing in the network. Results are averaged over 10^3 independent simulations.

3.1 The external field on the impact of public opinion distribution

According to the above regular, through numerical simulation we investigate the public opinion distribution changing with the adjustment parameter of node's inertia parameter γ, where τ is time step. The public opinion distribution

$$Q(g) = \frac{\text{the number of nodes with } g \text{ public opinion}}{\text{all the number of nodes}},$$

Where g is the serial number of public opinion. When τ is zero, the public opinion distribution is shown in Fig.2.

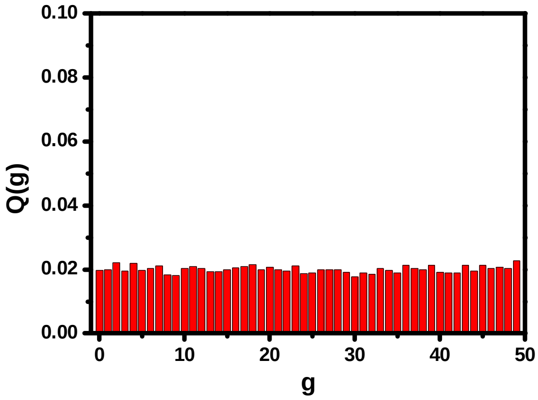


Fig.1 the initial public opinion distribution

As shown in Fig.2, the public opinion distribution $Q(g)$ is uniform and totally random before evolution. All of the public opinion distribution is about 0.02. In other words, the number of nodes with different public opinion is substantially the same.

The public opinion distribution with time evolution will change in the system under a constant external field and considering the impact of node's inertia. The public opinion distributions at the time stepτ=10000 are shown in Fig3-5, which represent different external field $\phi_0 = 0.1$, $\phi_0 = 0.5$, $\phi_0 = 0.9$. And the adjustment parameter of node's inertia are respectively (a) $\gamma = 0.3$; (b) $\gamma = 0$; (c) $\gamma = -0.1$; (d) $\gamma = -1$.

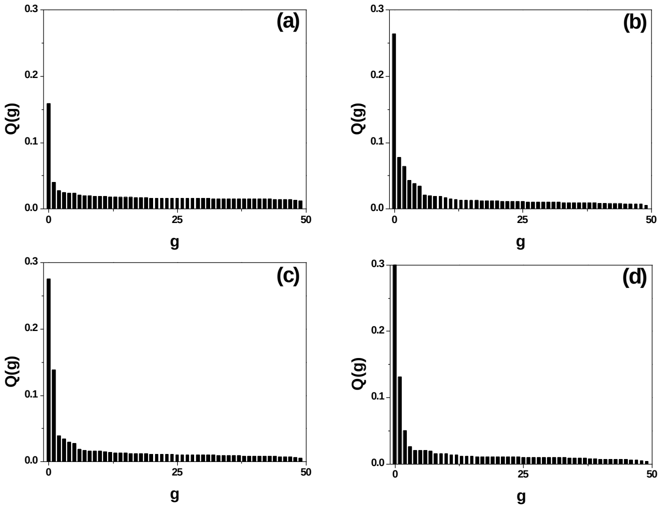


Fig.2 The public opinion distribution under the constant external field $\phi_0 = 0.1$

From Fig.3 it is shown that the public opinion distribution $Q(g)$ have obviously convergence effect with the time evolution in system under a constant external field $\phi_0 = 0.1$, and the convergence effect is relevent to the adjustment parameter of node's inertia γ. When $\gamma = 0$ without considering the influence of node's inertia in Fig.3(b), the public opinion distribution $Q(g)$ changed from the uniform distribution in initial (Fig.2) to non-uniform distribution. The largest value of Q with same public opinion serial number is 0.265, but the smallest Q is 0.003. And most of Q are 0.01. That is to say, even without the impact of node's inertia the public opinion distribution $Q(g)$ have obviously convergence effect with time evolution in system under a constant external field $\phi_0 = 0.1$. Therefore, the constant external field can lead to the uneven distribution.

When the adjustment parameter of node's inertia is 0.3 in system under the same constant external field $\phi_0 = 0.1$ as shown in Fig.3(a), at this time the public opinion distribution $Q(g)$ have great changes compared without considering node's inertia. We define the nodes having the same public opinion serial number as a association.

Here only one great association is about 0.158. The other associations are small, corresponding to 0.015. It is obtained that the non-uniformity distribution of public opinion is obviously smaller than that without considering node's inertia, which can fully show that the adjustment parameter of node's inertia at this moment have had a greater impact on the distribution of public opinion. The adjustment parameter of node's inertia γ plays an inhibitory effect of the convergence trend to the distribution of public opinion. In general this inhibitory effect is obvious when $\gamma > 0$。

When the adjustment parameter of node's inertia $\gamma = -0.1$ and $\gamma = -1$, as shown in Fig.3(c)and(d), there is obvious convergence effect of the public opinion distribution $Q(g)$ with the time evolution in system under a constant external field $\phi_0 = 0.1$. The cause to convergence effect is $e^{\gamma k_h}$ tending to zero as $\gamma < 0$. In this case the system have larger probability to implementation rule (2), so the system have more likely to change the public opinion of node but less likely to change the network structure. But the convergence effect of the public opinion distribution is almost no change compared with the case $\gamma = 0$ because of small constant external field, so there is no obvious impact on public opinion distribution with negative γ. That is to say, when constant external field is small, public opinion distribution have not significant difference in the case of negative γ and without considering node's inertia.

When the constant external field $\phi_0 = 0.1$ and $\phi_0 = 0.9$ as shown in Fig.4 and Fig.5，there is obvious convergence effect of the public opinion distribution $Q(g)$ with the time evolution in system. From Fig.4 and Fig.5 we can see that there is obvious convergence effect of the public opinion distribution under a constant external field when $\gamma = 0$ without considering node's inertia. When $\gamma = 0.3$ there isn't obvious convergence effect of the public opinion distribution, which will fully show that at this moment the node's inertia plays an inhibitory effect of the convergence trend to the public opinion distribution. When $\gamma = -0.1$ and $\gamma = -1$ in system, there is obvious convergence effect of the public opinion distribution under the constant external field. The cause to convergence effect is $e^{\gamma k_h}$ tending to zero as $\gamma < 0$. In this case the system have larger probability to implementation rule (2), so the system have more likely to change the public opinion of node but less likely to change the network structure. However, the convergence effect of the public opinion distribution is also obvious changes compared with the case $\gamma = 0$ because of large constant external field, so there is obvious impact on public opinion distribution with negative γ. That is to say, under large constant external field, public opinion distribution have significant difference in the case of negative γ and without considering node's inertia.

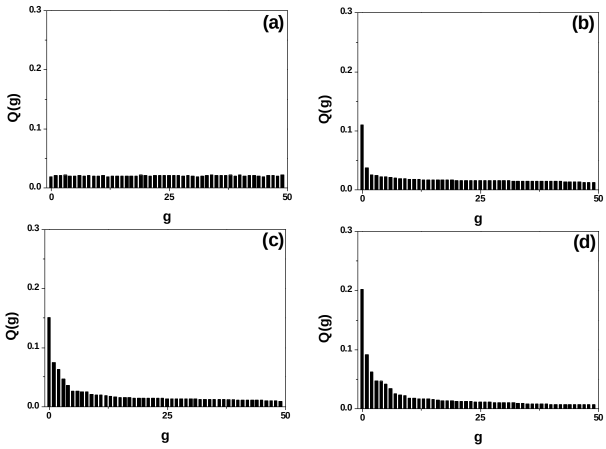


Fig.3 The public opinion distribution under the constant external field $\phi_0 = 0.5$

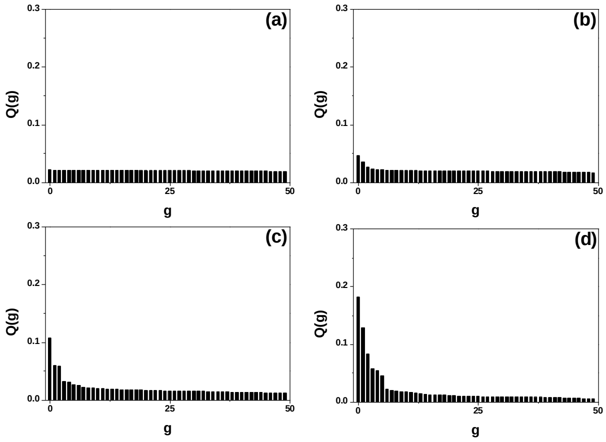


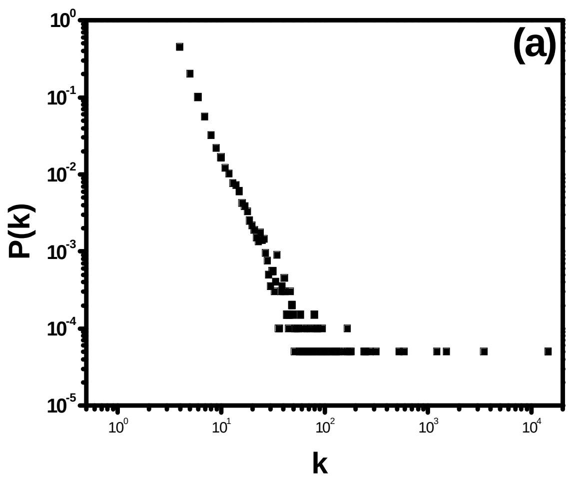
Fig.4 The public opinion distribution under the constant external field $\phi_0 = 0.9$

When the adjustment parameter of node's inertia γ is a constant, the convergence effect of the public opinion distribution $Q(g)$ with the time evolution in system under a constant external field may come into being. From Figs.3-5 it is shown that the public opinion distribution are becoming more and more uniform with the external field increasing, which can fully discript that the constant external field can control the convergence effect emergence of public opinion distribution. The cause to convergence effect is that the system under larger constant external field have larger probability to implementation rule (1) , so the system have more likely to change the network structure but less likely to change the public opinion of node, that is to say the public opinion of node have more likely to remain state.

In the all, the influence of nodes' inertia on the public opinion in system not only relates with the adjustment parameter of node's inertia γ but also depends on the constant external field ϕ_0, and the influence is very complicated. The time evolution in system under $\phi_0 < 1$ can result in the un-uniform distribution of public opinion (the convergence effect), and the different external field value have different results. The smaller external field ϕ_0 is, the greater heterogeneity of public opinion distribution is. In general the influence of nodes' inertia on the public opinion is that the the convergence effect of public opinion distribution can be strengthen with the adjustment parameter of node's inertia γ reducing, and with the greater value γ the inhibitory effect of the convergence trend to the public opinion distribution is more obvious. Two factors of node's inertia and constant external field acting on the system at the same time can lead to colorful results.

2.2 The influence of node's inertia on degree distribution

Network structure have changed with time evolution in system under constant external field considering node's inertia. Then we further investigate the influence of the adjustment parameter of node's inertia γ on degree distribution. In the following simulations, without loss of generality, constant external field $\phi_0 = 0.5$ We find that network structure trend to stable after 10^6 time steps. Degree distribution changed from initial power-law distribution to Poisson distribution in the stationary state. Fig.6 presents the degree distribution of network structure at $\tau = 0$ (Fig.6(a)) and $\tau = 10^8$ (Fig.6(b)). Here Log-Log plot is taken.



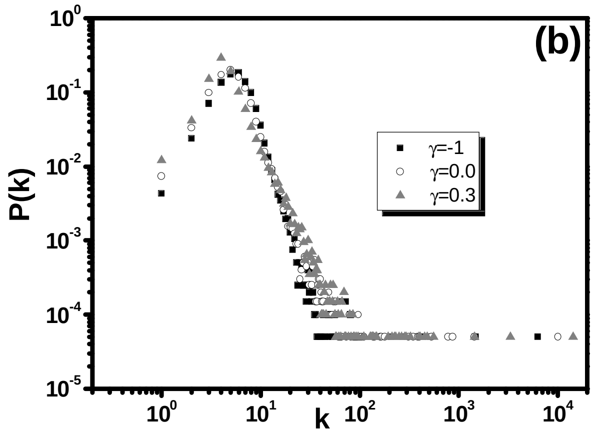
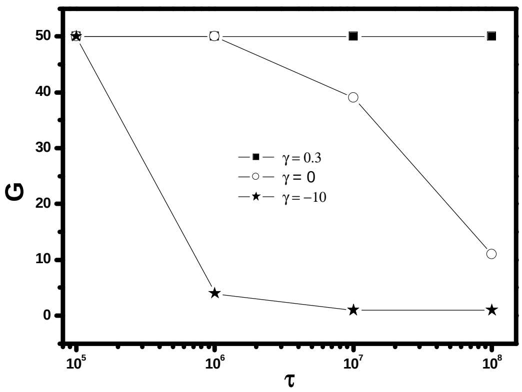


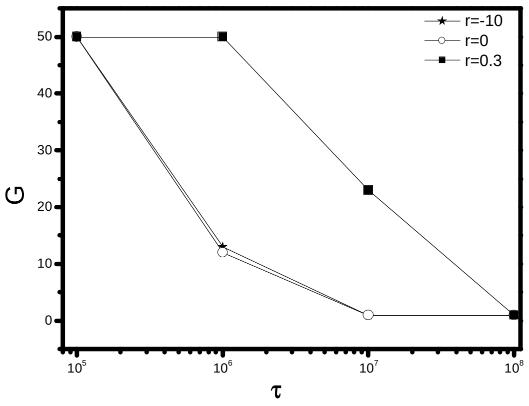
Fig.6 Degree distribution (a) $\tau = 0$; (b) $\tau = 1000000$

In initial state the network structure is BA scale-free network, so degree distribution in system exhibits a Power-law degree distibution as Fig.6.(a) shown. After $\tau = 10^8$ time step evolution of public opinion as shown in Fig.6.(b), it is found that degree distribution gradually gets away from typical power law distribution to Poisson distribution in our model. This is due to the network structure and the evolution of public opinion in system interacting and adapting with each other under the influence of the constant external field and node's inertia, which is in agreement with the previous simulation results in Fig.4.

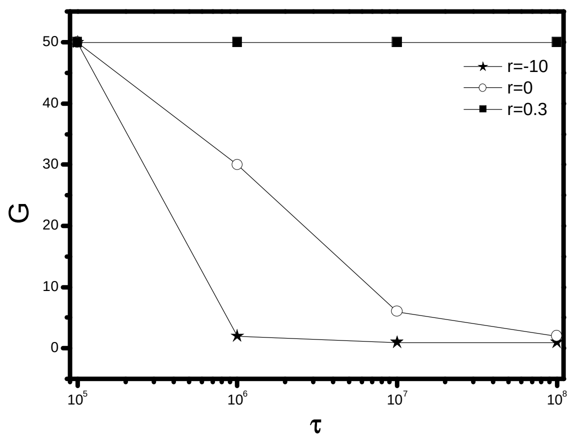
2.3 The time evolution of public opinion



(a) $\phi_0 = 0.5$



(b) $\phi_0 = 0.01$



(c) $\phi_0 = 0.1$

Fig.7 the time evolution of public opinion number under different adjustment parameter of node's inertia γ

From above results we know that the network structure and public opinion distribution in system under constant external field considering node's inertia all changed after 10^6 time step. Then we investigated the time evolution law of public opinion under different constant external field.

If not considering the influence of node's inertia, in the external field, the number of public opinion gradually decrease as time evolved. In other words there is con-vergenve trend. Then we researched, taking account of the influence of node's inertia, how public opinion would change along with time evolution. Fig. 7 shows the relationship between public opinion and time evolution, when the external field intensity values are 0.5, 0.01 and 0.1. And the horizontal axis represented the time step, expressed by τ; vertical axis that correspond to the time step of number public opinion, expressed by G.

As can be seen from Fig.7(a), in the four time steps 10^5、10^6、10^7and10^8, when γ=0, that is, the effect of node's inertia is not considered, when in the operation to step 10^6, basically the number of public opinion remains unchanged. But since then, a rapid decline in the number of public opinion until the step 10^8, the number of public opinion remained only 11, that is to say the evolution of the network itself must lead to some annihilation of public opinion. From our results, prior to step 10^6, though the evolution of public opinion causes its uneven distribution, the number of public opinion had not been changed. While after step 10^6, the evolution of public opinion will lead to reduction in the number of public opinion.

When γ =0, as time evolved, the number of public opinion in system remains unchanged, always 50, which showes that the system of the number of public opinion or the number of its corresponding associations would not be affected by the time evolution. And the result is identical to what can be seen in Figs. 2-4, the number of corresponding associations of public opinion uniformly distribute. That's because at this time the product of external field ϕ_0 and inertia $e^{\gamma k_i}$, the evolution of order parameter, in general is greater than or equal to 1, so that the implementation of the rules of network evolution are always the first step. that is only caused by changes in the network structure, without altering the network opinion distribution. Of course, the number of public opinion remained unchanged. That indicates the inertia inhibited its number of public opinion to change because of considering the effect of node's inertia, so at this time the inertia plays a role in inhibition.

At the time of $\gamma = -10$, which is equivalent to "negative inertia", it plays a role in accelerating convergence trend of the public opinion. From the figure we can see that as time increase, the number of public opinion gradually reduces, and it started to decrease faster, then slower, and finally tends to convergence, in the end there is only an opinion. It shows that when the inertia is small enough, the network evolution will lead to a rapid decline in the number of public opinion, and finally only an opinion. That is, convergence trend in Figs. 2-4 of the evolution over time will continue to strengthen, so that all public opinion will be the same. Another can be seen through Fig.7(a), at the same time step, with the decrease of γ, the number of public opinion is also reduced, therefore, at this moment the inertia accelerates the convergence trend effect of public opinion.

Here we must point out that there exists the least value γ, which lead to the total number of public opinion unchanged. In fact the least value γ relate with ϕ_0 and node’s degree k, which meet the equation $\phi_0 e^{\gamma k_i} = 1$. In our model the average degree is 8 in system, so when $\phi_0 = 0.1$, then $\phi_0 e^{\gamma k_i} \succ 1$ at the time $\gamma = 0.3$. If we take smaller ϕ_0, the total number of public opinion may be changed when $\gamma = 0.3$.

Fig.7 (b-c) shows the time evolution of public opinion under $\phi_0 = 0.01$ and

$\phi_0 = 0.1$. The three curves corresponding to different adjustment parameter of node's inertia γ. It can be seen that when $\phi_0 = 0.1$, the result is consistent with $\phi_0 = 0.5$. The total number of public opinion remains 50 in the case of $\gamma = 0.3$, in the other two cases, the total number of public opinion decrease sharply and have obvious convergence effect. And the total number of public opinion reduce quickly with γ reducing. When $\phi_0 = 0.01$, there is obvious convergence effect with different γ. Therfore, the constant external field for the evolution of public opinion have a great impact, The total number of public opinion reducing slow down with ϕ_0 increasing under a constant value γ, which confirms the above speculation.

In the all, The public opinion number changing with time evolution in system not only relates with node's inertia but also depends on the constant external field ϕ_0. With the time evolution in system, there is obvious convergence effect of the public opinions. The dozens of opinions in initial state evolve with time, Most of them perish and only a few of them remain and development after a long time. The total number of public opinion reducing slow down with ϕ_0 increasing under a constant value γ,which indicate the inhibitory effect of the convergence trend. When ϕ_0 is constant, the node's inertia can also have the inhibitory effect of the convergence trend. And the total number of public opinion reducing slow down with γ increasing. It can be obtained that adjusting the value of node's inertia factor γ and constant external field ϕ_0 can control the opinion value's number, which even controls the rate of change.

4. Conclusion

Public opinion evolution model is investigated on scale free network in which the topology structure is always changing in this paper. In the model we consider the influence of ϕ_0 and $e^{\gamma k_h}$ on public opinion evolution. Public opinion evolution and network topological structure interact with each other, and they are all influenced by ϕ_0 and γ. it is found that degree distribution gradually gets away typical power law distribution with time evolution in our model.

In summary, the influence of nodes' inertia on the public opinion in system not only relates with the adjustment parameter of node's inertia γ but also depends on the constant external field ϕ_0. The smaller external field ϕ_0 is, the greater heterogeneity of public opinion distribution is. In general the influence of node's inertia on the public opinion is that the the convergence effect of public opinion distribution can be strengthen with γ reducing.

The public opinion number changing with time evolution in system not only relates with node's inertia but also depends on the constant external field. It can be obtained that adjusting the value of node's inertia factor γ and constant external field ϕ_0 can control the opinion value's number, which even controls the rate of change. The sum of public opinion reducing slow down with ϕ_0 increasing under a constant value γ,which indicate the inhibitory effect of the convergence trend. When ϕ_0 is a constant, the node's inertia can also have the inhibitory effect of the convergence trend.

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