IOCFF 2026 Conference

The 1st International Online Conference on Fractal and Fractional

13–15 April 2026 | Online

MDPI

Bifurcation control and parametric influence analysis in a fractional bi-rhythm Van der pol-Rayleigh system driven by Gaussian colored noises

Yajie Li 1 , Zhiqiang Wu *2 , Yongtao Sun *2 , Ying Hao 3 , Shengli Chen 4 , Xiangyun Zhang 5

1 School of Mathematics and Physics, Henan University of Urban and Construction, Pingdingshan 467036, China

2 School of Mechanical Engineering, Tianjin University, Tianjin, 300072, China

3 School of Civil Engineering and Mechanics, Yanshan University, Qinhuangdao, 066104, China

4 School of Transportation Engineering, East China Jiaotong University, Nanchang 330013, China

5 Basic Course Department, Tianjin Sino-German University of Applied Sciences, Tianjin, 300354, China

INTRODUCTION & AIM

$$\begin{cases} {}_0^C D^{1+p} x-\mu(-\varepsilon+\alpha_1 x^2-\alpha_2 \dot{x}^4+\alpha_3 \dot{x}^6)\dot{x}+w^2 x+\alpha_0 x^3=[\alpha+(1-\alpha)]\eta(t) \\ \dot{\eta}(t)+\dfrac{1}{\tau}\eta(t)=\dfrac{1}{\tau}\xi(t)\ (i=1,2) \end{cases}$$

$$\begin{cases} E[\xi_i(t_1)\xi_i(t_2)]=2D_i\delta(t_1-t_2) \\ E[\xi_i(t)]=0 \end{cases} \quad (i=1,2)$$

$$\begin{cases} E[\eta_i(t_1)\eta_i(t_2)]=\dfrac{D_i}{\tau}\exp[-\dfrac{|t_1-t_2|}{\tau}] \\ S_i(w)=\dfrac{2D}{(1+\tau^2 w^2)} \end{cases}$$

METHOD

Minimum mean square error:

$${}_0^C D^{1+p} x=M(p,w)\ddot{x}+C(p,w)\dot{x}$$

$$\begin{cases} M(p,w)=w^{p-1}\sin(\dfrac{p\pi}{2}) \\ C(p,w)=w^p\cos(\dfrac{p\pi}{2}) \end{cases}$$

Isovalent oscillator for system：

$$\ddot{x}(t)-\mu_1\gamma\dot{x}+w_0^2 x+\beta_0 x^3=\beta_1\eta_1(t)+\beta_2 x\eta_2(t)$$

$$\begin{cases} \gamma=-\varepsilon-w^p\cos(\dfrac{p\pi}{2})/\mu+\alpha_1 x^2-\alpha_2\dot{x}^4+\alpha_3\dot{x}^6 \\ \mu_1=\mu w^{1-p}/\sin(\dfrac{p\pi}{2}),\ w_0^2=w^{3-p}/\sin(\dfrac{p\pi}{2}),\ \beta_0=\alpha_0 w^{1-p}/\sin(\dfrac{p\pi}{2}), \\ \beta_1=\alpha w^{1-p}/\sin(\dfrac{p\pi}{2}),\ \beta_2=(1-\alpha)w^{1-p}/\sin(\dfrac{p\pi}{2}) \end{cases}$$

Stochastic averaging method：

$$\rho(a)=4C(1+w^2\tau^2)aw_0^2(4\beta_1^2\sigma_1^2+a^2\beta_2^2\sigma_2^2)^{-\frac{\Delta_1}{\beta_2^{10}D_2^5}}\exp(\frac{\Delta_2}{768\beta_2^8 D_2^4})$$

$$\begin{cases} \Delta_1=2(1+w^2\tau^2)w_0^2[(\varepsilon+w^p\cos(\dfrac{p\pi}{2})/\mu+w^{p-1}\sin(\dfrac{p\pi}{2})/\mu)\beta_2^8 D_2^4+\alpha_1\beta_1^2\beta_2^6 D_1 D_2^3 \\ +10\alpha_2 w_0^4\beta_1^4\beta_2^4 D_1^4 D_2^4+35\alpha_3 w_0^6\beta_1^6\beta_2^2 D_1^3 D_2] \\ \Delta_2=a^2(1+w^2\tau^2)w_0^2[384(\alpha_1\beta_2^6 D_2^3+10\alpha_2 w_0^4\beta_1^2\beta_2^4 D_1 D_2^2+35\alpha_3 w_0^6\beta_1^4\beta_2^2 D_1^2 D_2) \\ -48\beta_2^2 D_2(10\alpha_2 w_0^4\beta_2^4 D_2^2+35\alpha_3 w_0^6\beta_1^2\beta_2^2 D_1 D_2)a^2+280\alpha_3\beta_2^6 D_2^3 w_0^6 a^4] \end{cases}$$

Singularity theory：

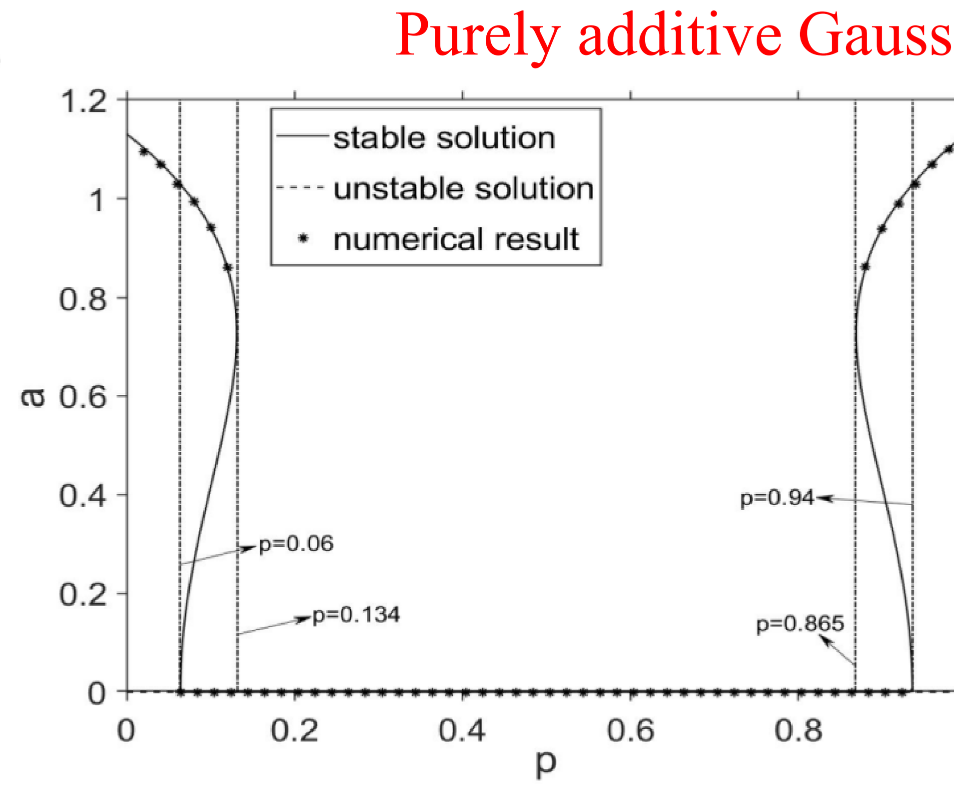
$$\begin{cases} \dfrac{\partial\rho(a)}{\partial a}=0 \\ \dfrac{\partial^2\rho(a)}{\partial a^2}=0 \end{cases}$$

Criticle condition for stochastic bifurcation：

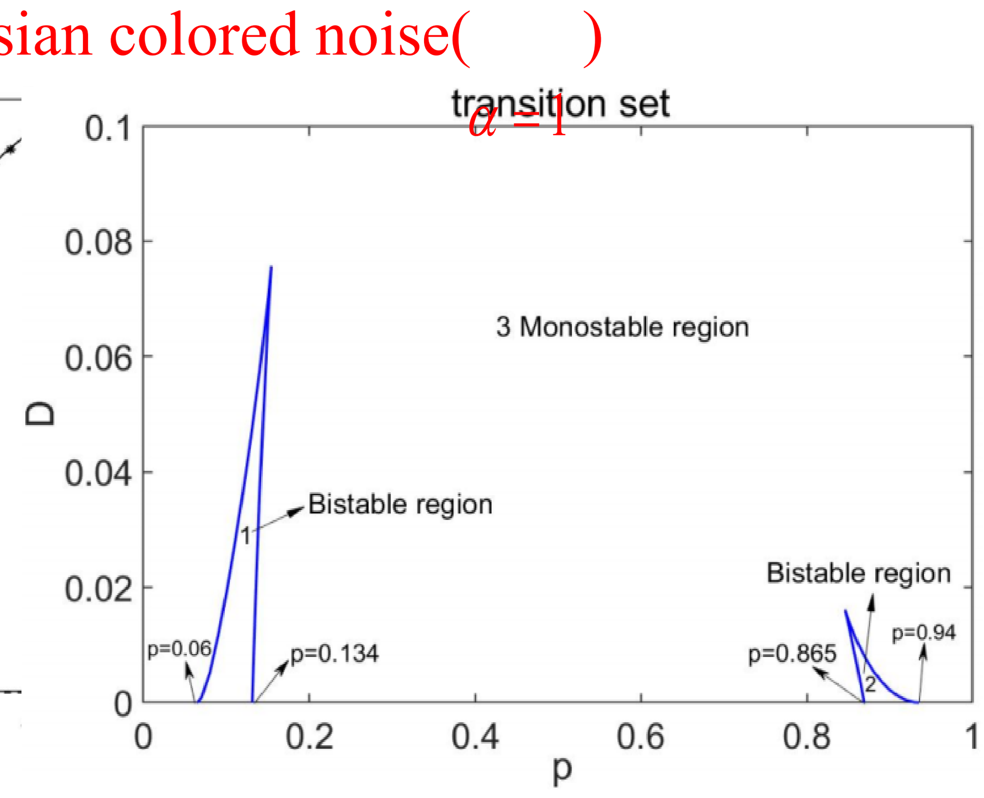
$$\begin{cases} H=\{R'+RQ'=0,R''+2R'Q'+RQ''+RQ'^2=0\} \\ R(a,\tau,\varepsilon,w,p,\alpha_1,\alpha_2,\alpha_3,D_1,D_2)=a(4\beta_1^2\sigma_1^2+a^2\beta_2^2\sigma_2^2)^{-\frac{\Delta_1}{\beta_2^{10}D_2^5}} \\ Q(a,\tau,\varepsilon,w,p,\alpha_1,\alpha_2,\alpha_3,D_1,D_2)=\dfrac{\Delta_2}{768\beta_2^8 D_2^4} \end{cases}$$

RESULTS & DISCUSSION

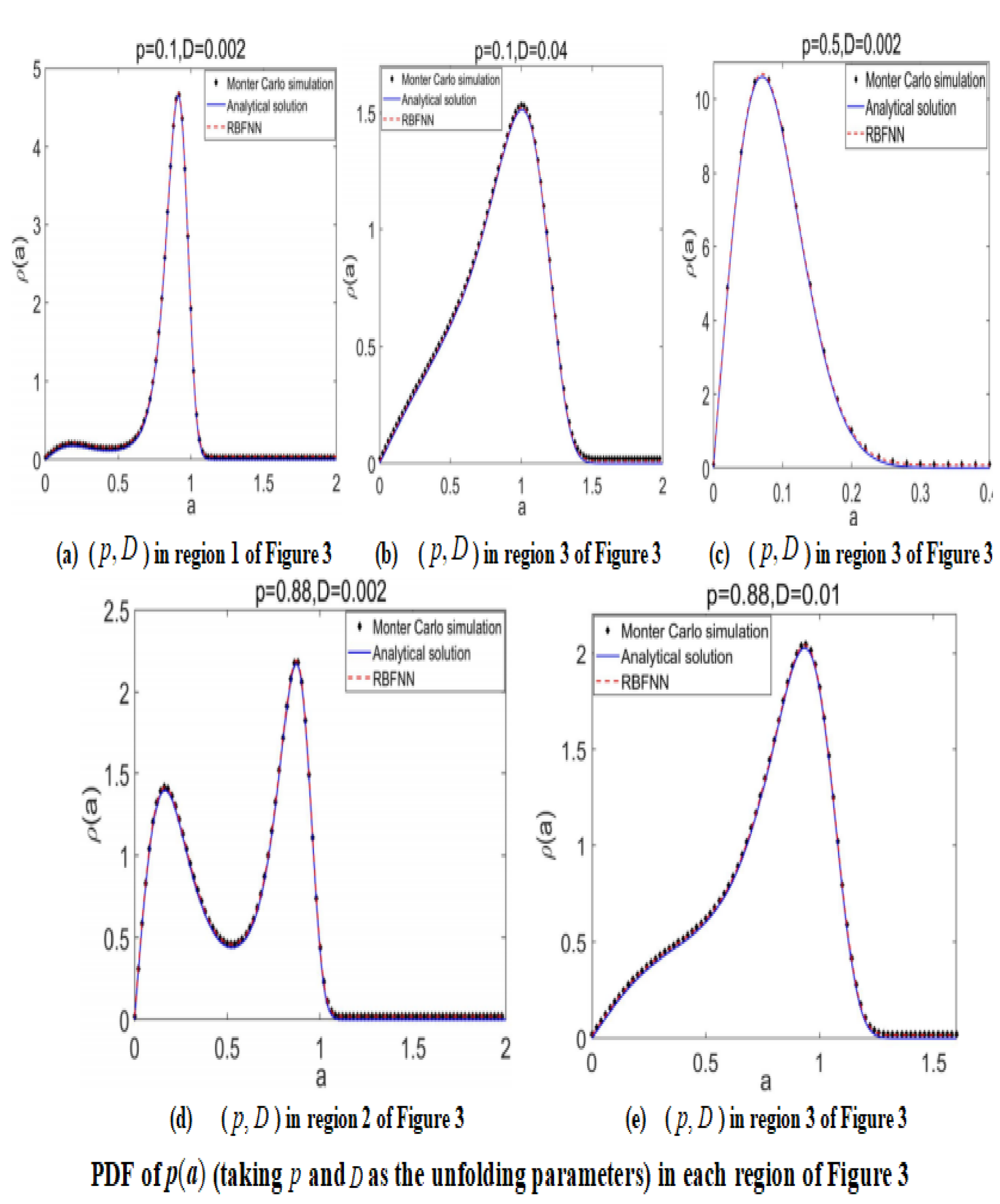
Purely additive Gaussian colored noise($\alpha=1$)



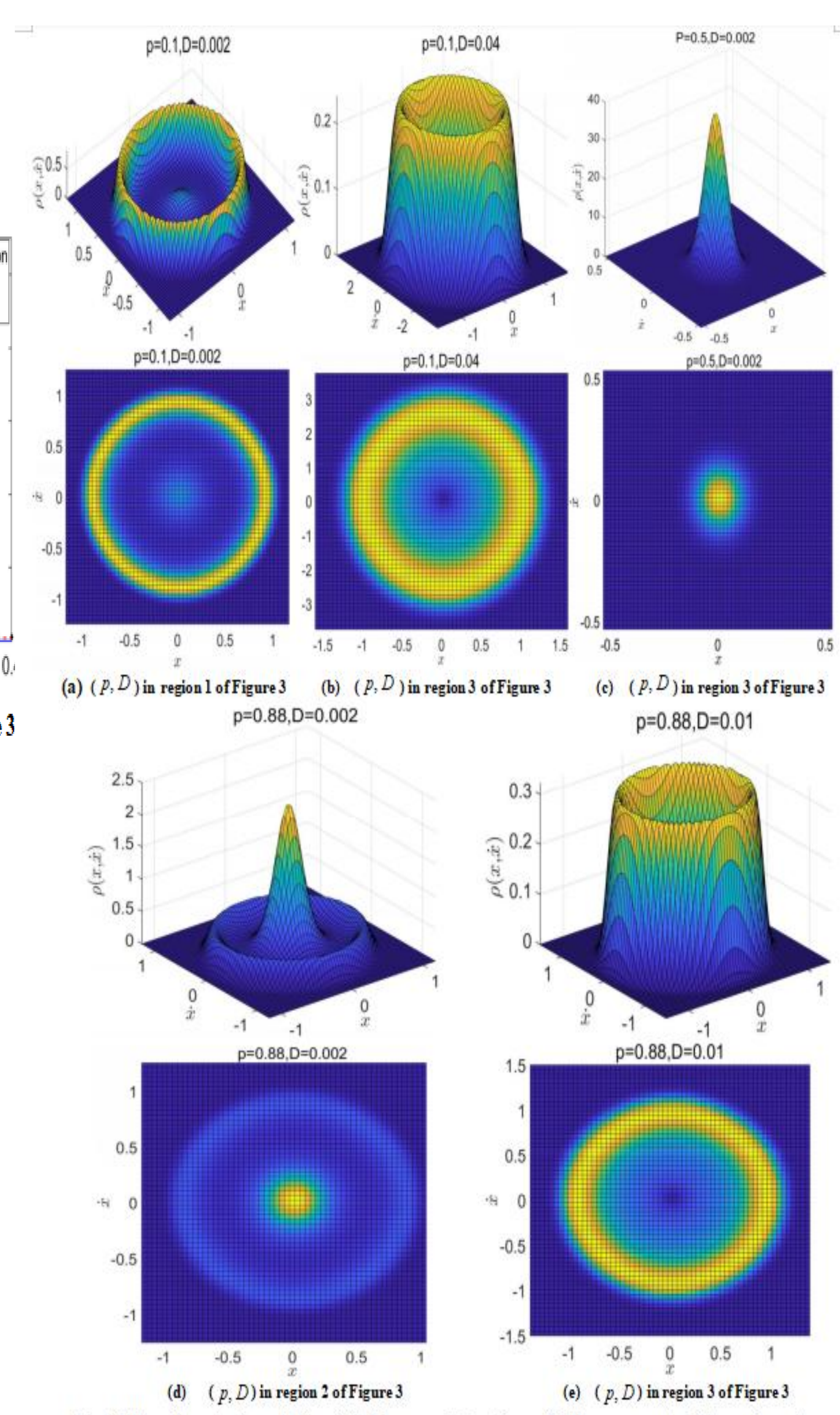
Bifurcation diagram of the deterministic system



Transition set curves



PDF of $p(a)$ (taking p and D as the unfolding parameters) in each region of Figure 3



Joint PDF and projection of $p(x,\dot{x})$ (taking p and D as the unfolding parameters) in each region

CONCLUSION

It shows that the fractional derivative's order p and noise intensity D can each arouse stochastic P bifurcation behavior of the system, and the number of peaks in the stationary PDF curves of system amplitude can be controlled from one to two by selecting the corresponding unfolding parameters.

FUTURE WORK / REFERENCES

However, the system studied in the article is the single degree of freedom system, and the complexity as well as the abstraction of state space increase the difficulty to analyze the high-dimensional dynamic system. The investigation of twodegree of freedom systems or even higher dimensional and coupled systems or even nonsmooth systems should be the next research focus in future.