

On the dynamics of a nutrient-plankton-fish ecosystem with Caputo fractional operator

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INTRODUCTION & AIM

Plankton are tiny aquatic creatures that live in bodies of water and can either float freely or swim slightly. Phytoplankton are extremely small unicellular plant organisms. Zooplankton, on the other hand, are creatures that feed on phytoplankton. Phytoplankton can transform mineral nutrients into biotic material using sunlight and improve seawater quality. They also supply half of the oxygen in the world through photosynthesis and absorb the harmful carbon dioxide. However, the phytoplankton can produce a harmful algal bloom that is toxic to humans and other sea animals. Fish are large aquatic animals that consume zooplankton. In this study, we propose a fractional order nutrient-plankton-fish interaction model which consists of four populations: nutrient, phytoplankton, zooplankton and fish. Our primary goal is to study the nutrient-plankton-fish interaction dynamics and regulation of algal blooms via fish predation while accounting for the memory effect.

METHOD

The proposed fractional-order model is formulated as follows:

$${}^cD^\mu N(t) = n_0 - a_1N - b_1NP + k_1c_1P$$

$${}^cD^\mu P(t) = a_2NP - c_1P - (\alpha_1PZ)/(g_1 + P)$$

$${}^cD^\mu Z(t) = (\alpha_2PZ)/(g_1 + P) - d_1Z - (\rho PZ)/(g_1 + P) - (\beta FZ)/(g_2 + Z)$$

$${}^cD^\mu F(t) = (a_3\beta FZ)/(g_2 + Z) - d_2F$$

with the initial conditions:
 $N(0) \geq 0$, $P(0) \geq 0$, $Z(0) \geq 0$ and $F(0) \geq 0$.

Here, $N(t)$ denotes the concentration of nutrients, $P(t)$ denotes the concentration of phytoplankton, $Z(t)$ denotes the concentration of zooplankton, and $F(t)$ denotes the concentration of fish at any time $t > 0$. To forecast ecosystem behavior and advance sustainable management, we find the biologically feasible equilibria of the system and investigate their stability.

RESULTS & DISCUSSION

The obtained results are as follows:

- The system's stability has decreased around the co-axial equilibrium as the mortality rate of zooplankton decreases.
- The stability of the system is inversely related to the maximal ingestion rate of fish, as the dynamics of the system become stable or unstable as the value of the maximal ingestion rate of fish increases.
- The nutrient consumed rate by phytoplankton negatively impacts the system's stability, as the system becomes unstable around the co-axial equilibrium as the nutrient consumption rate by phytoplankton increases.
- Moreover, the stability of the system decreases as the order of the fractional derivative increases. This implies that the dynamics become less stable when the model incorporates more memory or historical input.

CONCLUSION

This study proposed a nutrient-plankton-fish interaction model in a marine environment incorporating the Caputo fractional operator to investigate the role of various parameters, such as the mortality rate of zooplankton, the maximal ingestion rate of fish, and the nutrient consumption rate by phytoplankton, on stabilizing the system. Further, to investigate the effect of the order of the fractional derivative on system stability, we have arbitrarily chosen four values: 0.91, 0.94, 0.97, 1.

REFERENCES

- R. P. Kaur, A. Sharma, A. K. Sharma, Impact of fear effect on plankton-fish system dynamics incorporating zooplankton refuge, *Chaos, Solit. Fractals*, 2020,143, 110563, DOI: 10.1016/j.chaos.2020.110563