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Modulating Neuronal Entrainment via Distinct Fractional Derivative Orders in the FitzHugh–Nagumo Model

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INTRODUCTION & AIM

- Synchronization of neurons to external periodic inputs is vital for information processing and network coordination.
- Biological neurons exhibit history-dependent behaviors, meaning their dynamics rely inherently on past activity.
- This study analyzed a periodically forced Caputo fractional-order FitzHugh-Nagumo model to simulate neuronal dynamics:

$$
\begin{aligned}
{}^{C}\mathbf{D}_t^{\alpha} V &= V - \frac{V^3}{3} - W + I\cos(\omega t);\\
{}^{C}\mathbf{D}_t^{\beta} W &= \varepsilon(V + a - bW),
\end{aligned}
$$

- Because the fast membrane potential (V) and the slower recovery variable (W) reflect distinct biophysical processes, memory asymmetry was introduced by applying Caputo fractional derivatives of distinct orders (α and β) to the voltage and recovery variables.

Aim: To investigate how these distinct fractional orders influence an individual neuron's synchronization and phase-locking behavior when subjected to rhythmic external input.

RESULTS & DISCUSSION

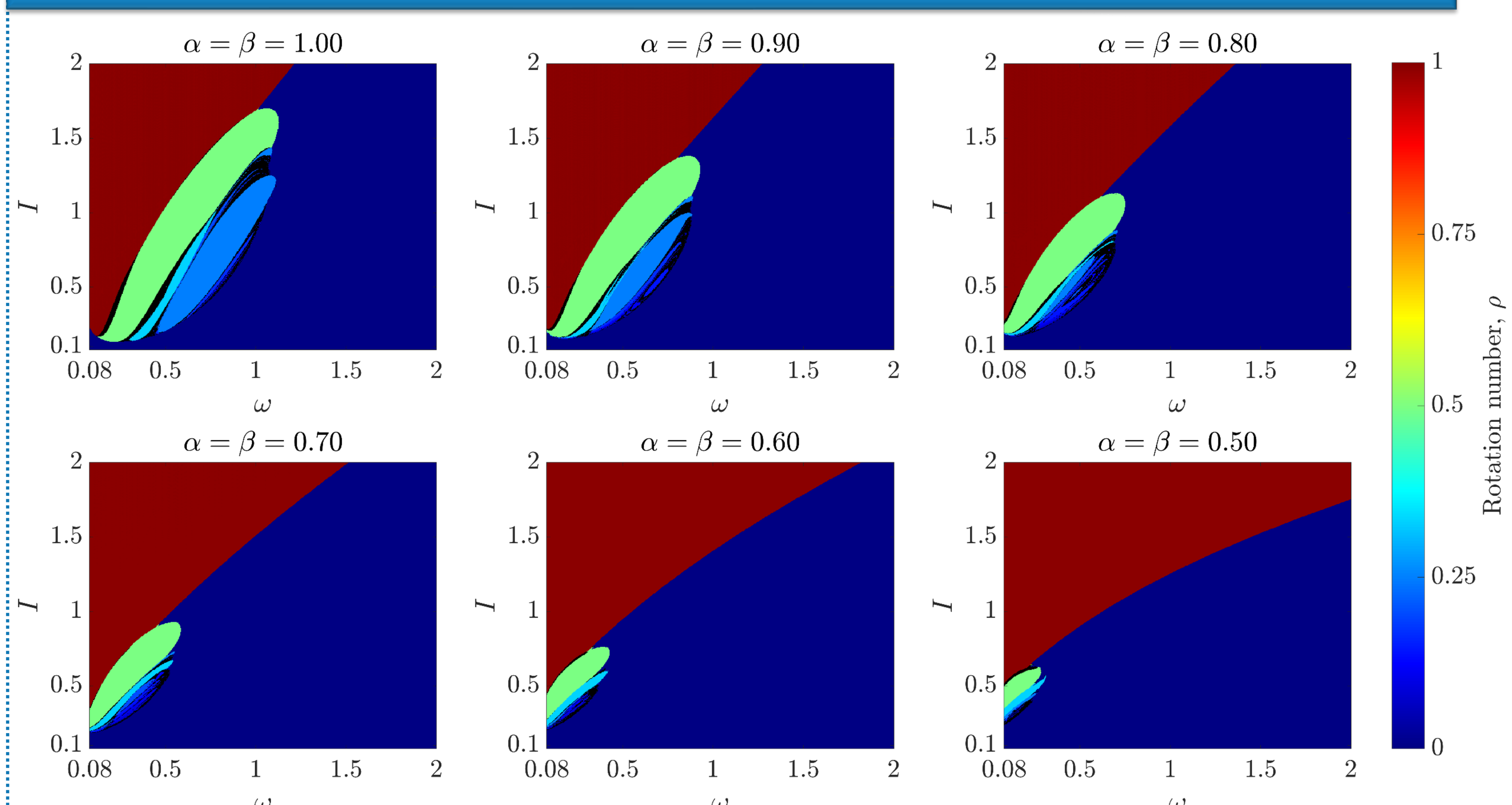

Fig. 1. Arnold tongue plots for the periodically forced FitzHugh-Nagumo model with equal fractional orders ($\alpha = \beta = \kappa, \kappa = 1.0, 0.9, \ldots, 0.5$). The color represents the rotation number ρ, with the gradient ranging from dark blue for quiescent states ($\rho = 0$) to dark red for 1:1 phase-locking ($\rho = 1$). Black regions correspond to complex dynamics. These plots show a consistent trend: as the fractional order κ decreases, the region of 1:1 phase-locking (dark red) progressively expands, while regions of other non-quiescent firing states contract. This demonstrates that introducing systemic memory promotes dynamical simplification.

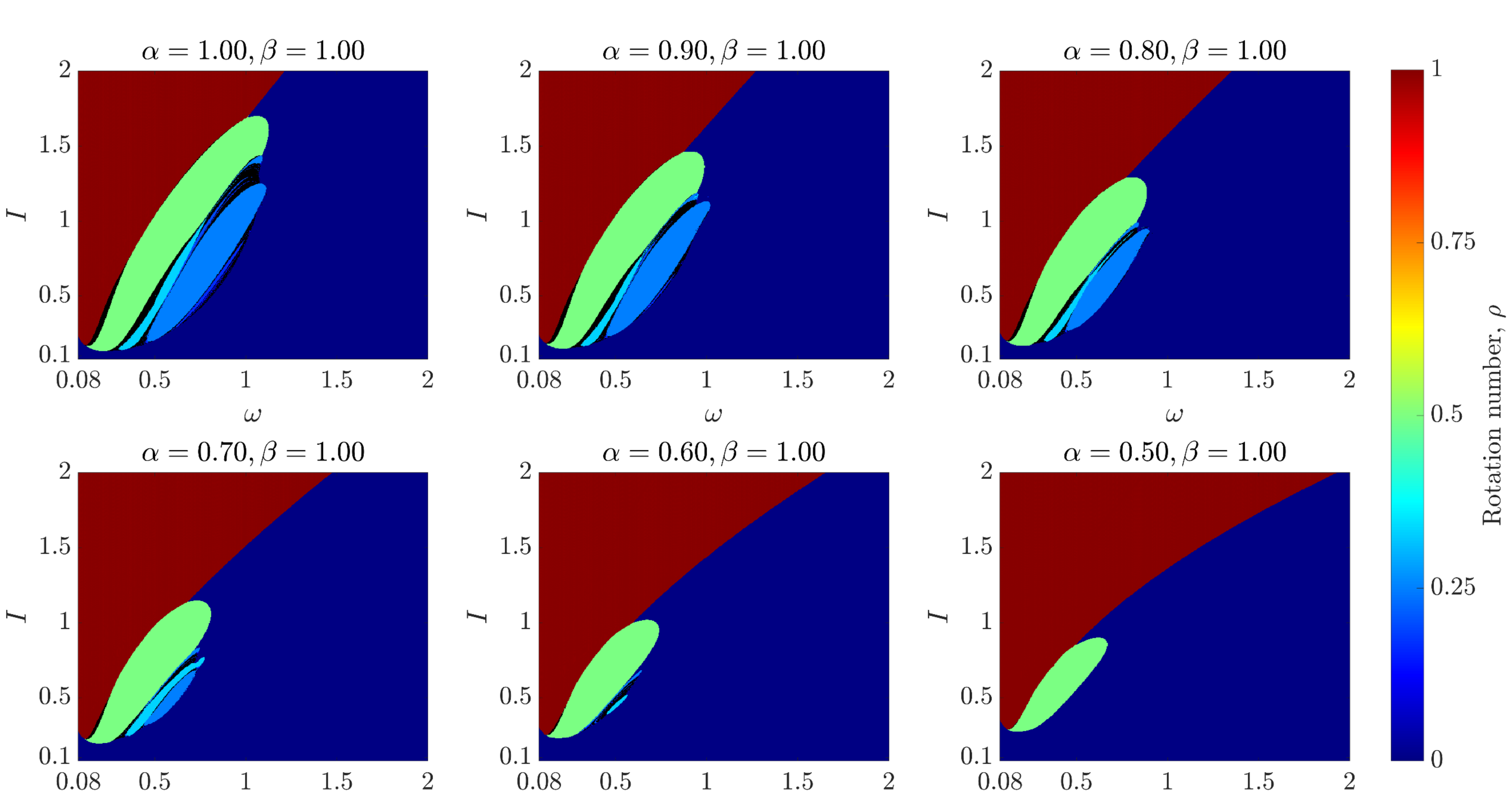

Fig. 2. Arnold tongue plots for the model with a fixed recovery order ($\beta = 1.0$) and varying voltage order (α). The color map is consistent with that used in Fig. 1. Lower voltage orders α cause the regions of higher-order locking and complex firing to contract significantly and becoming simpler in structure.

METHOD

- A state-based, two-threshold spike detection algorithm was implemented to ensure accurate spike registration.
- The degree of synchronization was quantified using the rotation number and inter-spike interval statistics to categorize trajectories into stable phase-locking or complex dynamics.
- The system's dynamics were systematically mapped across a two-dimensional parameter space of forcing amplitude and frequency to generate Arnold tongue plots.

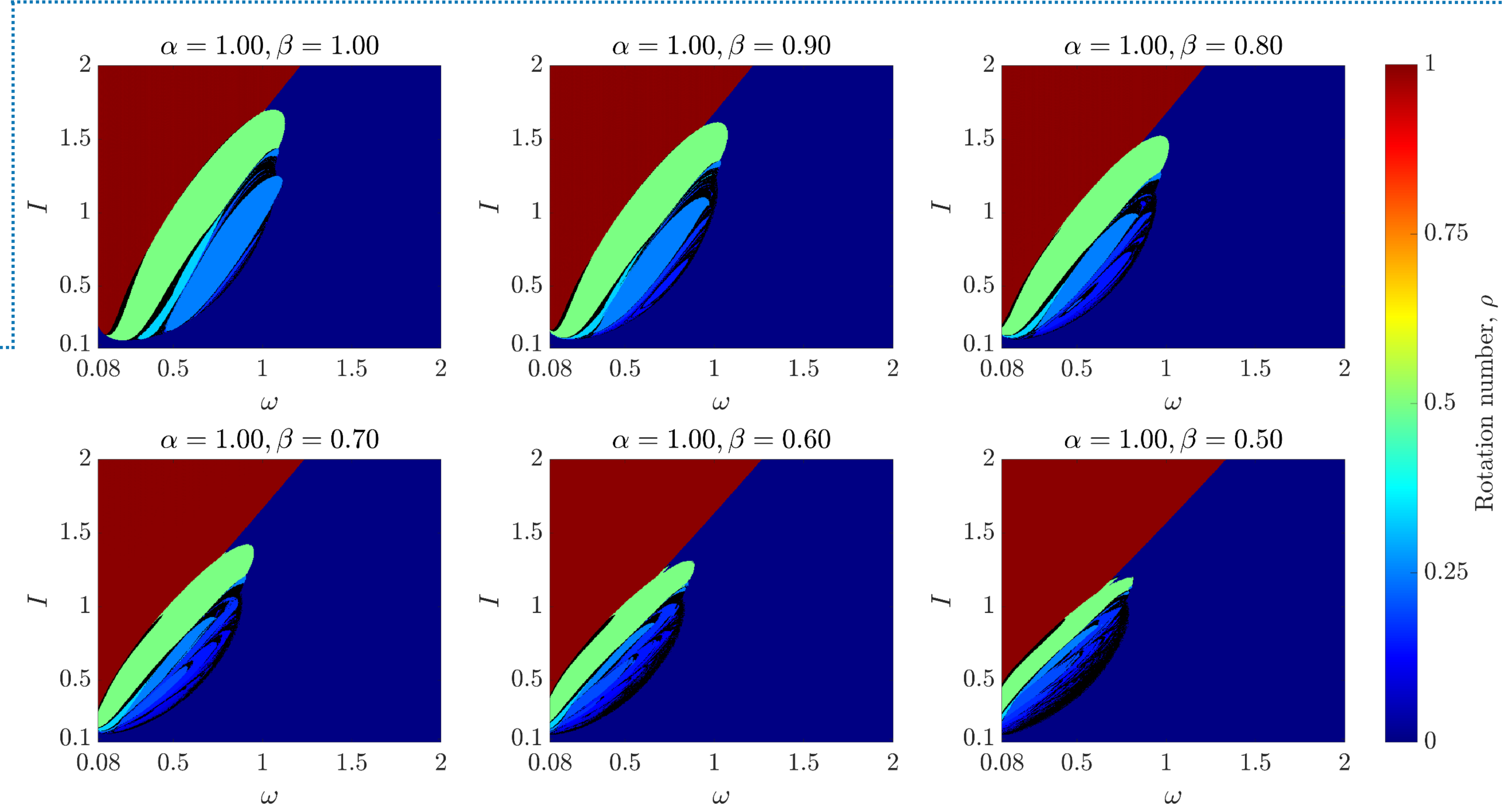

Fig. 3. Arnold tongue plots for the model with a fixed voltage order ($\alpha = 1.0$) and varying recovery order (β). The color map is consistent with that used in Fig. 1. Differently from the α-varying case, decreasing the recovery order β does not simply suppress non-1:1 firing responses, but rather reorganizes them and causes a notable shift of their domains toward lower forcing frequencies.

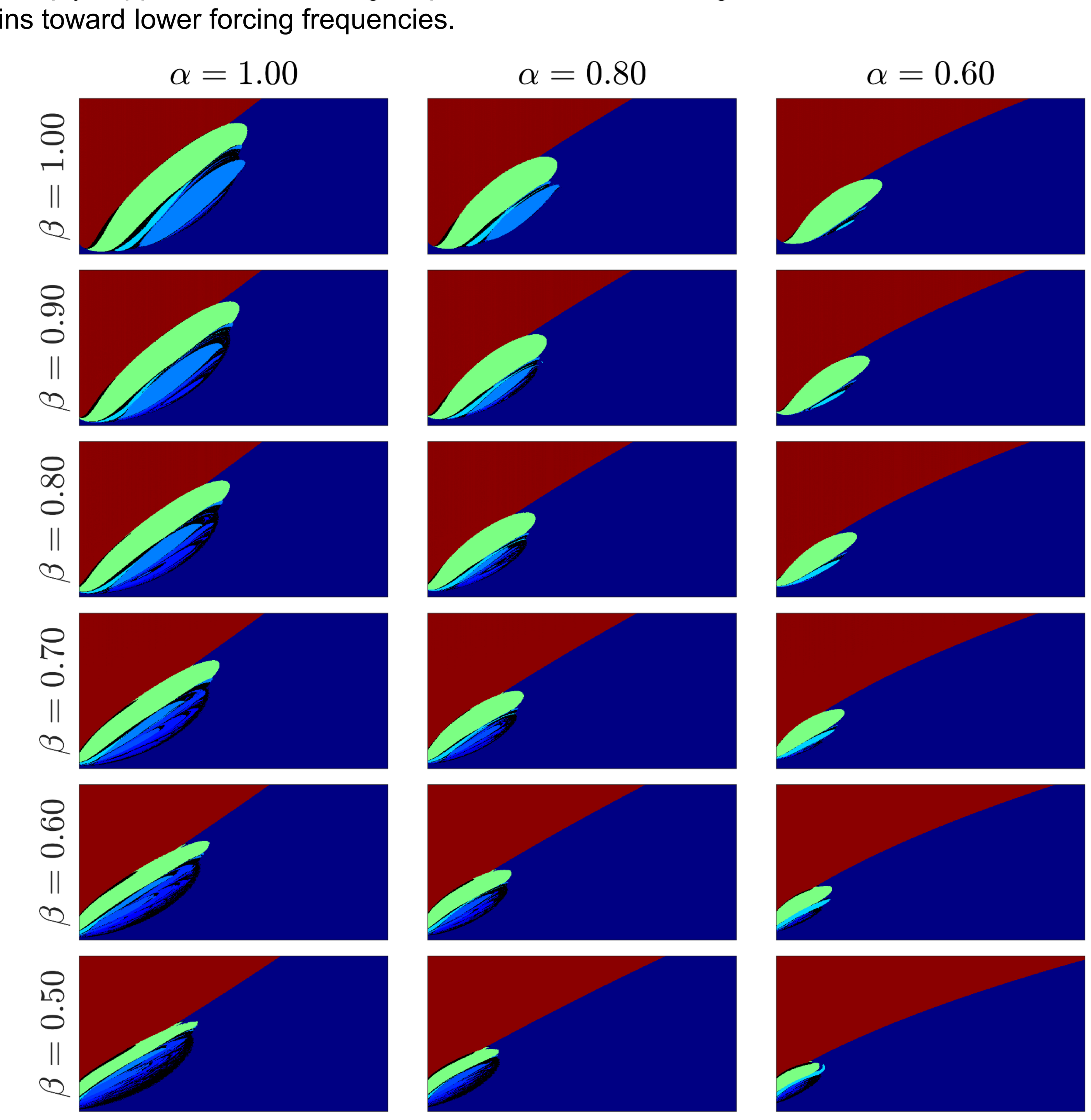

Fig. 4. Matrix of Arnold tongue plots for varying fractional orders $\alpha \in \{1.0, 0.8, 0.6\}$ (columns) and $\beta \in \{1.0, 0.9, \ldots, 0.5\}$ (rows). The axes and colormap, which are consistent with those used in previous figures, have been removed to maximize clarity. The horizontal axis for each plot represents the forcing frequency $\omega \in [0.08, 2]$, and the vertical axis is the forcing amplitude $I \in [0.1, 2]$. This composite view illustrates how the distinct effects of the fractional orders are superimposed. Decreasing α (moving horizontally left to right) systematically contracts the regions of non-1:1 firing, demonstrating a simplifying effect. Decreasing β (moving vertically top to bottom) shifts the entire dynamical structure toward lower frequencies. Notably, for a fixed low α (e.g., rightmost column), decreasing β causes the internal structure of the shrinking non-1:1 firing region to become proportionally richer in higher-order and complex dynamics.

CONCLUSION

- The fractional orders of the fast voltage and slow recovery variables assume distinct, non-interchangeable roles in shaping the neuron's response.
- Memory in the voltage dynamics acts primarily as a damping mechanism, while memory in the recovery dynamics acts as a timescale modulator that re-tunes the neuron's intrinsic frequencies.
- Fractional memory asymmetry could serve as a biophysical mechanism for neurons to selectively modulate both the complexity and the frequency selectivity of their responses.

REFERENCES

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