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Numerical Simulation of Caputo Fractional Oldroyd-B Fluid in a Top-Covered Square Cavity

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Introduction

- **Background:** Viscoelastic fluids are widely used in chemical engineering and material processing. The classic Oldroyd-B model is a fundamental constitutive equation, but it has limitations in characterizing fluids with complex microstructures. The Caputo fractional derivative shows significant advantages in describing the complex rheological properties and long-term memory effects of these fluids.
- **Problem:** Implementing non-local fractional derivatives in commercial CFD software is highly challenging due to memory storage requirements. Furthermore, simulating viscoelastic fluids at high Weissenberg (We) numbers often leads to severe numerical instability, known as the High Weissenberg Number Problem (HWNP).
- **Aim:** This study aims to develop a robust numerical framework within ANSYS Fluent to simulate the Caputo fractional Oldroyd-B fluid in a lid-driven cavity. By utilizing User-Defined Functions (UDFs) and an artificial viscosity strategy, we seek to analyze the effects of fractional order, Weissenberg number, and Reynolds number on the flow field topology and vortex center locations.

Method

- **Physical Model:** A 2D planar square cavity (1m × 1m)filled with a fractional Oldroyd-B fluid. The top wall is moving at a constant velocity (driven lid), while the other three walls impose no-slip boundary conditions.
- **Governing Equations:** The incompressible Navier-Stokes equations coupled with the Caputo fractional Oldroyd-B constitutive model.Caputo Fractional Derivative:

$$ {}_{0}^{C}D_{t}^{\alpha}\tau(t)=\frac{1}{\Gamma(1-\alpha)}\int_{0}^{t}(t-s)^{-\alpha}\frac{\partial\tau(s)}{\partial s}ds,\quad 0<\alpha<1 $$

- Numerical Implementation (The Core Framework):

1. **Finite Volume Method (FVM):** Solved using the Pressure-Based Coupled solver in ANSYS Fluent.
2. **UDS & UDF Coupling:** Three User-Defined Scalars (UDS) are allocated for the elastic stress tensor components ($\tau_{xx},\tau_{yy},\tau_{xy}$) . The constitutive equations are integrated into the momentum equations via source terms using UDFs.
3. **L1 Approximation Algorithm:** The time-fractional derivative is discretized using the L1 scheme. A sliding window of User-Defined Memory (UDM) is employed to store the historical stress states required for the memory effect.
4. **HWNP Stabilization:** To overcome numerical divergence at high We numbers, an artificial diffusion term (constant artificial viscosity, e.g., D=10^{-4}) is explicitly added to the UDS transport equations.

Results & Discussion

- Validation of the Numerical Model: The proposed FVM-UDF framework successfully converged. When the fractional order $\alpha \rightarrow 1$, the velocity profiles (U-velocity along the vertical centerline and V-velocity along the horizontal centerline) and primary vortex center coordinates show excellent agreement with existing integer-order benchmark data in the literature.
- Effect of Fractional Order alpha: Compared to the standard integer-order model, decreasing the fractional order alpha enhances the memory effect of the fluid. This leads to a delayed transient development of the flow field and alters the stress concentration patterns near the singular corners of the moving lid.
- Effectiveness of Artificial Viscosity: The introduction of the artificial diffusion coefficient effectively smoothed the steep stress gradients near the upper corners without compromising the overall physical accuracy of the bulk flow, allowing the simulation to remain stable at higher Weissenberg numbers.

Conclusions

- A comprehensive numerical framework for simulating Caputo fractional viscoelastic fluids was successfully established within ANSYS Fluent using FVM, UDS, and history-storing UDMs.
- The L1 approximation combined with the artificial viscosity stabilization strategy effectively overcomes the HWNP, ensuring robust convergence.
- The fractional order significantly regulates the flow topology and stress distribution, proving the necessity of fractional-order modeling for fluids with complex memory characteristics.

Future Work

- Implement non-uniform time-stepping for the L1 algorithm to optimize UDM memory allocation.
- Extend the current 2D fractional solver to 3D complex geometries.
- Explore the Log-Conformation Representation (LCR) as an alternative to artificial viscosity for extreme We numbers.

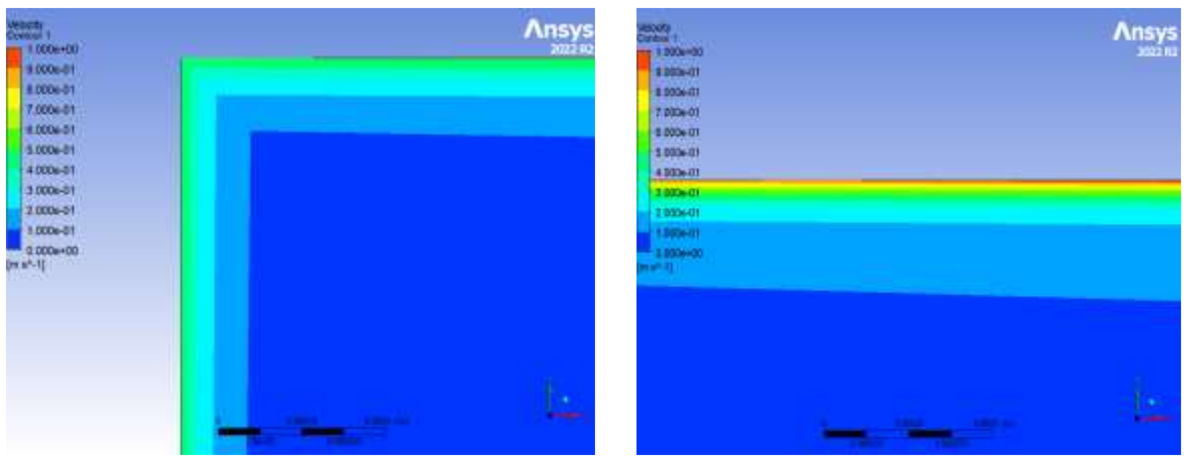

Figure 1: Enlarged velocity contour plot of the upper-left corner of the square cavity and an enlarged velocity contour plot of the driving wall at the top of the square cavity.