IOCFF 2026 Conference
The 1st International Online Conference on Fractal and Fractional
13–15 April 2026 | Online
MDPI

# Studying Variable-Order Caputo Fractional Differential Equations via Noncompactness and Fixed Point Theorem

**Fatma zohra Daoud, Zoubida bouazza**
**Departement of mathematics, University of Tiaret**
**Departement of informatics, University of Tiaret**

## INTRODUCTION & AIM

Fractional calculus extends differentiation to non-integer orders
Caputo derivative useful for physical interpretation
Goal: Study BVPs with variable-order derivatives
Use Darbo's theorem & Kuratowski measure

## METHOD

Transform variable-order problem into piecewise constant order
Work in Banach space of continuous functions
Apply measure of noncompactness
Use fixed-point approach

## PRELIMINARIES

Banach space $C(\Lambda, R)$ with sup norm
Definitions of CFIVO and CFDVO
Piecewise constant functions
Basic lemmas on fractional operators

## RESULTS & DISCUSSION

Main problem:Caputo variable-order differential equation
Boundary conditions: $\xi(0)=0$, $\xi(M)=0$
Order $\omega(s) \in (1,2]$
Nonlinear function $\Psi_1(s, \xi, D\xi)$
Existenceof solutions:Partition interval into subintervals
Reduce to constant-order problems
Construct operator S
Apply Darbo's Fixed Point Theorem
Key conditions:Continuity of $\Psi_1$
Lipschitz-type condition
Inequality ensuring contraction
Bounded, convex, closed sets

## CONCLUSION

Existence of solutions proven
Variable-order handled via piecewise approach
Framework useful for complex systems
Future applications in science & engineering

## FUTURE WORK / REFERENCES

Fractional calculus literature
Darbo fixed point theorem
Measure of noncompactness
Recent applied mathematics studies