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A Study on the Estimation of Lifetime Distributions and Its Application to Life Insurance

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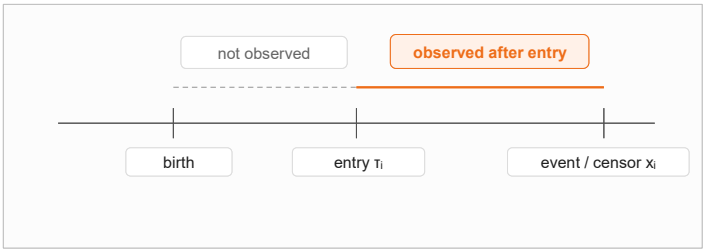
INTRODUCTION & AIM

Research objective

Accurate lifetime distribution estimation is essential for premium calculation, reserve valuation, and long-term mortality risk management.
Insurance data often combine delayed entry and incomplete follow-up. Ignoring LTRC mechanisms can distort fitted survival tails.
Aim: compare classical, generalized Marshall-Olkin (MO), and Phase-Type (PH) lifetime models under a unified LTRC likelihood and select complexity using EIC.

LTRC observation scheme



Observed lifetimes are conditional on survival beyond the entry age. Failures and censored records therefore require different likelihood contributions.

METHOD

1. LTRC likelihood

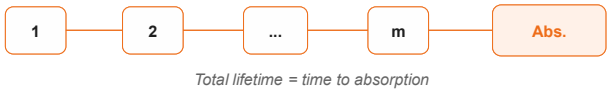
For observation i, x_i is the observed age/lifetime, τ_i is the entry age, and δ_i is the event indicator.

$$D = \{(x_i, \tau_i, \delta_i)\}_{i=1}^{n}, \quad \delta_i \in \{0, 1\}$$

$$\ell(\theta) = \sum_{i=1}^{n} \{\delta_i \log f(x_i \mid \theta) + (1-\delta_i)\log S(x_i \mid \theta) - \log S(\tau_i \mid \theta)\}$$

2. Phase-Type model

A PH distribution is the absorption time of a finite-state continuous-time Markov chain.
Transient phases represent latent lifetime stages; absorption represents death/end of lifetime.



Total lifetime = time to absorption

$$f(x) = \alpha \exp(Tx)\xi, \qquad S(x) = \alpha \exp(Tx)\mathbf{1}$$

3. Model selection

Increasing PH phases improves likelihood but may overfit. EIC estimates the prediction bias by bootstrap resampling and supports practical phase selection.

$$\mathrm{AIC} = -2\ell(\hat{\theta}) + 2k$$

$$\mathrm{EIC} = -2\{\ell(\hat{\theta}; D) - \hat{b}_{\mathrm{EIC}}\}$$

RESULTS & DISCUSSION

Main findings to communicate

- Flexible lifetime families capture actuarial lifetime patterns more effectively than overly rigid classical models.
- MO-type extensions improve advanced-age tail representation.
- PH distributions approximate complex hazard shapes through latent Markov phases.
- EIC balances goodness-of-fit and complexity, helping avoid unnecessary flexibility.

Model	Role	Interpretation
Classical	baseline	simple but may miss tail / hazard complexity
MO-type	tail extension	better advanced-age tail representation
PH / CF1	flexible model	captures complex hazard patterns
EIC	selection	balances fit and model complexity

Rank	Family	Model	AIC
1	**Geometric MO**	**Truncated Normal**	**61,425.18**
2	Geometric MO	Log-negative Gumbel	61,429.36
3	Geometric MO	Gamma	61,489.52
4	Standard	Truncated Negative Gumbel	61,494.72
5	Geometric MO	Trunc-neg-Gumbel	61,496.72
6	Log-series MO	Trunc-neg-Gumbel	61,496.72
7	Geometric MO	Exponential	61,581.92
8	Standard	Truncated Logistic	61,581.92

Table 1. Top AIC-ranked lifetime models under the LTRC setting. Geometric MO Truncated Normal gives the smallest AIC.

Order m	EIC	EIC CI lower	EIC CI upper	MAXITER
3	70,823.72	70,820.80	70,826.64	2,000
10	65,048.48	65,027.58	65,069.38	2,000
50	**63,352.37**	**63,305.40**	**63,399.34**	**5,000**
100	64,014.95	63,984.02	64,045.88	8,000
150	65,959.54	65,896.02	66,023.06	10,000
200	66,114.85	66,057.57	66,172.12	12,000

Table 2. EIC comparison for PH distributions; m = 50 gives the smallest EIC.

CONCLUSION

- LTRC likelihood enables fair comparison of standard, MO-type, and PH lifetime models.
- Geom. MO Truncated Normal gives the lowest AIC; PH with m = 50 gives the lowest EIC.
- Distribution choice matters for premium evaluation and long-term mortality risk management.

FUTURE WORK / REFERENCES

Future work: validate on larger/real actuarial datasets and quantify premium/reserve sensitivity.
Conflict of Interest: The authors declare no conflicts of interest.
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