IOCMA 2026 Conference
The 2nd International Online Conference on Mathematics and Applications
10–12 June 2026 | Online
MDPI

On Seasonal Autoregressive Processes Inference

Fatna Bensaber
Departement of Mathematics, Faculty of Science, University of Tlemcen, Algeria

INTRODUCTION & AIM

In the study of several real time series and beside the other major fluctuations such as the trend, the cycle and the noise, the presence of seasonal fluctuations is one of the most important issues.
The investigation has an established practice provided that seasonal variations have been regarded as a disruptive element and then must be eliminated. However, these fluctuations are an integral part that must be studied in order to evaluate and forecast the studied model. A well-known practice for modeling seasonal data is to utilise an autoregressive model that is able to handle the presence of the seasonal patterns in the data. Autoregressive models are kind of time series model that utilise lagged values of the target variable to make predictions about future values.
Notice that despite the fact that these data are obtainable in practice as sequences of discrete observed values, they are basically approached as functions. Hence their processing can not be achieved using classical statistical methods and then the appeal to a new modern and dynamically field of statistics was required whose subject of interest is data that can naturally be perceived as functions,the FDA (functional data analysis). It combines the methods and approaches of classical statistics and mathematical functional analysis and owns the similar goals in statistics. Its relevance is directly linked to the development of computer technology, which allows the acquisition, storage, and processing of large amounts of data.
we propose in this work a study of a functional modeling framework built on estimating the seasonality behaviour using the technique of dividing the time series into a deterministic component that comprises seasonality and a stochastic component that puts out of action a functional autoregressive model see[3].

METHOD

A seasonal functional autoregressive process of order one see [1] of mean m is a sequence $(Y_n, n \in \mathbb{Z})$ of r.v.s defined on a complete probability space(Ω, A, P) with values in a real separable Banach space B such that

$$Y_n - m = L(Y_{n-s} - m) + \varepsilon_n. \qquad (1)$$

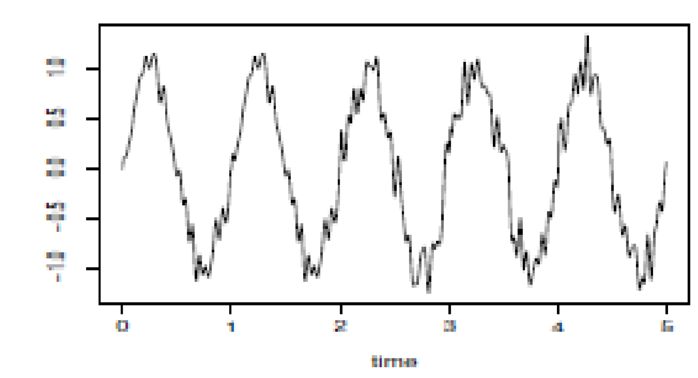
where L is a linear operator and ε is a strong white noise and s is a positive integer.
Another way of dealing with the seasonality is to consider it as a deterministic part perturbed by an autoregressive process. Suppose having a real almost sure continuous paths process $(\xi_t\,,\ t > 0)$ admitting the following decomposition

$$\xi(t) = S(t) + \chi(t), t \in \mathbb{R}, \qquad (2)$$

where S(t) is a non constant real continuous β-periodic function with $\beta > 0$ and $(\chi_t, t > 0)$ is a zero-mean process admitting a B-valued autoregressive representation ARB(1) see [2] i.e. the sequence $(\chi(n\beta + t), n \in N)$ satisfies (1) with $s = 1$, $B = C[0, \beta]$ the space of continuous functions on [0, β]. It clear that (ξ_t) has ARB(1) having the periodic function S as a mean.
When observing a trajectory of the process $(\xi_t\,,\ t > 0)$ over an interval $[0, n\beta]$, the observations over succesive intervals of length β are then rv's $Y_1, \ldots, Y_n$. Then, one can define the estimator that satisfies the strong law of large numbers

$$\overline{Y}^*_T = \frac{1}{T}\int_0^T Y(t)dt. \qquad (3)$$

RESULTS & DISCUSSION



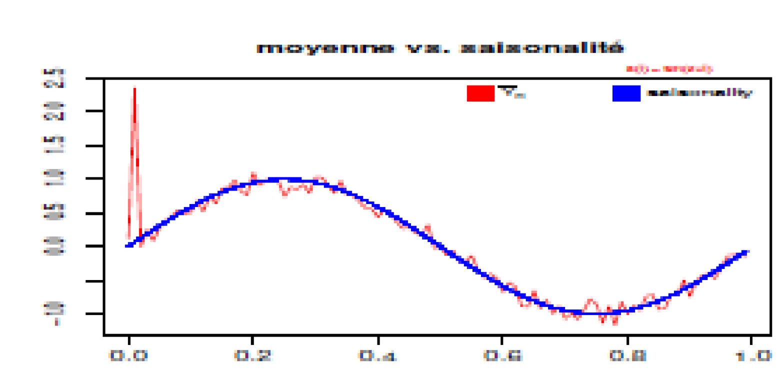
(a) Simulation of 5 observations at 100 discretisation points of an Sfar(1) with $s = 1$ perturbing the seasonality $S(t) = \sin(2\pi t)$

Seasonality Estimation

The observations of the process _x0011_ on successive intervals of length β can be interpreted as random variables $X_1; \ldots; X_n$ with values in C[0; β] in the sense of (2): for $i = 0; 1; \ldots; n = \frac{[T]}{\beta}$-1, set

$$X_n := \xi(i\beta + t);\ 0 \le t \le \beta$$

We define the empirical mean $\bar{X}_n$ from the observations $X_i; i = 1; \ldots; n$.
The following result gives its almost sure convergence to the seasonality m.
proposition:
(i) $\bar{X}_n$ is an unbiased estimator of m.
(ii) $\lim_{n\to+\infty} \bar{X}_n$ = m a.s. and in L^2



(b) Simulation showing the convergence of the empirical mean to the deterministic seasonality given 100 variables with 50 points of discretisation points

CONCLUSION

Investigating the seasonality in time series models is the consequence of a huge volume of published research in this era. In this work we propose a different way for dealing with seasonality when the pattern of seasonality is changing over time. Changing seasonal patterns impose special diffculties when attempting to separate seasonal and non-seasonal fluctuations. Accordingly, the development and enhancement of techniques to make such distinctions constitutes an important development in time series analysis. In order to obtain improvement in forecasting accuracy an efficient evaluation of the seasonal component using functional autoregressive models is provided.

FUTURE WORK / REFERENCES

[1] A. Zamani, H. Haghbin, M. Hashemi & R.J. Hydman. Seasonal Functional Autoregressive Models. Journal of Time Series Analysis, 43(2): 197-218, 2022.
[2] F. Bensaber. Functional autoregressive process with seasonality. Communications in Statistics-Theory and Methods, 52(20): 7131-7145, 2023.
[3] D. Bosq. Linear processes in function Spaces.. Springer-Verlag New York, Inc, 2000.