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An adaptive vector barrier interior point method Using majorant functions

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INTRODUCTION & AIM

This work proposes an adaptive vector barrier interior-point method for constrained convex optimization. The method combines vector logarithmic barriers, adaptive updates, and majorant function to calculate the step size instead of the line search.

Adaptive Vector Barrier approach	Advantages
Vector logarithmic barrier Adaptive componentwise updates Majorant step size	Faster convergence Numerical robustness Reduced CPU time

METHOD

1) Problem formulation

Consider the constrained convex optimization problem:

$$(P) \qquad \begin{cases} \min f(x) \\ Ax = b \\ x \geq 0 \end{cases}$$

Assumptions:

A1- The function $f: \mathbb{R}^n \to \mathbb{R}$ is twice continuously differentiable and convex.

A2- $A \in \mathbb{R}^{m \times n}$ has full rank matrix where $m < n$.

A3- The feasible set $K = \{x \in \mathbb{R}^n, Ax = b, x \geq 0\}$ is assumed to be nonempty.

2) Perturbed problem

In order to handle the positivity constraints while preserving strict feasibility of the iterates, we employ a logarithmic barrier approach. Interior-point methods are based on the replacement of inequality constraints by barrier terms that penalize the approach to the boundary of the feasible region. In the present framework, the positivity condition $x > 0$ is incorporated into the objective function through a logarithmic barrier term.

The associated logarithmic barrier function is defined by

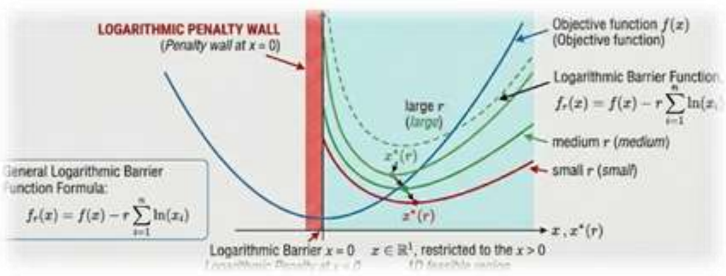
$$f_r(x) = f(x) - \sum_{i=1}^{n} r_i \ln x_i$$

Where $r \in \mathbb{R}^n$ is a vector valued barrier parameter such that:

$$r_i^{k+1} = \rho_i^k r_i^k \ ,\rho_i^k = \min\left(1, \frac{x_i^k}{|d_i^k| + \epsilon}\right)$$

The associated barrier subproblem is defined as

$$(P_r) \qquad \min\{f_r(x), \quad x \in \mathbb{R}^n\}$$



Existence and uniqueness of the solution of (P_r)

Theorem:

Let f be convex and twice continuously differentiable, and assume that the feasible set is nonempty. Then, the considered optimization problem admits a unique solution.

Descent direction and the step size

Theorem:

For each iteration k, there exists a step size t^k, computed by the majorant function technique, such that

$$f_r(x^k + t^k d^k) < f_r(x^k)$$

Where d^k is obtained by solving the system:

$$\begin{cases} \min \frac{1}{2} d^t \nabla^2 f(x) d + \nabla^t f(x) d \\ Ad = 0 \end{cases}$$

The search line function is given by

$\theta(t) = f_r(x + td)$-$f_r(x) = \mathrm{f}(\mathrm{x} + \mathrm{td}) - \mathrm{f}(\mathrm{x}) + \sum_{i=1}^{n} r_i ln\,(1 + \mathrm{t}y_i)$; $\qquad y = X^{-1}d$

Let $\rho = \min r_i \leq r_i$

Theorem

The Majorant function is given by:

$$\theta_1(t) = \frac{1}{\rho}\mathrm{f}(\mathrm{x} + \mathrm{td}) - \mathrm{f}(\mathrm{x}) - (\mathrm{n}-1)\ln(1 + \mathrm{t}\alpha) - \ln(1 + t\beta)$$

RESULTS & DISCUSSION

Convergence

Theorem.

Assume that f is convex and twice continuously differentiable, and that the feasible set K is nonempty. Then, the sequence $\{x^k\}$ generated by the proposed adaptive vector barrier interior-point method converges globally to the unique optimal solution.

Algorithm:

- Input: $x_0 \in K, r_0 \in \mathbb{R}^n, \varepsilon > 0.0 < \rho < 1$
- Iteration:
- Calculate d and $y^k = (X_k^{-1})d^k$
- If $||y|| > \epsilon$,
- Calculate t^*
- Calculate $x + t^*d$
- If $||y|| \leq \epsilon$, we have a good approximation
- If $||r|| \geq ||r_0||$, let $r = \rho r$, where $\rho r = (\rho_1 r_1, \rho_{21} r_2, , \ldots, \rho_n r_n,)$
- If $||r|| < ||r_0||$, Stop.

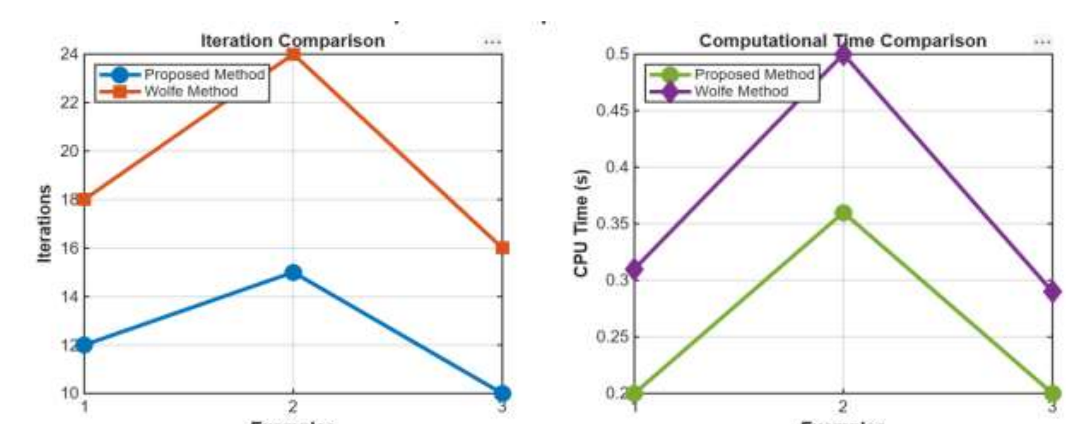


Figure 1. Performance comparison between the proposed adaptive vector barrier method and the Wolfe method.

Table 1. Comparative performance on three test examples

Exp	Iter	Time	Obj	Iter	Time	Obj
1	12	0.20	0.1252	18	0.31	0.1255
2	15	0.36	0.0817	24	0.50	0.0819
3	10	0.20	0.0321	16	0.29	0.019

CONCLUSION

An adaptive vector barrier interior-point method based on majorant functions is proposed for constrained convex optimization problems. The approach employs a componentwise adaptive barrier update together with an explicit step-size computation, avoiding classical line-search techniques. Theoretical analysis establishes descent, feasibility preservation, and global convergence. To evaluate its performance, the proposed method is compared with the Wolfe method, highlighting its efficiency and robustness, particularly for large-scale optimization problems.

FUTURE WORK/REFERENCES

Future work:

- Extension to Large-Scale Optimization Problems.
- Generalization to Nonlinear and Nonconvex Problems.

References:

1. J.-P. Crouzeix and B. Merikhi, A logarithmic barrier method for semidefinite programming, Mathematical Programming, 1990s.
2. B. Fellahi, B. Merikhi, A Logarithmic Barrier Approach via Majorant Function for Nonlinear Programming, J. Sib. Fed. Univ. Math. Phys. 16 (2023), 528–539.