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A Probabilistic Fuzzy Framework for Decision Making Under Uncertainty in Complex Systems

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INTRODUCTION & AIM

⚠ The Problem

Real-world decisions involve both vagueness (fuzzy ratings) and stochastic uncertainty (disagreement on importance). No single existing model handles both simultaneously.

🔍 What Exists

Probabilistic methods model randomness but use crisp weights. Fuzzy MCDA captures vagueness but uses fixed deterministic weights. Weight uncertainty is ignored in all existing methods.

💡 Our Solution

We assign an explicit probability distribution over q-ROFS fuzzy weight scenarios per criterion - the first framework to do so - with formal theoretical guarantees

Literature Gap

All existing probabilistic q-ROFS papers (Ranjan 2023, Attaullah 2023, Ashraf 2023, Yiarayong 2025) place probability inside ratings -- not on criterion weights.

METHOD

Fuzzy Sets & q-ROFS

q-Rung Orthopair Fuzzy Sets (q-ROFS), introduced by Yager (2017), extend classical fuzzy theory by assigning each element both a membership degree μ and a non-membership degree ν satisfying:

$$\mu^q + \nu^q \leq 1, q \geq 1$$

$$\pi = (1 - \mu^q - \nu^q)^{\frac{1}{q}} \text{ [hesitancy]}$$

When $q = 1$: *Intuitionistic Fuzzy Sets.*
When $q = 2$: *Pythagorean Fuzzy Sets.*
General q: *maximum flexibility in representing conflicting expert opinions as displayed in Figure 1*

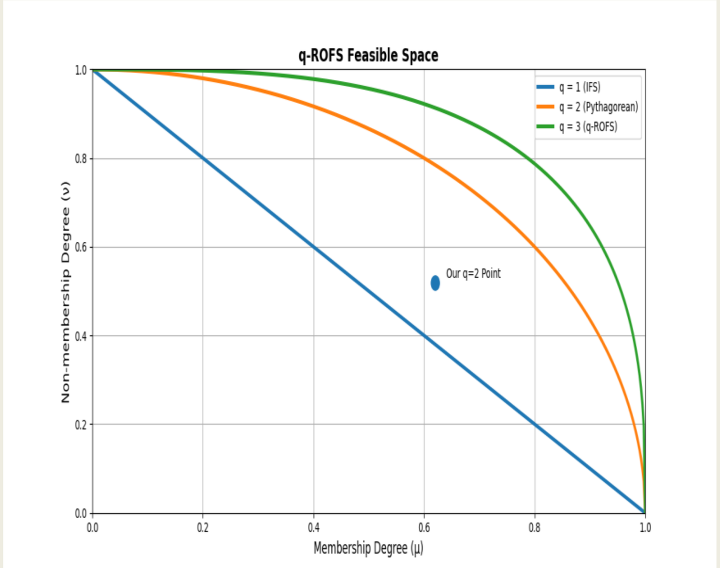

Figure 1: Feasible space under q-ROFS

Seven-step Decision Pipeline

Step 1: Problem Setup

Step 2: Linguistic Ratings
Words → q-ROFS pairs

Step 3: DM Aggregation
q-ROFWA operator

***Step 4: Prob. Weight Space**
$\mathcal{W}_j = (\widetilde{w_{jk}}, p_{jk})$

***Step 5: PFWA Operator**
$E[\widetilde{W_j}] \times \widetilde{x_{ij}}$

Step 6: Defuzzification
Score = $\mu^q - \nu^q$

Step 7: Ranking
Best alternative

***Probabilistic Fuzzy Weight Space**

$$\mathcal{W}_j = \{ (\widetilde{w}_{jk}, p_{jk}) : k = 1, \ldots, K \}$$

Probabilities derived endogenously from hesitancy:

$$p_{jk} = (1 - \pi_{jk}) / \Sigma_k (1 - \pi_{jk})$$

DMs who express more decisive weights receive higher probability automatically. No external calibration required.

***PFWA Aggregation Operator**

Output spread encodes decision uncertainty:
wide triangle = high disagreement,
narrow triangle = robust consensus.

Proven properties: Consistency · Boundedness · Monotonicity

Theorem 1 — Consistency

If all K weight scenarios are identical, $E[\widetilde{W}_j]$ reduces to that scenario regardless of the probability distribution.

Theorem 2 — Boundedness

PFWA output always satisfies μ, ν ∈ [0,1] and the q-ROFS constraint $\mu^q + \nu^q \leq 1$.

Theorem 3 — Monotonicity

Improving performance on any criterion under positive weight does not decrease the overall score $\widetilde{S}(A_i)$.

RESULTS & DISCUSSION

The proposed PFWA framework was evaluated using an oil supplier selection problem involving four alternative supplier nations A1,A2,A3 and A4 assessed across six strategic criteria provided in Table 1 under a q-rung orthopair fuzzy environment ($q = 2$). The obtained results demonstrate the effectiveness of probabilistic fuzzy weighting in capturing stakeholder uncertainty and improving ranking reliability.

TABLE 1. Evaluation Criteria for Oil Supplier Selection

Criteria	Evaluation Criterion	Description
C1	Price Stability	Consistency and predictability of crude oil pricing
C2	Supply Reliability	Ability to maintain uninterrupted supply under geopolitical uncertainty
C3	Transportation Cost	Logistics and shipping expenditure associated with import operations
C4	Environmental Compliance	Adherence to international environmental and sustainability standards
C5	Contract Flexibility	Adaptability of trade agreements and procurement contracts
C6	Strategic Alliance Strength	Long-term diplomatic and strategic relationship with importing nation

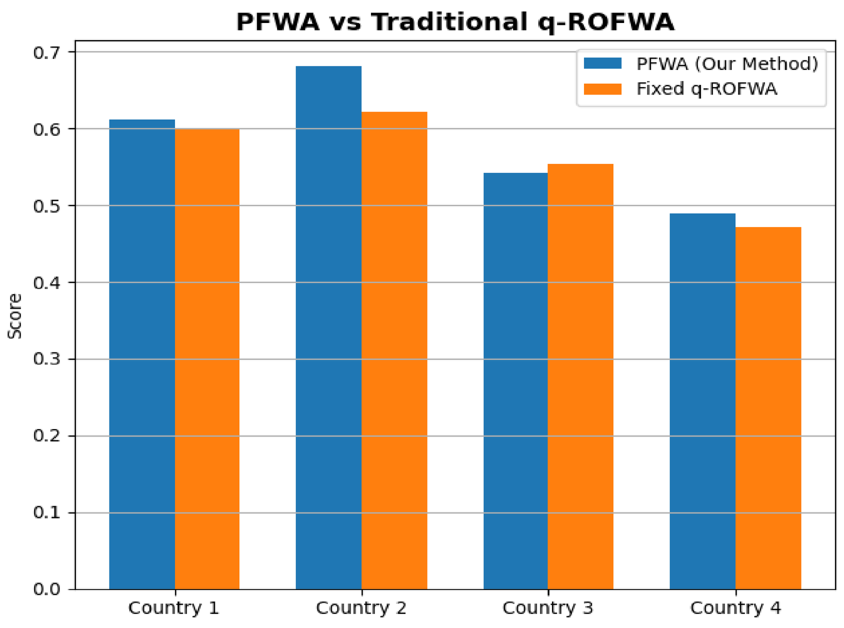

Figure 2. Comparative Analysis of PFWA and Traditional q-ROFWA Rankings

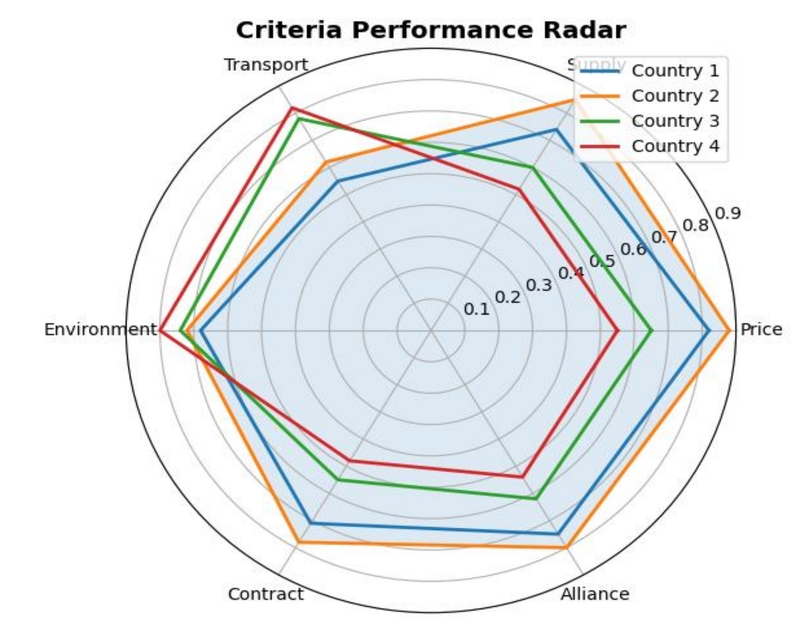

Figure 3. Radar Analysis of Criterion-Wise Performance of Supplier Alternatives

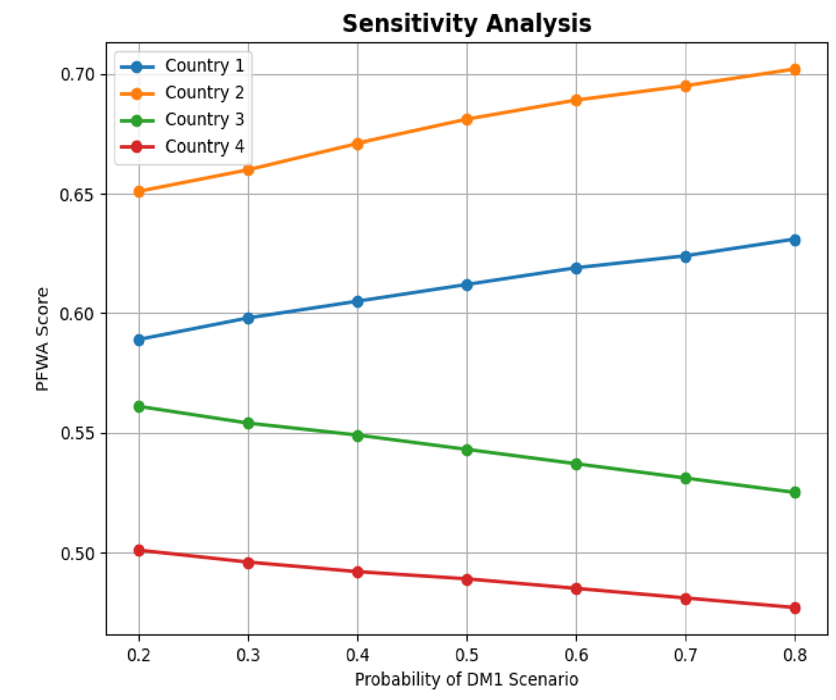

Figure 4. Sensitivity Analysis of PFWA Scores Under Probability Perturbation

Discussion

The final preference order obtained using the proposed PFWA framework is

$$A_2 \succ A_1 \succ A_3 \succ A_4$$

indicating that Country 2 is the most suitable oil supplier alternative under the probabilistic q-ROFS environment. Although both PFWA and traditional q-ROFWA methods produced the same ranking order, the proposed PFWA framework generated greater score differentiation among alternatives by incorporating probabilistic uncertainty in criterion weights. This demonstrates the capability of the proposed model to better represent stakeholder confidence and uncertainty during aggregation.

CONCLUSION

- First framework to model probability distributions over q-ROFS criterion weight scenarios.
- PFWA operator: proven Consistency, Boundedness, and Monotonicity.
- Hesitancy-derived probabilities eliminate need for external weight calibration.
- Fixed-weight methods produce rank reversals; our method avoids them

FUTURE WORK

- Extend to interval-valued q-ROFS settings
- Apply to supply-chain MCDA and healthcare decisions
- Develop software toolkit for practitioners

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