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Optimal Control Strategies for Predator-Prey Systems with Bang-Bang and Quadratic Control Terms

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INTRODUCTION & AIM

Classical predator-prey models such as the Lotka-Volterra framework have been widely used to describe ecological interactions, but their applicability to sustainable management is restricted due to the absence of intraspecies competition and simplistic assumptions regarding interaction intensity. To overcome these limitations, researchers have considered additional ecological features and applied optimal control theory as a powerful tool for developing sustainable management strategies. In our study, we consider an existing predator-prey system with internal competition to investigate two distinct management objectives. In the first strategy, our aim is to add the prey population in an optimal way so that the predator population grows in a finite time, reducing the risk of predator extinction while quadratically penalizing the control effort through a running cost functional. The second strategy aims to identify the optimal mixing time using a bang-bang control that maximizes the total population at a fixed final time T. Together, these two cases show how different cost structures and control types can result in different optimal strategies, providing a greater understanding of how continuous and bang-bang interventions influence ecological outcomes in predator–prey systems.

UNCONTROLLED SYSTEM, PRESENTED BY [1]

$$\frac{dx(t)}{dt} = a_1x(t) - b_1x(t)y(t) - a_2x(t)^2,$$
$$\frac{dy(t)}{dt} = -a_3y(t) + b_2x(t)y(t) - a_4y(t)^2.$$

x - prey population, y – predator population

a_1- prey growth rate b_1- predator consumption rate

a_3- predator death rate b_2- predator reproduction rate from prey

a_2- prey intraspecific competition

a_4- predator intraspecific competition

METHOD

- Develop two optimal control models of the predator–prey system with corresponding objective functionals for the two scenarios.
- Define the Hamiltonian, adjoint equations, transversality conditions, and optimality conditions for both cases.
- Apply Pontryagin's Maximum Principle to derive the necessary conditions for optimality.
- Present numerical results using MATLAB.

CONTROLLED SYSTEM & OBJECTIVE FUNCTIONAL

Quadratic control strategy:

$$\frac{dx(t)}{dt} = a_1x(t) - b_1x(t)y(t) - a_2x^2(t) + \mathbf{u(t)}x(t),$$
$$\frac{dy(t)}{dt} = -a_3y(t) + b_2x(t)y(t) - a_4y^2(t).$$

- Where $\mathrm{u(t)}$ represents the amount of prey addition. $(0 \le \mathrm{u(t)} \le \mathrm{umax})$

$$J(u(t)) = \int_0^T \left(-c_1y(t) + c_2u^2(t)\right) dt.$$

Bang-bang control strategy:

$$\frac{dx(t)}{dt} = a_1x(t) - b_1x(t)y(t)\mathbf{u(t)} - a_2x^2(t),$$
$$\frac{dy(t)}{dt} = -a_3y(t) + b_2x(t)y(t)\mathbf{u(t)} - a_4y^2(t).$$

- Where $(1-\mathrm{u(t)})$ represents the segregation rate at time t. $(0 \le \mathrm{u(t)} \le 1)$

$$J(u) = x(T) + y(T).$$

RESULTS & DISCUSSION

Quadratic control strategy:

For the parameters: a_1=2.0, b_1=1.1, a_2=0.1, a_3=1.0, b_2=0.9, a_4=0.1.

Figure 1: Prey population with and without control

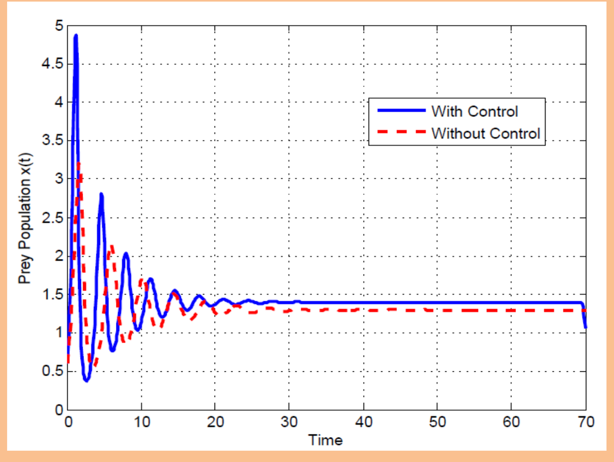
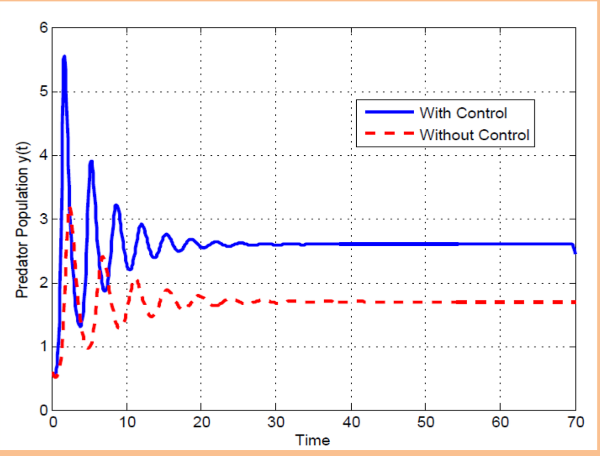


Figure 2: Predator population with and without control



For the parameters: a_1=0.9, b_1=0.1, a_2=0.1, a_3=0.1, b_2=0.1, a_4=0.1.

Figure 3: Prey population with and without control

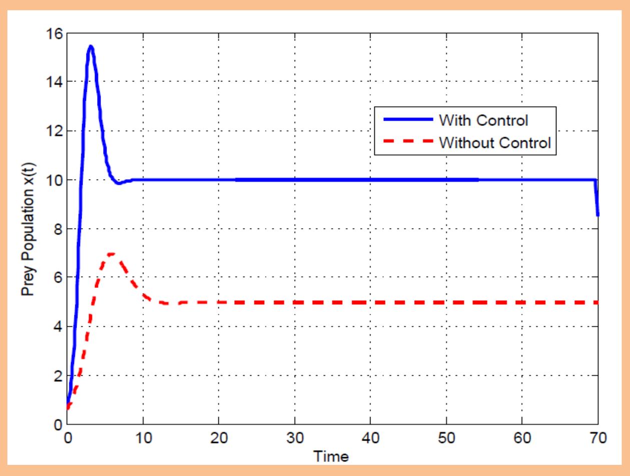
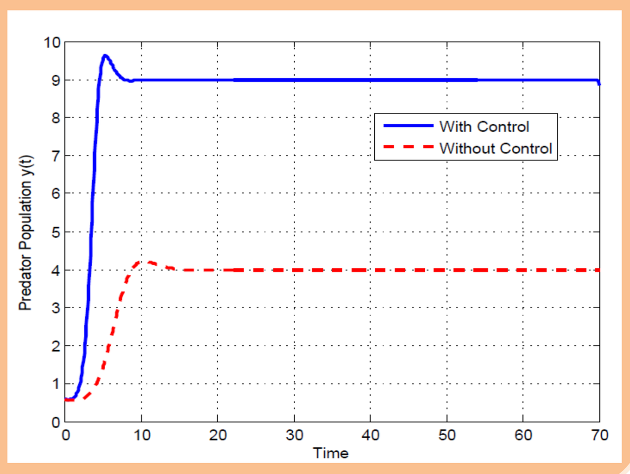


Figure 4: Predator population with and without control



Bang-bang control strategy:

For the parameters: a_1=1.0, b_1=0.2, a_2=0.3, a_3=0.2, b_2=0.3, a_4=0.3.

Figure 5: Cost functional with respect to the switching point

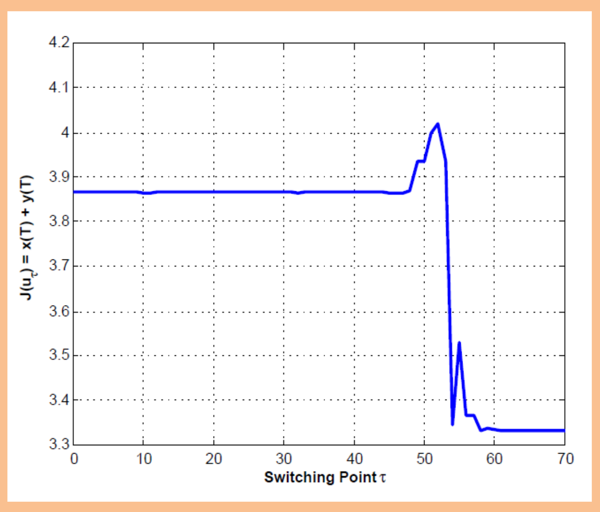
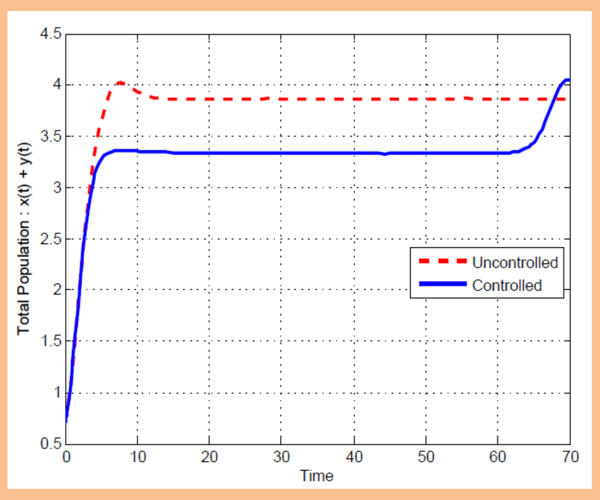


Figure 6: Total population with respect to time for both controlled and uncontrolled systems for $\tau = 52$



- For a final time of $T = 70$, Figure 5 shows that the optimal switching point occurs at approximately $\tau = 52$ and the maximal value of the cost functional is 4.0199.

CONCLUSION

- Quadratic control strategy**:** Effectively increases the predator population by adding prey over a finite time.
- Bang-bang control strategy**:** Maximizes the total population at the final time using an optimal switching strategy.

REFERENCES

1. A Wusu and O Olabanjo. "Dynamics of Sustainable Fisheries: A Mathematical Approach using Lotka-Volterra Equations". In: (2023).

FUTURE WORK

- Explore more information about the structure of the optimal control in our second case by analyzing the term $(\lambda_2b_2 - \lambda_1b_1)$ and find the number of switching points analytically.

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