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Performance Degradation and Stabilization of Semilinear Stochastic Systems Driven by α-Stable Lévy Noise

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1 INTRODUCTION & AIM

The Problem: Why does classical theory break? Stochastic control is built on continuous paths of Gaussian (Wiener) noise with finite variance. However, real systems often experience sudden, heavy-tailed, discontinuous jump behavior that this model cannot represent. Ubiquitous in:

- **Power grids** – fault cascades
- **Cyber-networks** – anomaly shocks
- **Finance** – price jumps

Main Challenge: For an α-stable Lévy process with index $\alpha \in (1,2)$, the second moment is infinite. Every L^2/mean-square tool, including Itô isometry, Lyapunov invariance, and Riccati design, fails at its core.

Our Aim: To refine the analysis in the Banach space $L^p(\Omega, H)$ with fractional moments $1 < p < \alpha$, and prove **finite-time stabilization** and **finite-approximate controllability** for semilinear impulsive stochastic evolution equations on a separable Hilbert space.

2 MATHEMATICAL FRAMEWORK

The controlled impulsive system on a separable Hilbert space H:

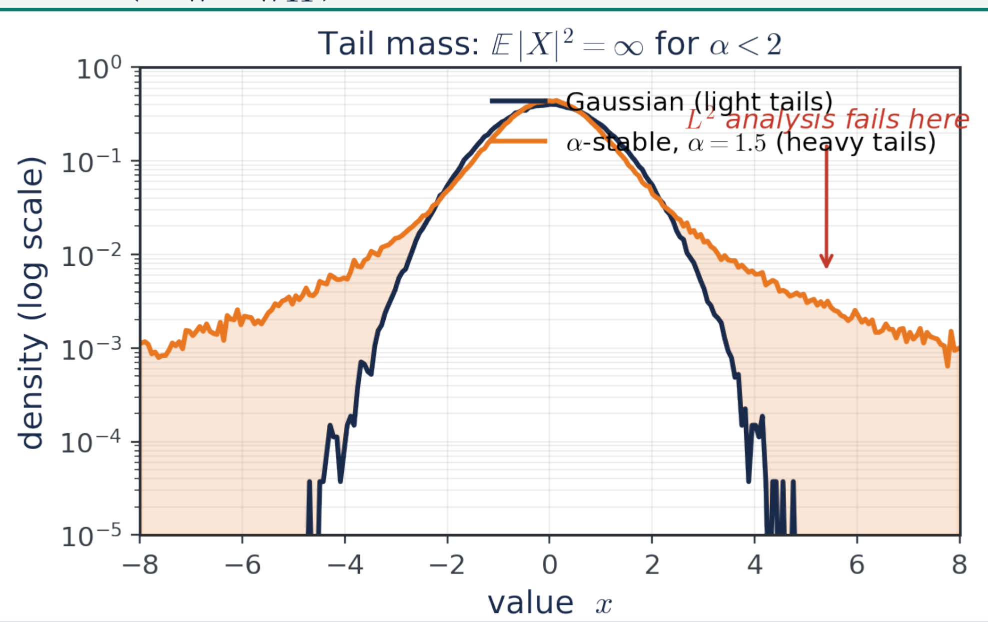
$$dX(t) = [\,A\,X(t) + f(t, X(t)) + B\,u(t)\,]\,dt + g(t, X(t))\,dL_\alpha(t), \quad t \neq t_k$$

$$\Delta X(t_k) = I_k(X(t_k^-)), \quad k = 1, \dots, m, \qquad X(0) = X_0$$

A: generates a C_0-semigroup S(t); **f, g:** nonlinear drift/diffusion; **I_k:** impulse maps; **L_α:** symmetric α-stable Lévy process.

KEY IDEA: work in the fractional-moment norm

$$\|X\|_{L^p} = (\mathbb{E}\,\|X\|_H^p)^{1/p} < \infty, \qquad 1 < p < \alpha < 2$$



3 METHODOLOGY

Mild-solution fixed-point operator Φ:

$$(\Phi X)(t) = S(t)X_0 + \int_0^t S(t-s)[f + Bu]\,ds + \int_0^t S(t-s)\,g\,dL_\alpha(s) + \sum_{0<t_k<t} S(t-t_k)\,I_k$$

1. **Reformulate:** Cast the dynamics in $L^p(\Omega, H)$, 1<p<α, where fractional moments of the α-stable driver stay finite.
2. **Construct:** Build the operator above from the mild solution; a fixed point of Φ is exactly the stabilizing trajectory.
3. **Contraction:** Picard iteration + Banach fixed-point theorem in the p-th moment norm yields a unique fixed point under Lipschitz hypotheses.
4. **Simulation:** Generate symmetric α-stable increments using CMS and verify the controller's behavior numerically.

4 RESULTS & DISCUSSIONS

THEOREM 1 [Existence & uniqueness]
Unique mild solution in Lp(Ω, H) for the impulsive system, proved via Picard iteration under contraction in p-th moment norm.

THEOREM 2 [Finite-approximate controllability]
If the linear part is approximately controllable and the resolvent condition holds, then for every ε>0 and target x_T, the feedback steers X(T) into the ε-ball about x_T.

THEOREM 3 [Finite-time stabilization]
There is a feedback law and settling time T*<∞ with $\mathbb{E}\|X(t)\|_H^p \to 0$ for t ≥ T*, despite heavy-tailed α-stable shocks.

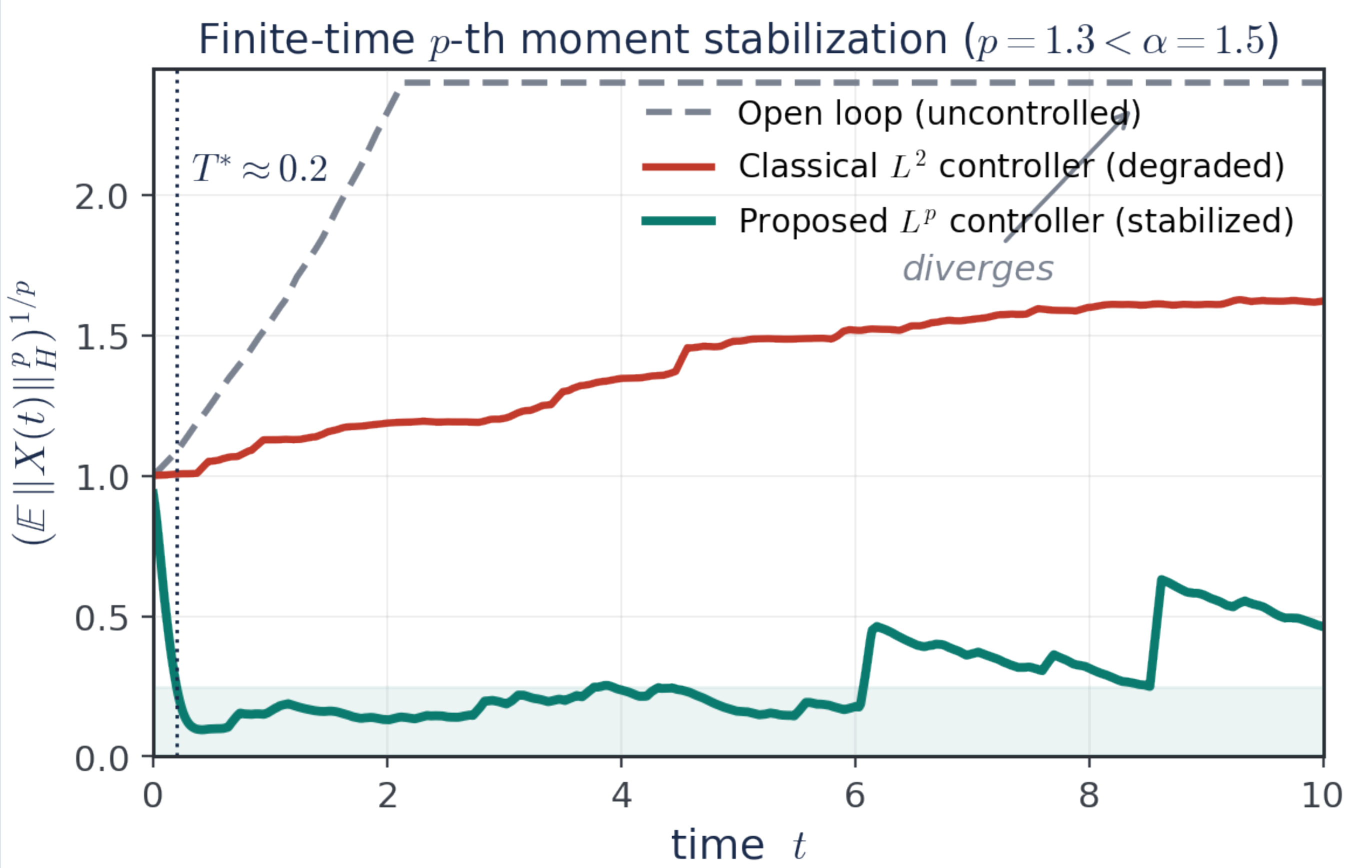


Fig. 2 · Monte-Carlo p-th-moment (p=1.3) under CMS-simulated α-stable noise (α=1.5). Open loop diverges; classical L^2 controller degrades to a large residual; the proposed L^p controller reaches a tight ball in finite time T and suppresses each jump.*

Reading the result: The gap between the red and teal curves **quantifies** the degradation of variance-based design under jumps and shows the fractional-moment law recovering stability where L^2 cannot.

5 CONCLUSION

Moving from L^2 to L^p (1<p<α) is not a technical convenience; it is the ***right home*** for control under heavy-tailed, impulsive noise. Within it, we obtain a unique stabilizing trajectory, finite-time decay of the p-th moment, and finite-approximate controllability, providing a rigorous foundation for **resilient controllers in safety-critical, non-Gaussian systems.**

6 FUTURE WORK & REFERENCES

Future directions

- Tempered and fractional-order operators for sub-diffusive jump regimes.
- Data-driven identification of the stability index α from grid/network telemetry.
- Output-feedback and observer design under partial, jump-corrupted observations.
- Multi-agent networks with α-stable inter-agent coupling.

References

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Keywords: stochastic control · Lévy noise · finite-time stabilization · impulsive systems · controllability · fractional moments

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