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Mathematical Modeling and Free Vibration Analysis of Nonhomogeneous Nanobeams with Axially Varying Nonlocal Parameter

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INTRODUCTION & AIM

Introduction:

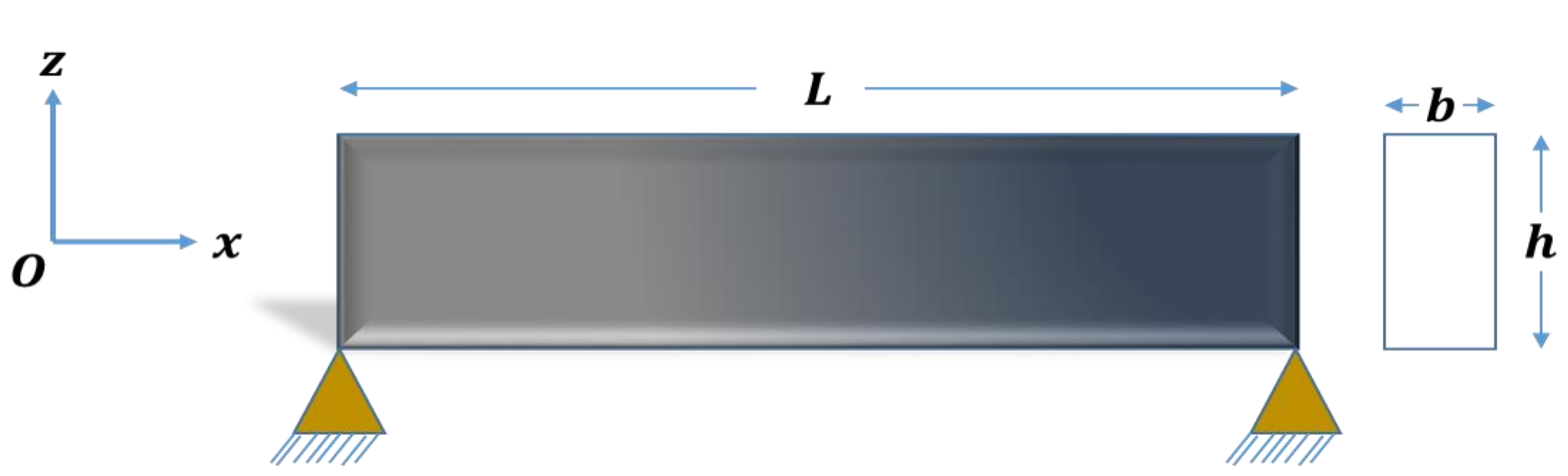
- Nonhomogeneous nanobeams, particularly axially functionally graded (AFG) nanobeams, possess continuously varying material properties, making them suitable for advanced nano-engineering applications.

Research Gap:

- Most existing studies assume a constant nonlocal parameter throughout the nanobeam.
- In practical AFG nanobeams, the nonlocal parameter may also vary along the grading direction.

Objective:

- Develop a vibration model for axially FG nanobeams with a linearly varying nonlocal parameter.
- Investigate the effects of power-law exponent (k), variable nonlocal parameters (a, b), and material gradation on the natural frequencies.



METHOD

Material Properties [1, 2]:

Young's modulus: $E(x) = E_l\, V(x) + E_r\,(1 - V(x))$,

Mass density: $\rho(x) = \rho_l\, V(x) + \rho_r\,(1 - V(x))$,

where the volume fraction $V(x) = \left(1 - \frac{x}{L}\right)^k$, subscripts l and r denote the left and right sides of the nanobeam, and k is the power-law exponent (non-negative).

Variable Nonlocal Parameter [2]:

$$\mu(x) = a + bx,$$

where a and b are constants.

The maximum strain (S) and kinetic (T) energies of the Euler-Bernoulli AFG nanobeam incorporating Eringen's nonlocal theory are expressed as [2]:

$$S_{max} = \frac{1}{2}\int_0^L \left(\begin{array}{l} J_0\left(\frac{dU_0}{dx}\right)^2 - J_1 \frac{d^2W_0}{dx^2}\frac{dU_0}{dx} - \omega^2\,(\mu(x))^2\left(I_0\left(\frac{dU_0}{dx}\right)^2 - I_1\frac{d^2W_0}{dx^2}\frac{dU_0}{dx}\right) - J_1\frac{dU_0}{dx}\frac{d^2W_0}{dx^2} \\ + J_2\left(\frac{d^2W_0}{dx^2}\right)^2 + \omega^2\,(\mu(x))^2\left(I_0\,W_0\frac{d^2W_0}{dx^2} + I_1\frac{dU_0}{dx}\frac{d^2W_0}{dx^2} - I_2\left(\frac{d^2W_0}{dx^2}\right)^2\right) \end{array} \right) dx,$$

$$T_{max} = \frac{\omega^2}{2}\int_0^L \left(I_0\left(U_0^2 + W_0^2\right) - 2I_1\,U_0\frac{dW_0}{dx} + I_2\left(\frac{dW_0}{dx}\right)^2 \right) dx.$$

where

$$(J_0, J_1, J_2) = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(x)(1, z, z^2)dz\,, (I_0, I_1, I_2) = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho(x)(1, z, z^2)dz$$

Implementation of Rayleigh-Ritz Method:

Here, the values of natural frequency ω are found using the Rayleigh-Ritz method. In this approach, the displacement functions $U_0(x)$ and $W_0(x)$ are expressed as (for simply-supported boundary conditions) [2]:

$$U_0(x) = x(L - x)\sum_{i=1}^{N} c_i\; x^{i-1}, \qquad W_0(x) = x(L - x)\sum_{j=1}^{N} d_j\; x^{j-1},$$

The Rayleigh Quotient is obtained by equating the expressions for the maximum strain energy with the maximum kinetic energy. Then by partially differentiating it by c_i and d_j, a system of $2N$ linear equations is generated, which is represented as:

$$([K] - \lambda^2[M])\{\triangle\} = 0,$$

where $[K]$ and $[M]$ are stiffness and inertia matrices, respectively, $\{\triangle\}$ denotes the column vector containing c_i's and d_j's, and the frequency parameter λ is defined by $\lambda = \omega L^2\sqrt{(\rho_l A)/(E_l I)}$, where I is the moment of inertia.

RESULTS & DISCUSSION

k	a	b	λ_1	λ_2	λ_3	λ_4
0	0	0	9.8692	39.4719	88.7936	157.8099
	0	0.2	9.3522	31.9999	60.0851	90.0130
	0.1	0.2	8.2992	24.0160	40.6329	57.1498
	0.1	0.3	7.6467	20.5155	33.6940	46.7374
1	0	0	7.0305	28.1832	63.4332	112.7592
	0	0.2	6.5858	21.8572	40.1720	59.4307
	0.1	0.2	5.7952	16.3291	27.3910	38.3995
	0.1	0.3	5.2750	13.7990	22.4935	31.1140
3	0	0	5.7321	23.7285	53.7589	95.8016
	0	0.2	5.3893	18.6316	34.6167	51.4234
	0.1	0.2	4.7542	13.9012	23.4582	32.9544
	0.1	0.3	4.3458	11.7645	19.2830	26.7355
5	0	0	5.4117	22.4032	50.8812	90.7772
	0	0.2	5.1055	17.7720	33.2388	49.5475
	0.1	0.2	4.5150	13.2701	22.4730	31.6111
	0.1	0.3	4.1416	11.2556	18.5074	25.6866

It is observed that higher values of the power-law exponent result in lower frequency parameter values. Similarly, increasing the nonlocal parameter (by increasing $(a, b,$ or both) also reduces the frequency parameters.

CONCLUSION

- A vibration model for axially FG nanobeams with variable nonlocal parameter is developed.
- Rayleigh-Ritz method is implemented to obtain the values of frequency parameters.
- Future research could explore more complex boundary conditions, multi-scale modeling, and the impact of environmental factors.

FUTURE WORK / REFERENCES

1. Alshorbagy, A. E., Eltaher, M. A., & Mahmoud, F. (2011). Free vibration characteristics of a functionally graded beam by finite element method. *Applied mathematical modelling*, *35*(1), 412-425.
2. Gartia, A. K., & Chakraverty, S. (2025, January). Effect of Variable Nonlocal Parameter on the Free Vibration of Axially Functionally Graded Nanobeams. In *International Conference on Computational Mathematics and Applications* (pp. 293-303). Singapore: Springer Nature Singapore.

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