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Strictly Anti-Diagonally Dominant Matrices in Systems of Yang–Baxter Matrix Equations

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INTRODUCTION & AIM

Discovered in the late 20th century, the Yang–Baxter equation is a parameter-free relation with extensive applications in both physics and mathematics Ding [2]. The earliest hint of this structure appeared in [3], where Onsager's 1944 solution of the Ising model, where he indirectly referred to the star-triangle relation, a precursor to the Yang–Baxter framework in statistical mechanics. The equation was formally introduced by C. N. Yang [4] in two landmark papers published in late 1967 on a one-dimensional quantum many-body problem. Yang established the form A(u)B(u + v)A(v) = B(u)A(u + v)B(v),where A(u) and B(v) are rational functions of the spectral parameters u and v. Later, in1972, R. J. Baxter [1] employed the same relation while solving the eight-vertex model in two-dimensional statistical mechanics. The term Yang–Baxter equation was coined by L. Faddeev [5] in the late 1970s to describe a unifying integrability principle connecting diverse areas of mathematics and physics. Conceptually, the equation describes a transformation F that governs how the states of two particles evolve after interaction. This study explores a system of Yang–Baxter-type matrix equations, XAX=BXB and XBX=AXA, which generalize the classical matrix Yang–Baxter equation. This work focuses on analyzing the existence of intertwining and commuting solutions using geometric and topological methods. To support this analysis, the notions of anti-diagonally dominant matrices (ADMs) and strictly anti-diagonally dominant matrices (SADMs) are introduced. It is shown that strictly anti-diagonally dominant matrices are nonsingular, ensuring stability and uniqueness in the associated linear systems. Furthermore, if the coefficient matrices of the system satisfy the SADM condition, then an intertwining solution X exists that fulfils both Yang–Baxter-type relations.

The following definition formalizes the concepts of an anti-diagonally dominant matrix (ADM) and a strictly anti-diagonally dominant matrix (SADM).

Suppose we have the matrix A = $[a_{ij}]n \times n$. Then

(i) A is called an anti-diagonally dominant matrix (ADM) whenever $|a_{i,n-i+1}| \geq \sum^{n}_{\substack{j=1 \\ j\neq n-i+1}} |a_{ij}|$, i = 1, 2, . . . , n.

(ii) A is called a strictly anti-diagonally dominant matrix (SADM) whenever $|a_{i,n-i+1}| > \sum^{n}_{\substack{j=1, \\ j\neq n-i+1}} |a_{ij}|$,i = 1, 2, . . . , n.

RESULTS & DISCUSSION

Theorem 3.1. Let A = $[a_{ij}] \in C_{n\times n}$be strictly anti-diagonally dominant by rows, i.e.,$|a_{i,n\text{-}i+1}| > \sum^{n}_{\substack{j=1, \\ j=n-i+1}} |a_{ij}|$, i = 1, 2, . . . , n.

Then A is nonsingular.

Theorem 3.2. Let B = $[b_{ij}]n$×n be a strictly anti-diagonally dominant matrix by rows such that XA = BX, where A=$[a_{ij}]n$×n. Then X is nonsingular matrix.

Theorem 3.3. If the coefficient matrix A of a system of n linear equations Ax = b is strictly anti-diagonally dominant matrix (SADM), then the system has a unique solution given by Cramer's rule: xi = $\frac{det(Ai)}{det(A)}$, i = 1, 2, . . . , n, where Ai is obtained from A by replacing its i-th column with the vector b.

Theorem 3.4. If A and B are coefficient matrices of the system of linear equations such that acts as a strictly anti-diagonally dominant matrix (SADM). Then X is an intertwining solution of the Yang-Baxter matrix equation, i.e. X · A · X = B · X · B and X. BX = AX · A

Corollary 3.5. Let B = $[b_{ij}]n \times n$ be a strictly anti-diagonally dominant matrix by rows such that XA = BX, where A = $[a_{ij}]$n×n. Then A and B are similar i.e. A = X^{-1}BX.

Theorem 3.6. Let A, B, X ∈$F_{n\times n}$ such that XA = BX. If either of the following conditions hold: **(i)** X commutes with A, i.e. XA = AX, **(ii)** X commutes with B, i.e. XB = BX, (iii) A = B, then X is an intertwining solution of the Yang–Baxter type matrix equations XAX = BXB and XBX = AXA.

Theorem 3.7. Let A = $[a_{ij}]_{n\times n}$ and B =$[a_{ij}]_{n\times n}$ be a anti-diagonal square matrices. Suppose X ∈ $C_{n\times n}$ satisfy the Yang-Baxter matrix equations X·A·X= B·X·B and X·BX = AX·A such that X acts as a Drazin inverse matrix. Then X = SJ, S = diag(s_1,s_2 . . . ,s_n).

CONCLUSION

In this work, we examined a system of Yang–Baxter-type matrix equations, XAX = BXB, and XBX = AXA, which extends the classical matrix Yang–Baxter framework. Through ge- ometric and topological analysis, we explored the structure and properties of their solutions, emphasizing the role of intertwining relationships between matrices A, B, and X. The con- ditions ensuring the existence of such intertwining solutions were identified and characterized. Moreover, we introduced and investigated the concepts of strictly anti-diagonally dominant and anti-diagonally dominant matrices, which provide a new perspective for understanding the solvability and stability of Yang–Baxter-type systems.

FUTURE WORK / REFERENCES

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