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The Alpha Group: Holonomic Structure and Dynamic Coupling of Dual Hopf Topologies in a Nontrivial Topological Space

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INTRODUCTION & AIM

The Alpha group, characterized by its matrix M_θ and intrinsic antisymmetry, reveals higher-order structures that enable the exploration of persistent paths and complex topology through graph-based representations.Foundational work by Donaldson, Kronheimer, and Connes establishes coherence as a key structural principle—whether via geometric connections on four-manifolds or through operator algebras in noncommutative geometry (NCG), where idempotents act as structural projectors replacing classical points with coherent modules. Both frameworks converge on the idea that intrinsic selection mechanisms constrain absolute freedom, aligning with the Frobenius approach to coherent structure emergence. This work argues that Alpha group geometry—dynamic and generated by algebraic operations—differs structurally from classical Riemannian geometry, moving from a restricted surface-based geometry to a global, dynamic exploration of space.

METHOD

The Adjacency Matrix A is the first crucial step in analyzing the system's topology. It acts as a simplifying filter, translating the complex operator M_θ into a binary graph G. A connection (edge) between two points (i and j) in G is established only if the intensity of their interaction in M_θ is greater than a predefined threshold ε (receiving a value of 1). If the interaction is weak, it receives a value of 0 (no edge). This process simplifies the system's complex geometry, focusing solely on the strong, coherent structures, thus enabling the use of basic graph theory tools to estimate the Betti numbers H(k) and quantify the underlying holes and cycles.

RESULTS & DISCUSSION

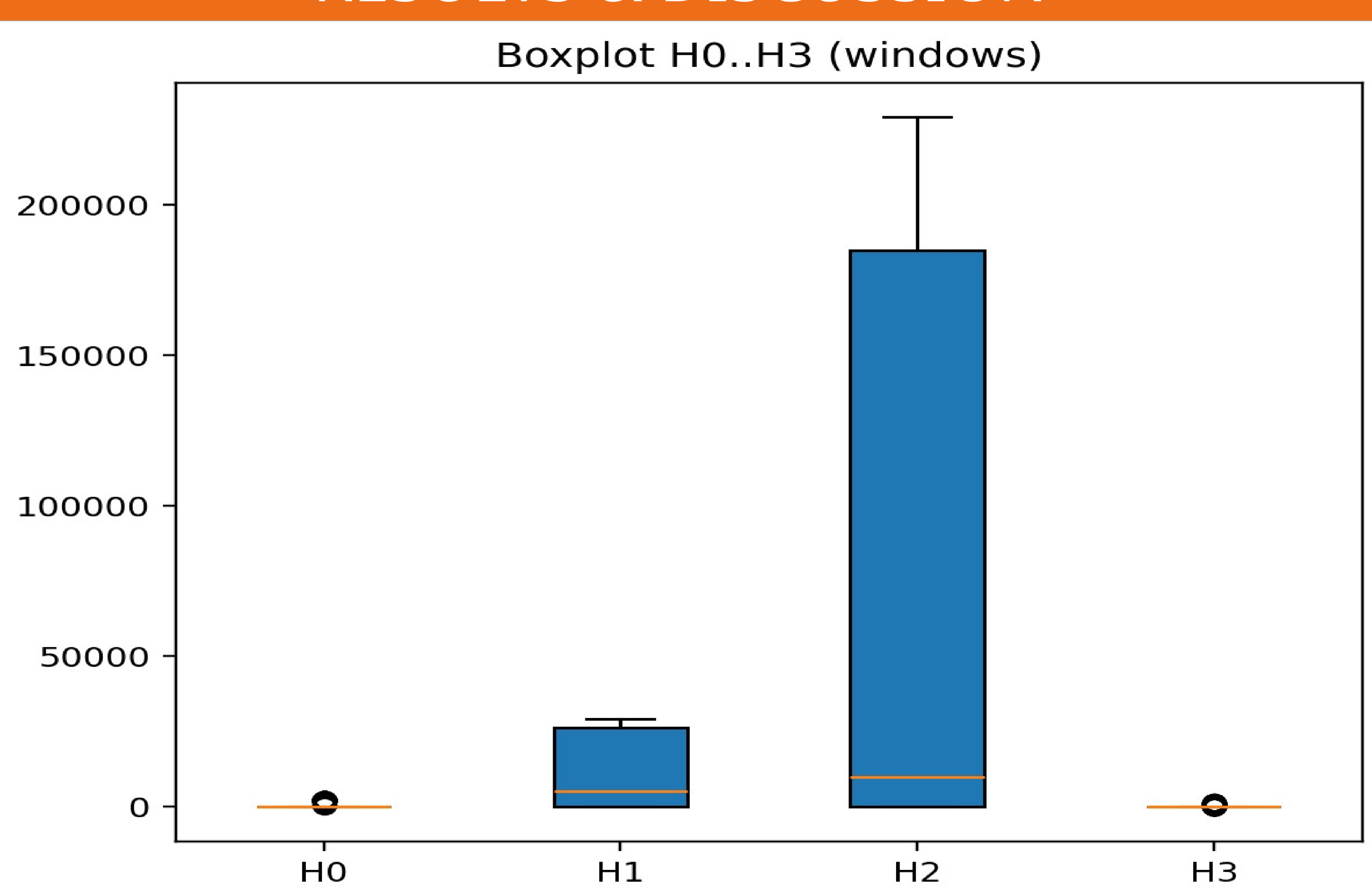

Figure 1: Dynamics of Betti numbers in the Alpha Group topology. The plot shows the stability of H3, the dynamic rise of H2, and the scarcity of H1, defining the characteristic non-Riemannian signature of the space. For θ = 0, the matrix M_θ (via the projective action of μ) fragments the topology, yielding $b_0 > 1$ and recovering the disconnected regime of Riemannian geometry. For θ = π/2, M_θ establishes planar links, producing $b_0 = 1$ and a single component locally homeomorphic to R^2. This connectivity shift, driven by μ, marks the geometric phase transition that defines the Alpha Group (non-Riemannian geometry). H2 is non-classical and emergent, while H3 acts as a topological invariant anchoring the Alpha structure.

The Alpha Group exhibits a unique homological signature: stable 3-cycles, explosive 2-cycle formation, and minimal 1-cycle presence.

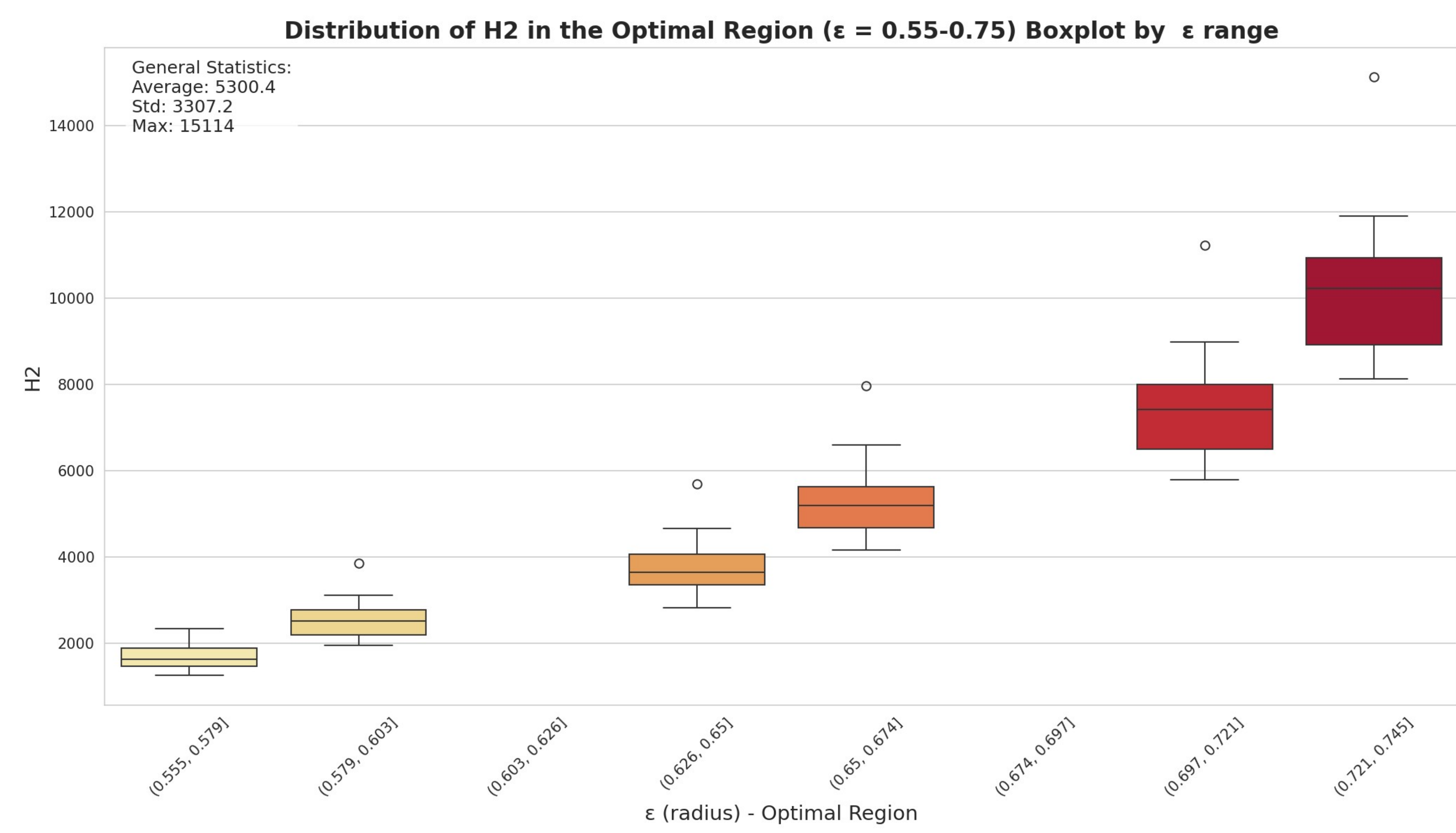

Figure 2: Distribution of H2 in the optimal region (ε = 0.55–0.75). The plot highlights the average (5,300 cycles), peaks exceeding 15,000, and the significant dispersion (standard deviation 3,307), illustrating a regime of maximal topological fertility in the Alpha Group.

CONCLUSION

Main Structural Result :The Alpha Group, generated by {1, i, μ, iμ} uses a division operation ⊘ realized by a 4×4 matrix M_θ. This matrix encodes space's four fundamental directions, imposes intrinsic anisotropy, and generates emergent dynamic topology. Persistent homology shows H3 as a topological anchor and H2 peaking dynamically, revealing evolving 2-cycles. The Alpha Group provides an alternative framework in topological exploration, moving beyond Riemannian constraints through algebraic operations. Using persistent homology and graph theory, we demonstrate dynamic H2 cycles and stable H3 structures that confirm its distinct structural behavoir for global geometric analysis. The Alpha Group geometry can be viewed as a sub-Riemannian Carnot–Carathéodory structure emerging from operator-based dynamics. The Alpha Group characterizes geometry via invariant structures under dynamic group transformations, following Klein's Erlangen Program.

FUTURE WORK / REFERENCES

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