IOCMA 2026 Conference

The 2nd International Online Conference on Mathematics and Applications

10–12 June 2026 | Online

MDPI

Structural Identities Determined by Prime Ideals through Generalized P-Derivations

Zakia Z. Al-Amery

Department of Mathematics, Aden University, Aden, Yemen

ABSTRACT

This paper focuses on the investigation of commutativity properties of the factor ring $\mathfrak{R}/P$, where P is assumed to be a prime ideal of an arbitrary ring $\mathfrak{R}$. The main objective is to determine sufficient algebraic conditions under which the quotient structure $\mathfrak{R}/P$ exhibit commutative behavior. To achieve this goal, the study employs the concept of generalized P-derivations F and $\sqcup$ associated with P-derivations χ and $\propto$, respectively. These generalized mappings are required to satisfy specific functional identities that create intrinsic links between the ring $\mathfrak{R}$ and its prime idea P

INTRODUCTION & AIM

Throughout this article, the symbol $\mathfrak{R}$ denotes an associative ring with center $Z(\mathfrak{R})$. A ring $\mathfrak{R}$ is said to be a prime ring if for any elements $\upsilon,\wp \in \mathfrak{R}$ the condition $\upsilon\mathfrak{R}\wp = \{0\}$ implies that at least one of the elements υ or $\wp$ must be zero. A prime ideal is a proper ideal P of a ring $\mathfrak{R}$ such that if $\upsilon\mathfrak{R}\wp \subseteq P$, then at least one of the elements υ or $\wp$ must be belong to P. A ring $\mathfrak{R}$ is said to be an integral domain if it is a commutative ring with unity and has no zero divisors. Every integral domain is a prime ring, but the converse is not true in general. For all $\upsilon,\wp \in \mathfrak{R}$, the symbols $[\upsilon, \wp] = \upsilon\wp-\wp\upsilon$ and $(\upsilon \circ \wp) = \upsilon\wp+\wp\upsilon$ denote the commutator and anticommutator, respectively. For a subset Θ of $\mathfrak{R}$, a mapping $\chi : \Theta \to \mathfrak{R}$ is said to be centralizing (or commuting) on Θ if $[\chi(\upsilon),\upsilon] \in Z(\mathfrak{R})$ (or $[\chi(\upsilon),\upsilon] = 0$) for all $\upsilon \in \Theta$. The mapping $\chi : \mathfrak{R} \to \mathfrak{R}$ is called P-additive if it satisfies $\chi(\upsilon +\wp)-\chi(\upsilon)-\chi(\wp) \in P$ for all $\upsilon,\wp \in \mathfrak{R}$. A P-additive mapping χ is called a P-derivation if it satisfies the relation $\chi(\upsilon\wp) -\chi(\upsilon)\wp-\upsilon\chi(\wp) \in P$ for all $\upsilon,\wp \in \mathfrak{R}$. A P-additive mapping $F: \mathfrak{R} \to \mathfrak{R}$ is called a generalized P-derivation associated with a P-derivation χ if it satisfies $F(\upsilon\wp) - F(\upsilon)\wp-\upsilon\chi(\wp) \in P$ for all $\upsilon,\wp \in \mathfrak{R}$. Additionally, assuming χ is a P-trivial (i.e., $\chi(\mathfrak{R}) \subseteq P$) in the last relation gives us a P-left multiplier concept, defined as $\Xi(\upsilon\wp) - \Xi(\upsilon)\wp \in P$ for all $\upsilon,\wp \in \mathfrak{R}$. The P-right multiplier is defined as $\Xi(\upsilon\wp) -\upsilon\Xi(\wp) \in P$ for all $\upsilon,\wp \in \mathfrak{R}$. Moreover, Ξ is considered a P-multiplier if it is both a P-left and P-right multiplier. It is clear that every generalized derivation is a generalized P-derivation, and that every left multiplier is also a P-left multiplier, but the converse may not be true in general.

In this work, we will further investigate the commutativity of a factor ring $\mathfrak{R}/P$. We will accomplish this by assuming that the arbitrary ring $\mathfrak{R}$ admits generalized P-derivations (F,χ) and $(\sqcup,\propto)$ that satisfy several identities.

For brevity, let (F,χ) and $(\sqcup,\propto)$ symbolize two generalized P-derivations associated with P-derivations χ and $\propto$, respectively.

MAIN RESULTS

Theorem 1

Let $\mathfrak{R}$ be a ring equipped with a P-derivation χ and a generalized P-derivation $(\sqcup,\propto)$ such that char$(\mathfrak{R}/P)$ not equal 2. Then, $[\chi(\upsilon),\chi(\wp)] \pm [\wp,\sqcup(\upsilon)] \in P$ for all $\upsilon,\wp \in \mathfrak{R}$ if and only if one of the following is true:

(i) $\mathfrak{R}/P$ is an integral domain;

(ii) (ii) $\chi(\mathfrak{R}) \subseteq P$, and $\sqcup(\mathfrak{R}) \subseteq P$.

Theorem 2

Let $\mathfrak{R}$ be a ring equipped with a P-derivation χ and a generalized P-derivation $(\sqcup,\propto)$ such that char$(\mathfrak{R}/P)$ not equal 2. Then, $\chi(\upsilon) \circ \chi(\wp) \pm[\wp,\sqcup(\upsilon)] \in P$ for all $\upsilon,\wp \in \mathfrak{R}$ if and only if one of the following is true:

(i) $\mathfrak{R}/P$ is an integral domain and $\chi(\mathfrak{R}) \subseteq P$.

(ii) $\chi(\mathfrak{R}) \subseteq P$ and $\sqcup(\mathfrak{R}) \subseteq P$.

Theorem 3

Let $\mathfrak{R}$ be a ring equipped with generalized P-derivations (F ,χ) and $(\sqcup,\propto)$ such that char$(\mathfrak{R}/P)$ not equal 2. Then $[F (\upsilon),\chi(\wp)] \pm\wp\circ\sqcup(\upsilon) \in P$ for all $\upsilon,\wp \in \mathfrak{R}$ if and only if one of the following satisfies

(i) $\mathfrak{R}/P$ is an integral domain and $\sqcup(\mathfrak{R}) \subseteq P$;

(ii) $F(\mathfrak{R}) \subseteq P$ and $\sqcup(\mathfrak{R}) \subseteq P$;

(iii) $\chi(\mathfrak{R}) \subseteq P$, and $\sqcup(\mathfrak{R}) \subseteq P$.

Theorem 4

Let $\mathfrak{R}$ be a ring equipped with generalized P-derivations (F,χ) and $(\sqcup,\propto)$ such that $[\upsilon,\sqcup(\wp)] \pm F([\upsilon,\wp]) \in P$ for all $\upsilon,\wp \in \mathfrak{R}$. Then, $\mathfrak{R}/P$ is an integral domain or $(\sqcup \pm F)(\mathfrak{R}) \subseteq P$.

Theorem 5

Let $\mathfrak{R}$ be a ring equipped with generalized P-derivations (F,χ) and $(\sqcup,\propto)$ such that one of the following identities holds for every $\upsilon,\wp \in \mathfrak{R}$:

(i) $F([\upsilon,\wp]) \pm F(\wp)\sqcup(\upsilon) \in P$,

(ii) $F([\upsilon, \wp]) \pm F(\upsilon)\sqcup(\wp) \in P$.

Then either $\propto(\mathfrak{R}) \subseteq P$ and $\mathfrak{R}/P$ is an integral domain, or $F(\mathfrak{R})$ is a subset of P.

Theorem 6

Let $\mathfrak{R}$ be a ring equipped with generalized P-derivations (F,χ) and $(\sqcup,\propto)$ such that $[\upsilon,F (\wp)] \pm F(\wp)\sqcup(\upsilon) \in P$ for all $\upsilon,\wp \in \mathfrak{R}$. Then one of the following is true:

(i) $F(\mathfrak{R}) \subseteq P$;

(ii) $\chi(\mathfrak{R}) \subseteq P$ and $\propto(\mathfrak{R}) \subseteq P$;

(iii) $\propto(\mathfrak{R}) \subseteq P$ and $\mathfrak{R}/P$ is an integral domain.

CONCLUSION

In this work, we explored the commutativity of the quotient ring $\mathfrak{R}/P$ and the behavior of two generalized P-derivations F and $\sqcup$ associated with P-derivations χ and $\propto$, respectively. We investigated that $\mathfrak{R}/P$ is an integral domain, and one of χ, $\propto$, F , $\sqcup$ and $F\pm\sqcup$ mapping the whole ring $\mathfrak{R}$ to prime ideal P.

FUTURE WORK

For a suggestion future studies, the scholar may be expanded current studied identities to generalized (α,β)-P-derivations, multiplicative P-derivations, and multiplicative generalized P-derivations.