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Structural Properties of Fuzzy β-Continuous and M-Fuzzy β-Continuous Mappings

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INTRODUCTION & AIM

Context: Fuzzy topological spaces generalize classical topology via gradual membership. Various continuity classes fuzzy continuous, pre-continuous, semi-continuous have been studied extensively.

Gap: The broader class of fuzzy β-continuous mappings unifies these notions, but its structural properties under composition and hierarchy remain to be fully characterized.

Aim: Establish equivalent characterizations, compositional behavior, and hierarchical relationships of fuzzy β-continuous and M-fuzzy β-continuous mappings.

KEY DEFINITIONS

Fuzzy β-Continuous Map

A map f : (X,τ) → (Y,σ) is fuzzy β-continuous if $f^{-1}(V)$ is a fuzzy β-open set in X for each fuzzy open set V in Y.

M-Fuzzy β-Continuous Map

A map f : (X,τ) → (Y,σ) is M-fuzzy β-continuous if $f^{-1}(V)$ is fuzzy β-open in X for every fuzzy β-open set V in Y.

CHARACTERIZATIONS OF FUZZY β-CONTINUOUS MAPS

Characterization via Closed Sets

f is fuzzy β-continuous ⟺ $f^{-1}(F)$ is fuzzy β-closed in X for every fuzzy closed set F in Y.

Characterization via β-Closure

f is fuzzy β-continuous if: $\beta cl(f^{-1}(V)) \leq f^{-1}(\beta cl(V))$ for every fuzzy set V in Y.

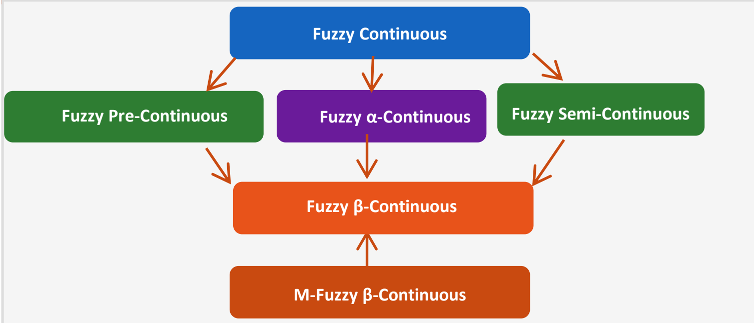
5 Equivalent Conditions (Bijective f)

(1) f is fuzzy β-continuous
(2) $f(\beta cl(A)) \leq cl(f(A))$
(3) $int(f(A)) \leq f(\beta int(A))$
(4) $\beta cl(f^{-1}(B)) \leq f^{-1}(cl(B))$
(5) $f^{-1}(int(B)) \leq \beta int(f^{-1}(B))$

COMPOSITION RESULTS

- f fuzzy continuous, g fuzzy continuous → g∘f is fuzzy β-continuous ✓
- f fuzzy pre-continuous, g fuzzy continuous → g∘f is fuzzy β-continuous ✓
- f fuzzy α-continuous, g fuzzy continuous → g∘f is fuzzy β-continuous ✓
- f fuzzy semi-continuous, g fuzzy continuous → g∘f is fuzzy β-continuous ✓
- f, g both fuzzy β-continuous → g∘f need NOT be fuzzy β-continuous ✗

HIERARCHY OF FUZZY MAPPINGS



M-FUZZY β-CONTINUOUS MAPPINGS: KEY RESULTS

M-Fuzzy β ⟹ Fuzzy β-Continuous

Every M-fuzzy β-continuous map is fuzzy β-continuous.

The converse does not hold in general (see Example in illustrative section).

Characterization via β-Interior

f is M-fuzzy β-continuous ⟺ $f^{-1}(\beta int(V)) \leq \beta int(f^{-1}(V))$ for every fuzzy set V in Y.

Bijective Characterization

Bijective f is M-fuzzy β-continuous ⟺ $\beta int(f(A)) \leq f(\beta int(A))$ for every fuzzy set A in X.

ILLUSTRATIVE EXAMPLES

- **β-continuous ⇏ fuzzy continuous:** X=Y={a,b,c}, A={⟨a,0.7⟩,⟨b,0.7⟩,⟨c,0.8⟩}, B={⟨a,0.4⟩,⟨b,0.6⟩,⟨c,0.6⟩}, τ={0,A,1}, σ={0,B,1}. Identity f is fuzzy β-continuous but NOT fuzzy continuous.
- **β-continuous ⇏ α-continuous or semi-continuous:** A={⟨a,0.1⟩,⟨b,0.3⟩,⟨c,0.2⟩}, B={⟨a,0.1⟩,⟨b,0.1⟩,⟨c,0.1⟩}, C={⟨a,0.4⟩,⟨b,0.4⟩,⟨c,0.5⟩}, τ={0,A,B,A∧B,A∨B,1}, σ={0,C,1}. Identity is β-continuous but neither α-continuous nor semi-continuous.
- **β-continuous ⇏ M-fuzzy β-continuous:** Same spaces as above. Identity is fuzzy β-continuous but NOT M-fuzzy β-continuous.

CONCLUSION

✓ f is fuzzy β-continuous ⟺ $f^{-1}(F)$ is β-closed for all fuzzy closed F, and ⟺ $\beta cl(f^{-1}(V)) \leq f^{-1}(\beta cl(V))$.

✓ For bijective maps, five equivalent characterizations of fuzzy β-continuity are established.

✓ Composition g∘f is β-continuous when g is fuzzy continuous and f satisfies any stronger continuity condition; fails for two general β-continuous maps.

✓ Strict inclusions: fuzzy continuous ⊂ fuzzy α/pre/semi-continuous ⊂ fuzzy β-continuous.

✓ M-fuzzy β-continuity implies fuzzy β-continuity; characterized via βint condition; bijective form also established.

REFERENCES

[1] Zadeh, L.A. (1965). Fuzzy sets. Information and Control, 8(3), 338–353.

[2] Chang, C.L. (1968). Fuzzy topological spaces. J. Math. Anal. Appl., 24, 182–190.

[3] Lowen, R. (1976). Fuzzy topological spaces and fuzzy compactness. J. Math. Anal. Appl., 56, 621–633.

[4] Abd El-Monsef, M.E., El-Deeb, S.N. & Mahmoud, R.A. (1983). β-open sets and β-continuous mappings. Bull. Fac. Sci. Assiut Univ., 12, 77–90.

[5] Bin Shahna, A.S. (1991). On fuzzy strong semicontinuity and fuzzy precontinuity. Fuzzy Sets Syst., 44, 303–308.

[6] Caldas, M. & Jafari, S. (2004). On some applications of fuzzy β-open sets. Sci. Math. Jpn., 59, 539–545.