IOCMA
2026
Conference

The 2nd International Online Conference on Mathematics and Applications

10–12 June 2026 | Online

MDPI

The world is a Category of transformations

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INTRODUCTION & AIM

The world is inherent dynamic, even the physical vacuum is filled with oscillations of energy. Hence, if we are to understand any of the emergent phenomenon of the world at a profound level, it is necessary to achieve a basic understanding of the properties of these dynamics. Dynamism is presented in many forms, all of them are conjugations or particular kinds of **transformations** which I identify as the basic principle of nature.
The focus of this research will be the study of the mathematical properties of transformations of natural entities. Moreover, it will also be noted how patterns of resistance to change can be understood within this theory. In this way, we refining our understanding of the wholeness and unity of nature (Bohm, 1980).

METHOD

First, we need to define three classes in the sense of NBG set theory. All of the natural entities that exists will be grouped under the class $\mathbf{E}$. This class is extended to $\mathbf{E}^p$ which includes all the natural entities that have existed, exists or may exists. Each of the elements of $\mathbf{E}^p$ have aspects which are fundamental characteristics of them. We record all of the aspects of all of the elements of $\mathbf{E}^p$ in the class $\mathbf{K}$. We set a primitive relation $\succ$ in $\mathbf{E}^p \times \mathbf{K}$, thus we write $e \succ k$ whenever $k \in \mathbf{K}$ is an aspect of $e \in \mathbf{E}^p$.
With this, we define formally a transformation as follows.

Definition: A **transformation** is a duple $t:=(K,s)$ where

1. $K \subset \mathbf{K}$ is non-empty,
2. s is a set-class function $s : I_n \to \mathbf{E}^p$ where I_n is the set $I_n := \{1, \dots, n\}$ with n as the number of states of the transformation which may not all be different, i.e., s is not necessarily injective.
3. There are no constant steps, i.e., $\forall i \in I_n \setminus \{n\}, s(i) \neq s(i+1)$.
4. $\forall e \in s(I_n)\ \exists \kappa \in K, e \succ \kappa$.
5. $\forall i \in I_n \setminus \{n\} \exists \kappa \in K, s(i) \succ \kappa \wedge s(i+1) \succ \kappa$.

All of the transformations will be collected in the class $\mathbf{V}$. Whenever the final state of a transformation is equal to the first state of another, we can compose them according to the following procedure.

Composition
Let $t_1 = (K_1, s_1)$ and $t_2 = (K_2, s_2)$. Let $K_1 \cup K_2 = K_3$. Being $s_1 : I_n \to \mathbf{E}^p$ and $s_2 : I_m \to \mathbf{E}^p$, we construct $s_3 : I_p \to \mathbf{E}^p$ with $I_p = \{1, \dots, p = n + m - 1\}$. If $m = 1$, we define $s_3 = s_1$. If $m \geq 2$, we define s_3 as

$$s_3(i) = \begin{cases} s_1(i), & 1 \leq i \leq n \\ s_2(i-n+1), & n+1 \leq i \leq n+m-1 \end{cases}$$

It is straightforward that $t_3 = (K_3, s_3)$ is a transformation which is denoted by $t_3 = t_2 \circ t_1$.

Define $\mathbf{V}c$ as the subset of $\mathbf{V} \times \mathbf{V}$ such that $(t_1, t_2) \in \mathbf{V}c$ if and only if $R(t_1) = D(t_1)$. With this, we can define the following function $\circ$

$$\begin{aligned} \circ \ :\ \ & \mathbf{V}c \to \mathbf{V} \\ & (t_1, t_2) \to \circ(t_1, t_2) = t_2 \circ t_1 \end{aligned}$$

Definition: A non-empty subset $S \subset \mathbf{V}$ is called a **subspace** of $\mathbf{V}$ if and only if for any pair $t_1, t_2 \in S$ if the final state of t_1 is equal to the first state of t_1 then $t_2 \circ t_1 \in S$.

Hence subspaces are subsets of $\mathbf{V}$ that are closed under composition. For any transformation $t=(K,s)$, we denote the first state $s(1)=D(t)$ and the last state $s(n)=R(t)$.

Definition: For every $e \in \mathbf{E}^p$, the following set is called **space of transformations of** e.

$$\blacktriangleleft e \blacktriangleright := \{t \in \mathbf{V} : D(t) = e\}$$

RESULTS & DISCUSSION

Proposition 1: For any $t \in \mathbf{V}$, $t := (K, s : I_n \to E^p)$ with $R(t)=D(t)$, $t \circ t = t$ if and only if $n = 1$.

Proposition 2: For any $e \in \mathbf{E}^p$, $\blacktriangleleft e \blacktriangleright$ is a subspace of $\mathbf{V}$.

Theorem: There is a category W whose $Ob(W) = \mathbf{E}^p$. For any pair of objects $e_1, e_2 \in \mathbf{E}^p$, the set of morphisms $Mor(e_1, e_2)$ is the set of transformations $t \in \mathbf{V}$ for which $D(t)=e_1$ and $R(t)=e_2$. For every triple $e_1, e_2, e_3 \in E^p$, there is a function $\circ_{e1,e2,e3} : Mor(e_1, e_2) \times Mor(e_2, e_3) \to Mor(e_1, e_3)$ that is the restriction of the function $\circ$ to the subset $Mor(e_1, e_2) \times Mor(e_2, e_3)$. Then $\circ_{e1,e2,e3}$ is the composition of morphisms. Finally, every identity morphism is a trivial transformation with only one step which vacuously satisfies the definition of transformations.

In this context, it is possible to formally define patterns of resistance to change. First, every identity morphism will be collected into the subset $Id \subset \mathbf{V}$. It can be shown easily that Id is a subspace of $\mathbf{V}$.

Definition: A subset SC of V such that $Id \subset SC$ is called a **standard condition**.
Choosing SC reflects an expectation of some transformations and a prohibition of others. In other words, it sets the «environmental conditions».

Proposition 3: $\forall e \in E^p$ and any standard condition SC, $\blacktriangleleft e \blacktriangleright \cap SC \neq \emptyset$.

Definition: We say that $k \in \mathbf{K}$ is a **pattern of resistance to change** relative to **SC** if and only if

$$\forall t=(K_t, s_t) \in \blacktriangleleft e \blacktriangleright \cap SC, \forall i \in Domain(s_t), s_t(i) \succ \kappa.$$

Definition: If $e \in E^p$ possesses a pattern of resistance to change relative to some SC, then we say that e is a **whole** relative to SC.

CONCLUSION

The world in which we live navigates in a categorical structure. Admitting a certain degree of relativity, we can define wholes.

FUTURE WORK / REFERENCES

It is natural to ask what type of category is W. Then apply all of the concepts of category theory to the understanding of the world. It will also be interesting to continue to explore the properties of wholes.

- D. Bohm. Wholeness and the implicate order. Routledge & Kegan Paul,1980.