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Zero-Divisors in Commutative Rings: A Graph-Theoretic Perspective

Assila Khaireddin1,* · **Khalid Louartiti**2

1 MASD, ENSA-Tetouan, Abdelmalek Essaadi University, Tetouan, Morocco
2 Abdelmalek Essaadi University, Tetouan, Morocco | * khaireddin.assila@etu.uae.ac.ma

POSTER PRESENTATION
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Core message: zero-products become a computable graph model of ring structure.

INTRODUCTION & AIM

Zero-divisors mark the failure of cancellation: a nonzero element a satisfies ab = 0 for some nonzero b. They reveal structure in ideals, factorization, and modular arithmetic.

Build Gamma(R): vertices are Z*(R); edge x--y means xy = 0.

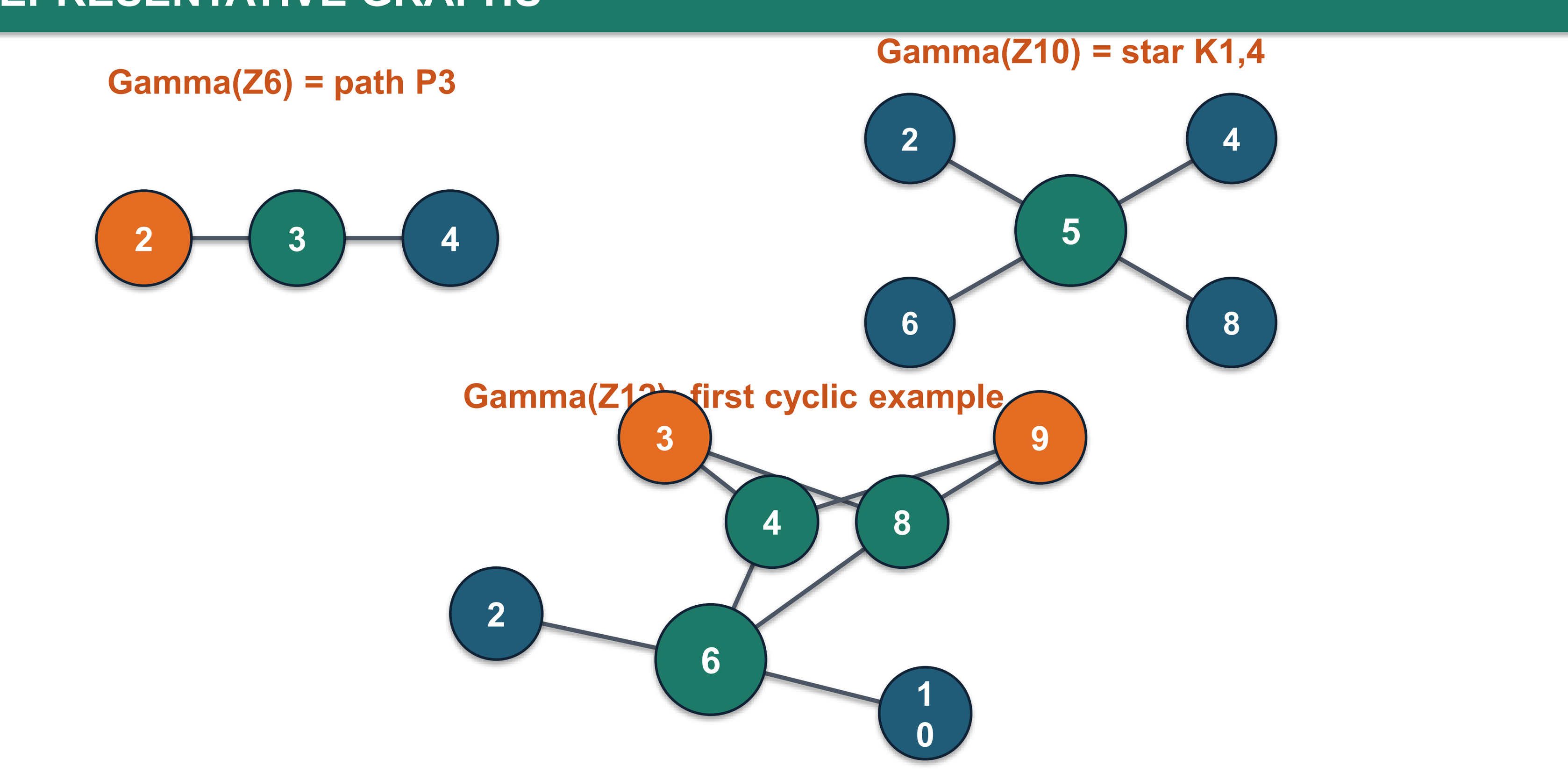
Aim: compare rings using diameter, girth, clique number, chromatic number, spectrum, annihilators, and equitable partitions.

METHOD

1. **Find Z*(R)**
   Use gcd in Z_n or solve ab = 0.
2. **Build edges**
   Join x and y iff xy = 0.
3. **Compute invariants**
   diameter, girth, omega, chi, spectrum.
4. **Compress**
   Group equal annihilators.

In Z_n: a is a zero-divisor iff gcd(a,n) > 1.

REPRESENTATIVE GRAPHS



The visual shift is clear: path -> star -> cyclic bipartite graph.

CONCLUSION

- The graph translates algebraic products into visible structure.
- The gcd test makes Z_n examples fast and transparent.
- Compression by annihilators preserves the main zero-product pattern.

Final message: the graph is not decoration; it is a model of multiplication failure.

RESULTS & DISCUSSION

Main findings from modular examples

- Z6 gives a path P3 with central annihilator 3.
- Z10 gives a star K1,4 with center 5.
- Z12 gives a 4-cycle; the graph remains bipartite.

Ring	Z*(R)	Graph	diam	girth	chi
Z6	{2,3,4}	path P3	2	inf	2
Z10	{2,4,5,6,8}	star K1,4	2	inf	2
Z12	{2,3,4,6,8,9,10}	bipartite, 4-cycle	3	4	2

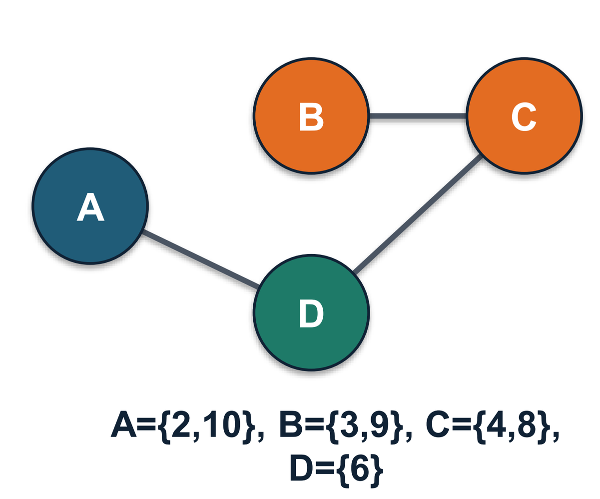
Interpretation: small graph invariants already distinguish different zero-product patterns and expose algebraic symmetry.

Award-level strength of the poster

- One clear question.
- Three worked examples.
- One compressed structural explanation.

ANNIHILATOR COMPRESSION

Equal annihilators mean equal zero-product behavior.



A={2,10}, B={3,9}, C={4,8}, D={6}

For Z12:
ann(3)=ann(9), ann(2)=ann(10), ann(4)=ann(8).

Seven vertices become four structural classes.

Compression makes symmetry computable.

FUTURE WORK / REFERENCES

Future work: compute larger finite rings and compare spectra of compressed graphs for Noetherian/Artinian quotient families.

References: Beck, J. Algebra, 1988; Anderson & Livingston, J. Algebra, 1999; Anderson & Badawi, J. Algebra, 2008.

Conflict of Interest: The authors declare no conflicts of interest.