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The Canonical Triple-Graph: A Structural Organization of the Positive Integers

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INTRODUCTION & AIM

Pythagoras held that integers are fundamental relational primitives. CTG returns to this view.

The positive integers are usually understood through their linear ordering, 1, 2, 3, 4, 5 … This ordering conceals a richer internal structure independent of numerical size.

What algebraic structure is intrinsic to the positive integers?

For odd m, define the admissible associator: $E_k(m) = (2^k m - 1) / 3$, whenever $2^k m \equiv 1 \pmod 3$

This assigns every positive integer a unique parent. The resulting directed graph—the Canonical Triple-Graph (CTG)—is rooted, acyclic, and self-similar.

Integers admit two distinct modes of description. As magnitudes, they carry linear order and size — the perspective that conceals internal structure. As algebraic carriers, they are characterized solely by parity, congruence classes, and 2-adic valuation. CTG operates entirely in the second mode: it assigns each integer a structural address — parent, block membership, and pillar position — fully determined by the admissibility condition and independent of numerical size.

Every integer has a unique place in the structure—timeless, not built by any process, yet completely ordered by depth, ancestry, block position, and pillar position. No iteration, no traversal, and no convergence assumption is required.

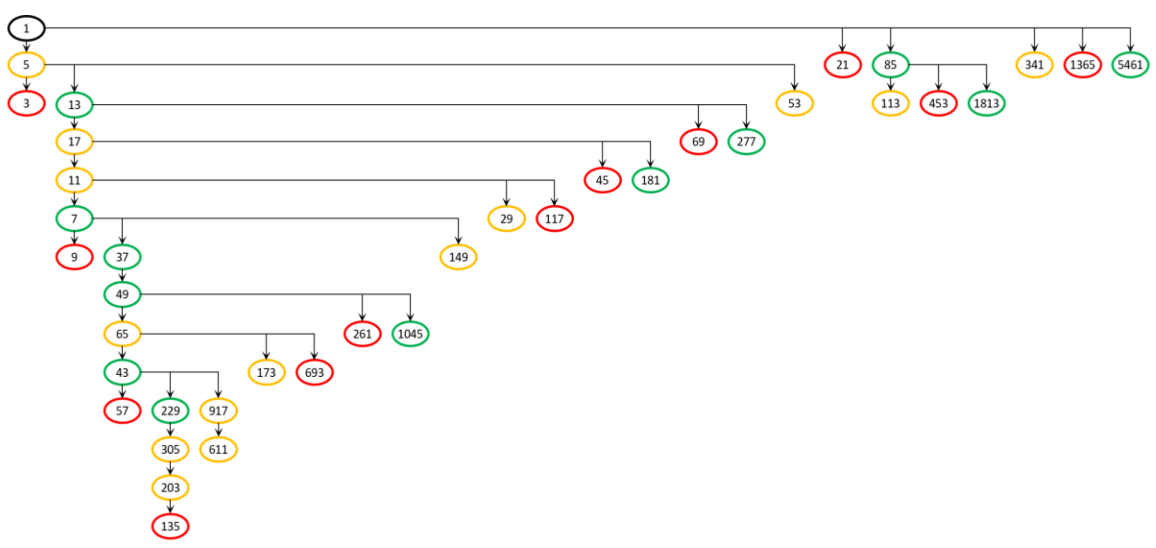

Fig. 1: The Canonical Triple-Graph, rooted at 1. Red: inactive; yellow, green: active entries. Structural depth increases downward.

KEY DEFINITIONS

Admissible associator: For odd m, define $E_k(m) = (2^k m - 1)/3$, whenever $2^k m \equiv 1 \pmod 3$. This is purely algebraic; no iterative or dynamical interpretation is assumed.
Block(m): The infinite set of children of m: $\{ E_k(m) : k \geq 1, 2^k m \equiv 1 \pmod 3 \}$. All members share a common parent. Block(m) is an intrinsically n-ary relation—not a chain of pairwise links.
Canonical triple: Each block decomposes into consecutive triples (n, 4n+1, 16n+5). Exactly one entry per triple is inactive (≡ 0 mod 3); two are active. The residue layout is constant along the entire block.
Vertical pillar: $P(m) = \{ 2^r m : r \geq 0 \}$ for each odd integer m. Distinct pillars are disjoint. Every positive integer $n = 2^r m$ lies on exactly one pillar, uniquely addressed by its odd part m and its 2-adic valuation r.

RESULTS & DISCUSSION

1. **Unique parenthood:** Every positive integer has exactly one parent under the admissible associator. Parenthood is determined algebraically, not by traversal.
2. **Acyclicity:** Block-layer depth strictly increases along directed edges. No directed cycle can occur in the principal component.
3. **Root uniqueness:** The integer 1 is the unique self-parenting element: $E_2(1) = 1$. No other integer satisfies this condition.
4. **Deterministic residue grammar:** The residue pattern mod 3 is constant within each block. At each level t, all residue classes mod 3^t are represented. Groups of 3, 9, 27, … close exactly at successive powers of 3.
5. **Extension to all positive integers:** The odd structure extends canonically via unique 2-adic factorization. Every even integer lies on exactly one pillar P(m). Vertical pillars introduce no branching and do not alter the odd hierarchy.
6. **Self-similar structure:** The same local configuration—one inactive entry, two active—recurs uniformly at every depth. This is structural recurrence, not traversal.
7. **No generative process:** CTG does not construct or enumerate the integer hierarchy. The admissible associator explicates relations among existing objects. All admissible edges are defined simultaneously and a priori—not produced by iteration or traversal.

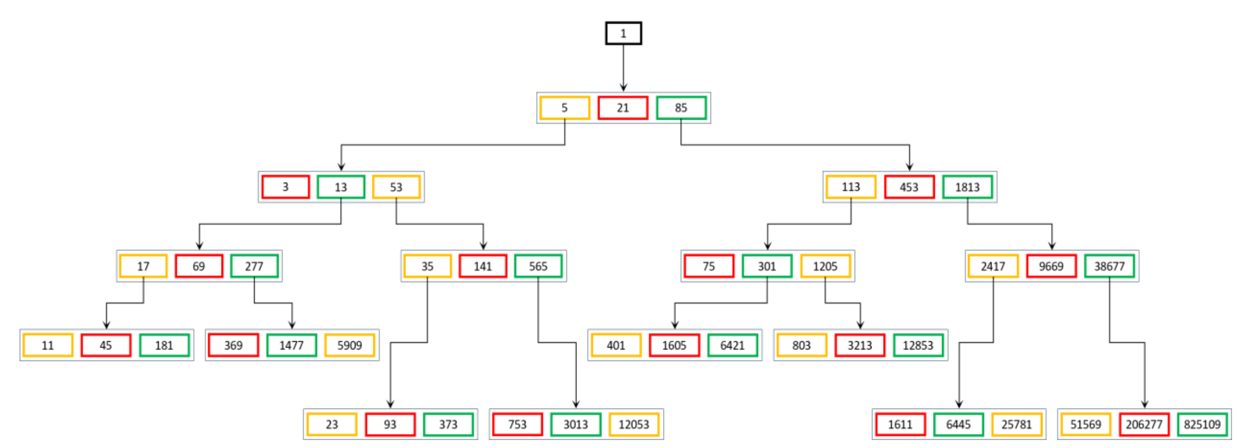

Fig. 2. Every block decomposes into canonical triples (n, 4n+1, 16n+5) with exactly one inactive entry (red), recurring uniformly at every depth.

CONCLUSION / REFERENCES

Every integer m ≠ 1 lies in exactly one block, placing it within one block-closed component C(x).
Depth is defined intrinsically: each integer in Block(m) has depth equal to the depth of m plus 1; an integer outside every block has depth 0.
Every upward chain strictly decreases depth, terminating at a block-free integer x of depth 0.
The terminus satisfies $E_k(x) = x$; solving yields $x(2^k - 3) = 1$, forcing k = 2 and x = 1.
Therefore C(1) is the only possible component, every positive integer belongs to it, and the Collatz conjecture follows as a structural corollary.
Formal proofs of all stated properties appear in the preprint below.

Q&A: https://doi.org/10.5281/zenodo.20281413

Preprint: https://doi.org/10.5281/zenodo.17909390