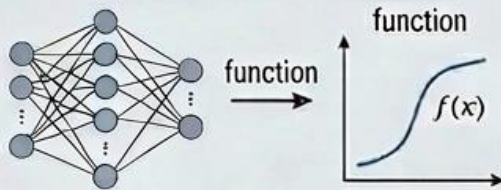


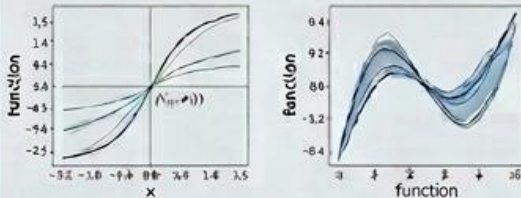
Topological Stability of Neural Operators: A Nonlinear Functional–Geometric Theory for Controlling Infinite-Dimensional Learning Systems

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AND WMO ARTS AND SCIENCE COLLEGE

INTRODUCTION & AIM



Neural Network → Neural Operator



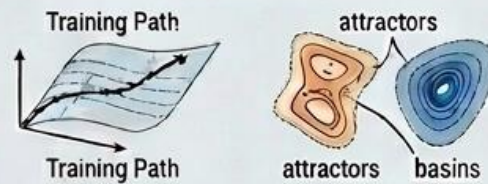
Vector Representation → infinite, ..., \mathcal{N}^1
~ infinite basis sets

Function Space Representation → \cdot is finite
* vectors

- Developing a rigorous unified framework for stability, controllability, and generalization in function spaces.

METHOD

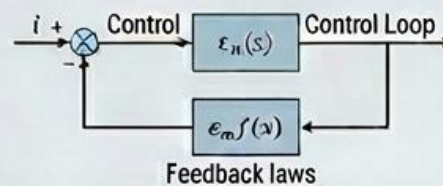
Learning as a Topological Flow



Topological Loss Function Design: Integrating degree constraints for robust optimization.

$$T_{\text{topological}} + f(x) = \frac{1}{2m} \sum_{i=1}^n ((\text{topological loss}))^2$$

Control Theory Application

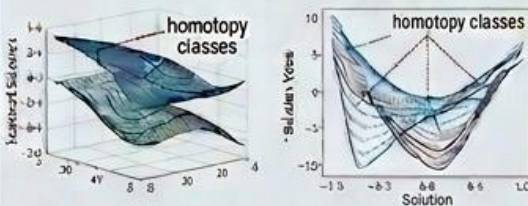


- Training algorithms as feedback control laws (e.g. the usalient woriossional, ...Route to stability-certified and robustly controllable infinite-dimensional state space.

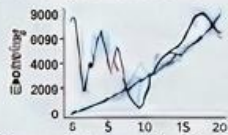
RESULTS & DISCUSSION

TOPOLOGICAL GENERALIZATION & STABILITY

Learned Solution Manifolds



- Generalization Preventing Chaotic Overfitting governed by homotopy class, navigated unseen test regimes

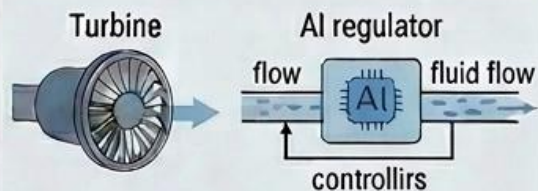


- Architectural choices enforce global geometric constraints



- Degree theory used to quantify global geometric structure and avoid underdetermined learning instabilities.

CONCLUSION



- Verified framework for designing AI controllers with provable stability guarantees for physical systems.

FUTURE WORK

- Extension to higher-order manifolds
- Applications to quantum many-body problems
- Integration with causal inference for robust, safe AI control of unmodeled systems.