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The characters of involutive automorphisms of simple Lie algebras

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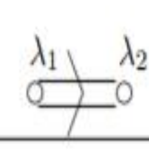
INTRODUCTION & RESULTS

The paper deals with the arbitrary finite-dimensional irreducible representations of simple complex Lie algebras g of types B2 and G2 and real forms so(1,4), so(3,2) and G of these algebras. Involutive automorphisms θ on Cartan subalgebras of these algebras are considered. Formulae for characters value χ(θ) are obtained. That allows to find the number of linearly independent space-like and time-like vectors for these algebras in the representation space.

Consider a simple complex Lie algebra g and an irreducible finite-dimensional representation φ : g → sl(V). Denote by gr the real form of inner type for algebra g. Then φ(gr) ⊆ su(p,q), where p − q = δ(gr) is the signature of the invariant Hermitian form on V. It is possible to find δ(gr) using Weyl character formula for the representation φ. In the paper [1] F.I.Karpelevich derived formulae for δ in the case of classical Lie algebras. In [2] Lie algebras su(p,q) are considered and convenient tables for δ in the case of small rank of g are found. In [5] the algebras so(4,1) and so(3,2) are explored and formulae for δ(gr) are presented as the summation taken over all the orbits of the system of weights for the representation φ.

In [3, 4] formulae for |δ| in the case of gr = G,FI,FII,so(p,q) where presented. Nevertheless in applications the exact tables including the sign of δ are necessary. We obtain in this paper the tables of δ in terms of the marks of the highest weight of representation φ. Besides similar table for any representation of exceptional Lie algebra of type G2 are derived. And it is not necessary to know the system of all weights of the representation φ.

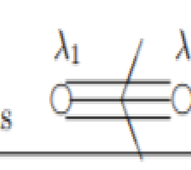
The main results can be presented n the tables

Table 1. Character values for the representations  .

λ_1	λ_2	$\mathfrak{g}_\sigma = \mathfrak{so}(3,2)$	$\mathfrak{g}_\sigma = \mathfrak{so}(1,4)$
a	o	0	0
e	e	$(-1)^{\frac{1}{2}\lambda_2} \cdot \frac{1}{2}(\lambda_1+\lambda_2+2)$	$\frac{1}{2} \cdot (\lambda_1+1)(\lambda_1+\lambda_2+2)$
o	e	$(-1)^{\frac{1}{2}\lambda_2} \cdot \frac{1}{2}(\lambda_1+1)$	$-\frac{1}{2} \cdot (\lambda_1+1)(\lambda_1+\lambda_2+2)$

Symbol $e(o)$ in the columns λ_i denotes an even(odd) λ_i,

symbol a denotes any λ_i independent of whether it is even or odd.

Table 2. Character values for the representations 

λ_1	λ_2	$\mathfrak{g}_\sigma = G$
o	o	0
e	e	$\frac{1}{8} \cdot (\lambda_1+3\lambda_2+4)(\lambda_1+\lambda_2+2)$
e	o	$-\frac{1}{8} \cdot (\lambda_2+1)(2\lambda_1+3\lambda_2+5)$
o	e	$\frac{1}{8} \cdot (\lambda_2+1)(\lambda_1+2\lambda_2+3)$

Symbol $e(o)$ in the column λ_i denotes an even(odd) λ_i.

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