A Communication-Free Parallel Screened Poisson Solver for Incompressible Navier–Stokes

JUNLONG XING[1,*], Zheng-an Yao[2], Qiru Wang[1]

[1] School of Mathematics, Sun Yat-sen University, Guangzhou 510275, China [2] Institute of Advanced Studies in Hong Kong, Sun Yat-sen University, Hong Kong, China [*] xingjlong@mail2.sysu.edu.cn

1. Motivation: the pressure projection is a global bottleneck

Projection methods split the incompressible Navier–Stokes update into a local momentum predictor and a pressure correction. The predictor is local, while exact incompressibility leads to the classical pressure Poisson equation

$$-\Delta\phi^{n+1} = -\frac{1}{\Delta t}\nabla\cdot \boldsymbol{u}^*$$

whose Green response is long-ranged. On distributed machines this step requires halo exchange, synchronization, global FFT transposes, or Krylov reductions. The objective is to redesign the pressure correction so that the pressure solve has an intrinsic finite interaction length.

2. Narrative bridge: screened Poisson locality → CFD pressure step

The construction follows the same locality logic used in screened Poisson normalization for large images: adding a zero-order term changes a Poisson-type global response into a screened response with exponential decay.

Screened Poisson in large-scale reconstruction
$(-\Delta + \lambda_c I)u_c = f_c$

↓ locality transfer

Screened pressure correction in CFD
$(-\Delta + \kappa^2 I)\psi = -\Delta t^{-1} d^*$

$$E_{\mathrm{core}} \lesssim C e^{-\delta/\ell_\lambda} = C e^{-\sqrt{\lambda_c}\delta}$$

The pressure step is therefore formulated not as a globally exact projection, but as a locality-certified screened projection.

3. LCSP screened projection formulation

Relaxed incompressibility.

$$\boldsymbol{u}_\kappa^{n+1} = \boldsymbol{u}^* - \Delta t\,\nabla\psi_\kappa^{n+1},$$
$$\nabla\cdot \boldsymbol{u}_\kappa^{n+1} + \eta\psi_\kappa^{n+1} = 0, \qquad \kappa^2 = \eta/\Delta t.$$

Screened pressure equation.

$$(-\Delta + \kappa^2 I)\psi_\kappa^{n+1} = -\frac{1}{\Delta t}d^*, \qquad d^* = \nabla\cdot \boldsymbol{u}^*$$

Taking the divergence of the corrected velocity gives

$$\nabla\cdot \boldsymbol{u}_\kappa^{n+1} = -\Delta t\,\kappa^2\psi_\kappa^{n+1}.$$

Thus κ is simultaneously the pressure screening strength and the incompressibility-relaxation parameter. As $\kappa \to 0$, the screened pressure equation degenerates to the classical PPE.

4. Communication-free Overlap–Restrict assembly

The domain is partitioned into disjoint cores C_p; each core is enlarged by r halo layers. Local screened PPEs are solved independently on C_p^{+r}, and only the core region is retained.

partition cores C_p → extend halos C_p^{+r} → local screened PPE solves → restrict to cores

$$L_{\kappa,h}\psi_{p,h}^{+r} = b_h|_{C_p^{+r}}, \qquad \psi_h^{\mathrm{OR}} = \sum_p E_p^c R_p^c \psi_{p,h}^{+r}.$$

No inter-subdomain trace exchange, subdomain iteration, or global reduction is used during the pressure solve.

5. Locality certificate and error budget

Artificial boundary error satisfies a homogeneous screened equation inside each halo. A discrete exponential barrier yields

$$\|\psi_h^{\mathrm{OR}} - \psi_h\|_{\ell^\infty(\Omega_h)} \le 4e^{-\alpha r h}\max_p M_p.$$

The nondimensional overlap parameter is

$$\theta_{\kappa,r} = \frac{rh}{\ell_\kappa} = \kappa r h.$$

The divergence residual is decomposed as

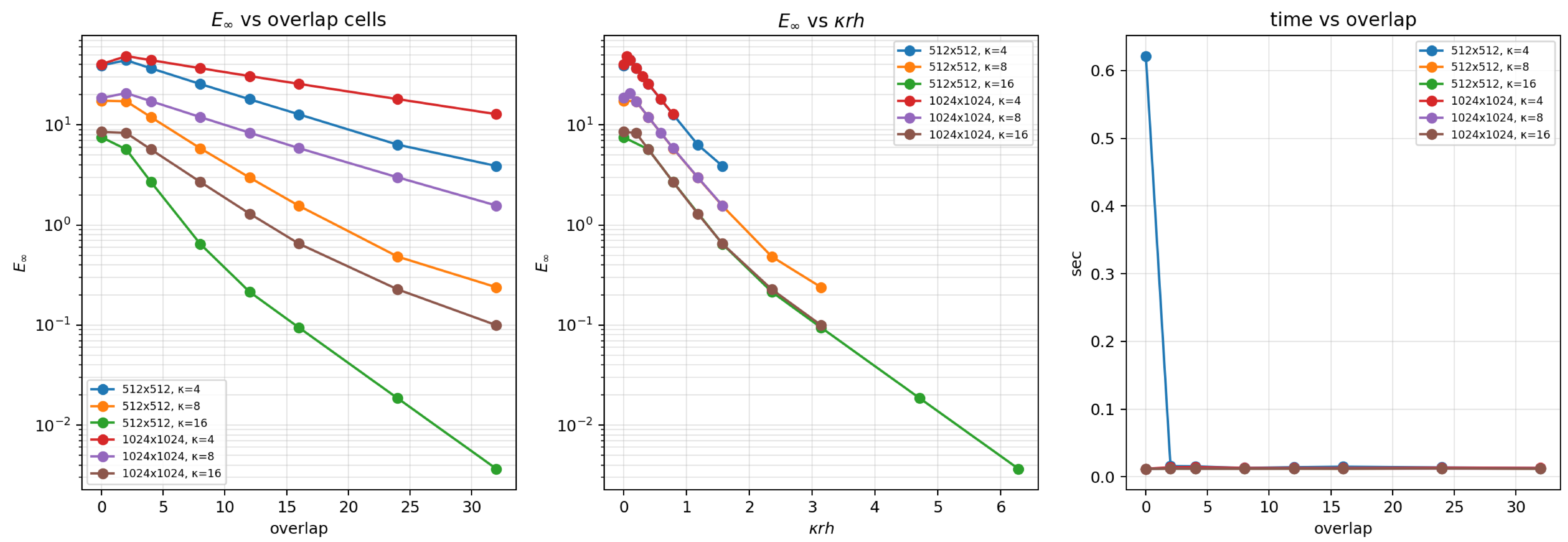
$$\|\nabla_h\cdot \boldsymbol{u}_h^{n+1,\mathrm{OR}}\|_2 \le \underbrace{\Delta t\kappa^2\|\psi_h\|_2}_{\text{global screened penalty}} + \underbrace{C\Delta t h^{-2}e^{-\alpha r h}(\cdots)}_{\text{local assembly error}}.$$

Overlap controls the artificial-boundary contribution; κ controls the incompressibility relaxation.

6. Take-home message

LCSP replaces the globally coupled Poisson projection by a screened, locality-certified pressure correction. The design variable is $\theta_{\kappa,r} = \kappa r h$: κ sets pressure influence length, while r controls artificial-boundary decay before core restriction. The method makes the CFD trade-off explicit: exact incompressibility versus communication-free parallel efficiency.

7. Taylor–Green locality evidence



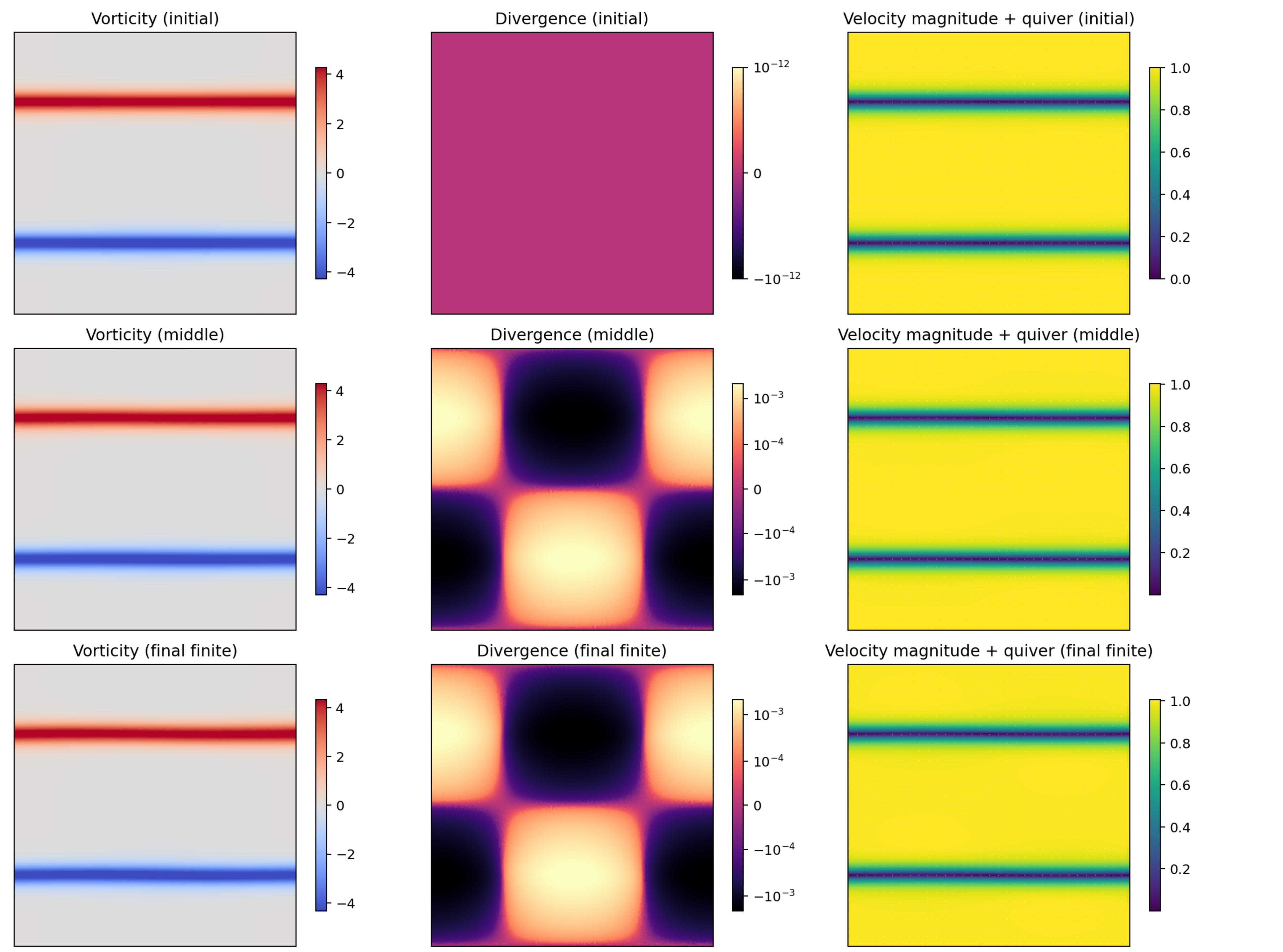
The core pressure error decreases rapidly with overlap. When the horizontal axis is rescaled as κrh, curves from different grid sizes collapse more consistently, matching the locality certificate.

8. Pressure-solver comparison at a fixed Taylor–Green state

Method	time/s	p error	u error	comm/red.
Global PPE (PCG)	2.97×10^{-1}	6.08×10^{-4}	4.53×10^{-6}	400/800
Global screened PCG	2.11×10^{-1}	2.48×10^{-6}	2.15×10^{-7}	365/730
Local non-overlap PPE	6.10×10^{-1}	6.21×10^{1}	2.32×10^{-1}	0/0
Overlap screened PPE	1.30×10^{-2}	7.21×10^{-1}	3.88×10^{-2}	**0/0**
Iterative overlap-sync	6.33×10^{-2}	9.46×10^{-3}	5.77×10^{-4}	5/0

Interpretation: global PPE is accurate but communication-heavy; local non-overlap PPE is communication-free but inaccurate; overlap screened PPE gives a fast, zero-communication, controllable compromise.

9. Double shear layer validation



The screened pressure step is tested under sharp shear layers. Initial divergence is at floating-point scale; after time integration the divergence residual remains around 10^{-3}, while vorticity and velocity magnitude preserve the two-layer flow structure without visible numerical instability.

Keywords: Incompressible Navier–Stokes; Screened Projection; Pressure Poisson Equation; Domain Decomposition