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On Curvatures Inheritance in Generalized Fifth Recurrent Finsler Space

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INTRODUCTION & AIM

Finsler geometry is a kind of differential geometry originated by P. Finsler in (1918). It is an extension of Riemannian geometry, Riemann studied the distance between points in n-dimensions using only positional coordinates, while Finsler generalized Riemann's idea and studied the distance between points in n-dimensions by using two coordinates: positional and directional. the fundamental function of Finsler space is defined as:

$$F = \sqrt{g_{ij} y^i y^j} ,$$

where the metric tensor g_{ij} and the vectors y^i and y^j are non-zero.

In differential geometry the Lie- derivative evaluates the rate of change of avector field or a tensor field along the smooth vector field $v^i(x)$. It is named after Souphus Lie who introduced it in the 19^{th}century. The symbol L_v denoted to Lie-derivative. The Lie-derivative of a general mixed tensor field T^i_{jkh} is given by following formula :

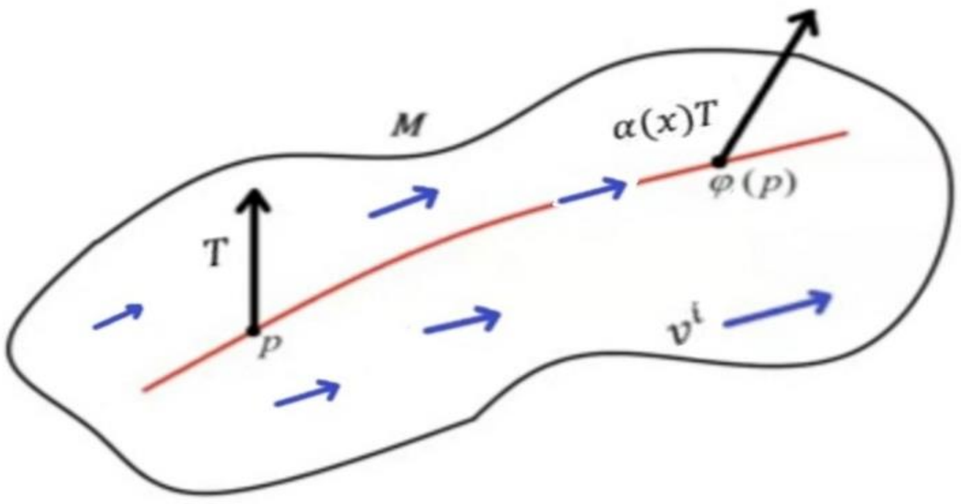
$$L_v\, T^i_{jkh} = v^m \mathcal{B}_m T^i_{jkh} - T^m_{jkh}\mathcal{B}_m\, v^i + T^i_{mkh}\mathcal{B}_j\, v^m + T^i_{jmh}\mathcal{B}_k\, v^m + T^i_{jkm}\mathcal{B}_h\, v^m$$
$$\dot{\partial}_m T^i_{mkh}\, \mathcal{B}_r v^m y^r.$$

The tensor T whose components are related to the components of a vector field $v^i(x)$ during the application of a force, and which inherits it to a new tensor $\alpha(x)T$, where $\alpha(x)$ is a non-zero scalar function is called the inheritance tensor and defined as:

$$L_v\, T\, = \alpha(x)T.$$

The aim of this work is to study the contraction and vanishing of curvature tensors in generalized fifth recurrent (fifth slope) Finsler space for Cartan's fourth curvature tensor K^i_{jkh} in sense of Berwald ($G\mathcal{B}K - 5RF_n$) that admits *K* and *H*-curvatures inheritance as they flow along a smooth vector field.

GEOMETRIC FIGURES



The above figure shows the infinitesimal change of a tensor T as it flows along a vector field $v^i(x)$ on a manifold M, during this flow the tensor components are associated with the vector components at an arbitrary point P and are inherited (transferred) to point $\varphi(P)$ (trajectory of P) via a transformation mapping $\varphi: T \to \alpha(x)T$ (where $\alpha(x)$ is a non zero scalar function).The tensor T that satisfies the above transformation mapping is called an inheritance tensor.

MAIN RESULTS

Theorem 1

In $G\mathcal{B}K - 5RF_n$, the Lie- derivative of K-curvature inheritance and the Berwald covariant derivative of the fifth order are commutative if and only if the Lie- derivative of non-zero covariant vector field of fifth order a_{sqlnm} vanishes simultaneously with the vanishing of the scalar function $\alpha(x)$ by Berwald covariant derivative of the fifth order.

Theorem 2

In $G\mathcal{B}K - 5RF_n$ that admits K-curvature inheritance, the scalar function of Cartan's fourth curvature tensor K^i_{jkh} is vanishing [provided (Theorem1) holds].

Theorem 3

In $G\mathcal{B}K - 5RF_n$ that admits K-curvature inheritance and H-curvature inheritance, the Lie-derivative of Cartan's fourth curvature tensor K^i_{jkh} and Berwald's curvature tensor H^i_{jkh} are equal if the Berwald's covariant derivative of fifth order for these curvature tensors are equal.

Theorem 4

In $G\mathcal{B}K - 5RF_n$ that admits K-curvature inheritance and H-curvature inheritance, the Lie-derivative of Cartan's fourth curvature tensor K^i_{jkh} in the direction of vector y^h is vanishing

Theorem 5

In $G\mathcal{B}K - 5RF_n$ that admits H-curvature inheritance, the scalar value of the smooth vector field $v^j(x) = \sum_{k=1}^{n} H_k y^k$ if the Lie-derivative and the Berwald's covariant derivative of fifth order of the $h(v)$ -torsion tensor H^i_{kh} are commutative.

CONCLUSION

This study has established new identities and values for certain type of curvature tensors in generalized $\mathcal{B}K$-fifth recurrent Finsler space that admits K-curvature inheritance and H-curvature inheritance. This paper has examined the directional and extensional changes that occur to curvature tensors in fifth recurrent Finsler space when a force acts upon them. We obtained the tensors are vanishing when flowing in a certain direction under specific condition. Our study aligns with the mysteries of the universe, as any extended particles within black holes vanish, a phenomenon of disappearance that scientists continue to investigate to this day.

FUTURE WORK

A comparative study of contraction of curvature tensors with in Finsler space: fifth – order vs. higher – order slopes.