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Qualitative Analysis and Phase Portraits of an Epidemic Model

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INTRODUCTION & AIM

Mathematical modeling provides a powerful framework for understanding the transmission dynamics of infectious diseases. This study focuses on a Susceptible-Infected-Recovered-Susceptible (SIRS) compartmental model featuring a specific non-linear rational incidence rate

$$G(I)S = \frac{k_1 I + k_2 I^2}{1 + \alpha I^2}$$

This rate accounts for the psychological effect within the population via the denominator, while the numerator measures both linear and non-linear hazards of infection. While previous research explored local dynamics, the rational nature of this model severely restricts global analysis. This work aims to overcome these limitations by providing a comprehensive global analysis, specifically focusing on the system's behavior near infinity.

METHOD

The study employs the qualitative theory of planar differential equations to investigate the system's state space. To overcome the analytical limitations of the original fractional model, algebraic rescalings are applied to reduce it to a topologically equivalent two-dimensional cubic polynomial system

$$\begin{cases} \dfrac{dx}{dt} = -ax^3 - rx^2y + sx^2 - exy + bx \\ \dfrac{dy}{dt} = (x-y)(x^2+1), \end{cases}$$

where a, r, e are strictly positive parameters. A central component of the methodology is the application of the Poincaré compactification technique, which extends the polynomial vector field from the finite plane to the Poincaré disc. This enables a rigorous analysis of the system's behavior at infinity. All singular points are characterized by evaluating Jacobian matrices for finite equilibria and utilizing specialized coordinate transformations to investigate infinite singularities at the boundary of the disc, providing a complete topological mapping of the disease dynamics.

RESULTS & DISCUSSION

Theorem.

Assume that $a, r > 0$, and the fundamental threshold $s > \sqrt{1 + 2a + r}$, is satisfied. Let E denote the relevant non-trivial equilibrium of the system, which corresponds to E_* if b>point 0, E_c if b=0, or E_1 if $0 > b \succ -\frac{2+3a+r}{1+2a+r}$. The system undergoes a Hopf bifurcation at the equilibrium E if and only if $\boldsymbol{e = e_H}$ with $\boldsymbol{e_H}$ defined by

$$\frac{\boldsymbol{s(a(2b+3) + br + b + r + 2) \pm (a + b + 2ab + r + br)\sqrt{s^2 - 4(2a + r + 1)}}}{\boldsymbol{2(2a + r + 1)}}$$

Furthermore, this Hopf bifurcation is supercritical. Consequently, a stable limit cycle bifurcates from the equilibrium E for parameter values $e < e_H$ sufficiently close to e_H.

CONCLUSION

The SIRS model reveals complex features where limit cycles indicate that temporary immunity leads to periodic outbreaks. By employing global analysis at infinity, this study provides a complete topological description of the model. These findings offer valuable insights into nonlinear disease dynamics, establishing a robust theoretical foundation for public health interventions.

FUTURE WORK / REFERENCES

Dumortier F, Llibre J, Artés JC. Qualitative theory of planar differential systems. New York: UniversiText, Springer--Verlag; 2006.