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SPECTRAL COLLOCATION SOLUTION OF SOME STRONGLY NONLINEAR HEAT TRANSFER PROBLEMS

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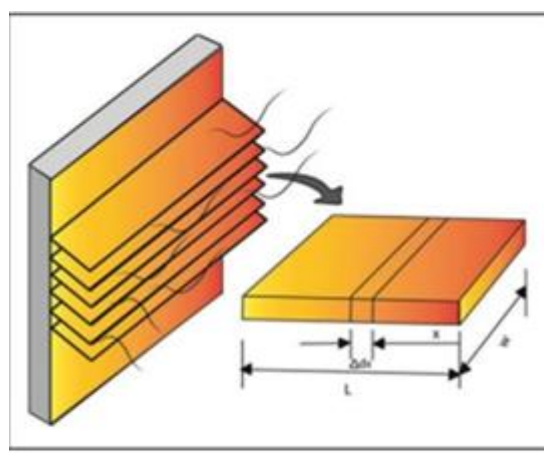
INTRODUCTION & AIM



Heat transfer through extended surfaces, commonly known as fins, is widely used in engineering applications such as heat exchangers and air-conditioning systems. Fins are thin, elongated structures attached to surfaces like walls, pipes, or plates to increase surface area and enhance heat dissipation. In this work, we aim to solve the following steady-state heat transfer problems using a spectral collocation scheme based on Fibonacci polynomials.

Model problem 1. We consider heat transfer within a straight rectangular fin by taking into account convective, radiative or combined convective-radiative surface in a power-law form and temperature dependent surface heat flux. This assumption results in the following one-dimensional two-point mixed boundary-value problem [Haq & Ishaq, 2012]:

$$Y''(x) - \kappa^2\big(Y(x)\big)^{m+1} = 0, Y'(0) = 0, Y(1) = 1,$$

where, $Y(x)$ is the temperature, κ is the conductive-convective fin parameter, and m is nonlinearity parameter.

Model problem 2. We consider convective-radiative cooling of a lumped system. It is assumed that the specific heat depends on temperature linearly. The mathematical model for this case gives the following initial-value problem [Haq & Ishaq, 2012]:

$$\big(1 + \mu Y(x)\big)Y'(x) + Y + \nu\big(Y(x)\big)^4 = 0, \qquad Y(0) = 1,$$

where μ, ν are constants.

METHOD

The Fibonacci polynomials can be generated via following three-term recurrence relation [Mirzaee & Hoseini, 2016]:

$$\varphi_j(x) = x\varphi_{j-1}(x) + \varphi_{j-2}(x), \qquad \forall j \geq 2$$

where $\varphi_1(x) = x, \varphi_0(x) = 1$. Any square integrable function, $Y \in L^2[a,b]$, can be approximated by $(n+1)$th degree Fibonacci polynomial at the point $x = x_i$ as

$$Y(x) = \sum_{j=0}^{n} C_j\varphi_j(x_i) = \mathbf{\Phi}^{\mathrm{T}}(x_i)\mathbf{C} = D_0\mathbf{C},$$

where $\mathbf{C}^T = [C_0, C_1, \cdots, C_j, \cdots, C_n]$ and

$$D_0 = \begin{bmatrix} \mathbf{\Phi}^{\mathrm{T}}(x_0) \\ \vdots \\ \mathbf{\Phi}^{\mathrm{T}}(x_n) \end{bmatrix} = \begin{pmatrix} \varphi_0(x_0) & \cdots & \varphi_n(x_0) \\ \vdots & \ddots & \vdots \\ \varphi_0(x_n) & \cdots & \varphi_n(x_n) \end{pmatrix}.$$

Similarly, the first- and second-order derivatives can be approximated at each $x = x_i$ $(i = 0,1,2 \cdots, n)$ as:

$$Y'(x_i) = \big(\mathbf{\Phi}^{\mathrm{T}}(x_i)\big)'\mathbf{C} = D_1\mathbf{C};\ Y''(x_i) = \big(\mathbf{\Phi}^{\mathrm{T}}(x_i)\big)''\mathbf{C} = D_2\mathbf{C}.$$

We now consider the initial/boundary value problem in general form approximated at $x = x_i$ $(i = 0,1,2, \cdots, n)$ as

$$Y''(x)\Big|_{x=x_i} = f\left(x_i, Y(x)\Big|_{x=x_i}, Y'(x)\Big|_{x=x_i}\right).$$

Using the approximations for Y, Y' and Y'', the above equation reduces to the following nonlinear system of algebraic equation

$$D_2\mathbf{C} - \mathbf{f}(\mathbf{x}, D_0\mathbf{C}, D_1\mathbf{C}) = \mathbf{0},$$

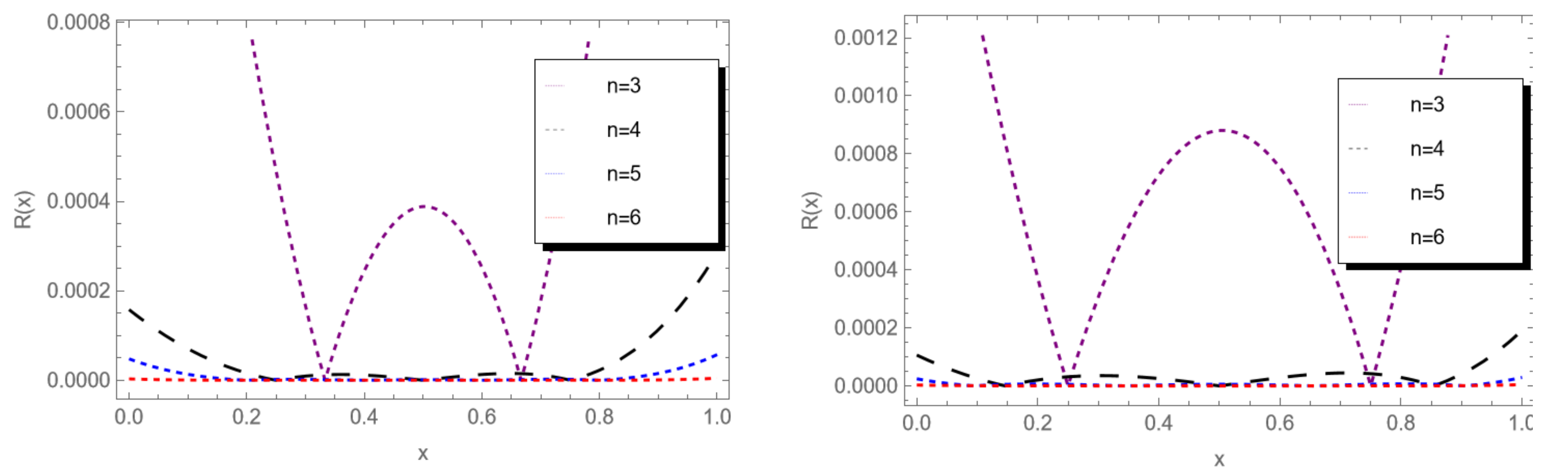
where $\mathbf{0} = [0,0,0, \cdots, 0]^T, \mathbf{x} = [x_0, x_1, x_2, \cdots, x_n]^T$.

Solving the nonlinear system of equations for the unknown vector $\mathbf{C}$, the approximate solution can be obtained at any point $x \in [a,b]$.

Remark: The collocation points x_i used in this study are uniform and Chebyshev points. The boundary/initial condition(s) are implemented by replacing the first and last rows of the final system corresponding to $x_0 = a$ and $x_n = b$. The Mathematica built-in function NSolve was used for solving the nonlinear system of equations.

RESULTS & DISCUSSION

The figures below show the method convergence over uniform (left) and Chebyshev points (right) for increasing nodes/polynomial order n when $\kappa = \sqrt{0.09}$, $m = 3$ in model problem 1.



The data below displays solution comparison against OHAM [Haq & Ishaq, 2012] and numerical solutions when $\kappa = \sqrt{0.09}$, $m = 3$ and $n = 6$ in model problem 1.

x	Present	OHAM	Numerical
0.0	0.960624	0.9606237	0.960955
0.1	0.961008	0.9610071	0.961326
0.2	0.962159	0.9621582	0.962449
0.3	0.964081	0.9640810	0.964335
0.4	0.966782	0.9667814	0.966998
0.5	0.970269	0.9702683	0.970451
0.6	0.974553	0.9745530	0.974705

The data below displays percentage of error comparison against OHAM [Haq & Ishaq, 2012] and DTM [Yaghoobi & Torabi, 2011] for $n = 4$ at $x = 0.2$ in model problem 2.

μ	ν	Present	OHAM	DTM
0.4	0.4	0.116621	0.0168998	0.001888708
0.4	0.6	0.217827	0.0139921	0.005815298
0.4	0.8	0.337273	0.0033507	0.013804869
0.8	0.4	0.0464139	0.1190960	0.000418251
0.8	0.6	0.09609	0.1056940	0.001349424
0.8	0.8	0.160923	0.0682416	0.003305231

CONCLUSION

A spectral collocation method is proposed for strongly nonlinear initial and mixed boundary-value problems. It yielded accurate results for both small and large parameters without linearization or initial guesses. Chebyshev points improved mixed-condition accuracy at boundaries. The study also examined temperature behavior under varying thermal parameters, proving the method efficiency for high-order nonlinear problems.

FUTURE WORK / REFERENCES

[1]. S. Haq, M. Ishaq, International Journal of Heat and Mass Transfer, 55 (2012) 5737–5743.
[2]. H. Yaghoobi, M. Torabi, International Communication in Heat and Mass Transfer, 38(6) (2011) 815-820.
[3]. F. Mirzaee, S.F. Hoseini, Applied Mathematics and Computation, 273 (2016) 637–644.