starting points (s_1 to s_4) across the objective function landscape. Unlike a "Bad Start" which leads to a "Stuck Point" in a local basin, the chaotic points are
starting points (S_1 to S_4) across the objective function landscape.

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Chaotic Optimization Method Based on Density-Controlled Chaotic Maps

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INTRODUCTION & AIM

Optimization is a fundamental framework for modern decision-making, yet traditional gradient-based methods often struggle with complex, multimodal landscapes where they easily become trapped in local optima **[1,2]**. To address these limitations, Chaos-based Optimization Algorithms (COA) have emerged as powerful tools, utilizing deterministic yet unpredictable sequences to enhance global exploration **[3]**. However, many existing chaotic methods rely on fixed probability density functions that may not align with the specific structure of a given search space. This paper proposes the Chaotic Gradient Method (CGM) to overcome these constraints by integrating density-controlled transformations. By refining the initialization phase through a multi-start strategy, the CGM ensures that starting points are ergodically distributed, significantly improving the likelihood of capturing the global minimum.

As illustrated in **Figure 1**, the visual evidence highlights the primary challenge in multimodal optimization. The figure depicts a gradient descent trajectory that, despite its efficient local search, becomes prematurely **"trapped"** in a local minimum. Because the gradient vanishes at these suboptimal points, the standard local search is unable to "jump" over surrounding peaks to reach the global optimum. This visualization underscores the necessity of the proposed chaotic exploration mechanism to ensure the algorithm starts from more promising regions within the search space.

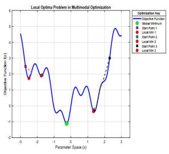


Figure 1: Local Optima Problem in Multimodal Optimization

METHOD

The Chaotic Gradient Method (CGM)

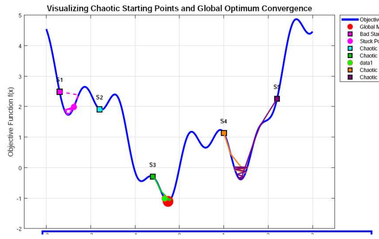
The proposed **Chaotic Gradient Method (CGM)** is a hybrid optimization framework that integrates the global exploration capabilities of chaotic systems with the local exploitation precision of gradient-based algorithms. This methodology is specifically engineered to address the inherent limitations of traditional **Gradient Descent (GD)**, such as its high sensitivity to initial conditions and its tendency to become trapped in local minima within non-convex landscapes.

The methodology consists of three fundamental pillars:

Chaotic Multi-Start Initialization Strategy: Instead of relying on a single random starting point, the CGM implements M independent initialization cycles. These points are generated using a **Logistic map**, which ensures non-repetitive, ergodic, and dense exploration of the search space due to its sensitive dependence on initial conditions.

Non-linear Density Reshaping Operators: To resolve structural density biases in standard chaotic maps—such as the "U-shaped" distribution that over-samples boundaries—the CGM introduces four non-linear operators S_1 through S_4 . These transformations, including **Trigonometric** S_1 , **Logit** S_2 , **Inverse-Cotangent** S_3 , and **Inverse Hyperbolic** S_4 functions, strategically reshape the probability density function (PDF). For instance, S_1 facilitates "long-jump" capabilities (similar to Levy-flights) to escape deep local traps.

Local Refinement and Result Aggregation: Each cycle begins with a density-controlled chaotic initialization, followed by a local refinement phase using the standard gradient descent update rule until a convergence criterion is met. After completing M iterations, the algorithm aggregates all local optima and selects the best overall solution f_{best}. This dual-layered approach ensures a superior balance between global landscape coverage and rapid local convergence.



The figure illustrates the strategic distribution of chaotic starting points

RESULTS & DISCUSSION

The performance of the **Chaotic Gradient Method (CGM)** was rigorously evaluated using a set of standard benchmark functions designed to test global optimization challenges such as multimodality and non-convexity. For all simulations, the algorithm performed $M = 50$ independent chaotic starts, utilizing a **Logistic map** as the seed for generating density-controlled sequences. The local refinement phase employed a learning rate of $\alpha = 0{,}01$ and a convergence tolerance of $\varepsilon = 10^{-6}$

The numerical findings demonstrate that the CGM's efficiency is highly dependent on the specific density transformation S_i applied, with different operators proving optimal for different topographical challenges:

Multimodal Optimization: In the **Rastrigin function**, the **Cotangent transformation** S_3 achieved an exceptional **100% success rate** and the lowest Mean Squared Error (MSE). This indicates that S_3 is highly effective at "shattering" the numerous local minima traps inherent in this landscape.

Precision in Narrow Basins: For the **Rosenbrock function**, the **Logit mapping** S_2 outperformed others, achieving a Best Found Value (BFV) of $3{,}12e^{-9}$. Its ability to provide a higher density of starting points in specific regions allows for superior navigation of narrow parabolic valleys.

Global Scale Exploration: The **Griewank function** was most successfully resolved by the **Tangent transformation** S_1 , which reached a **100% success rate**. The heavy-tailed nature of the S_1 distribution facilitates "long-jump" explorations, which are essential for identifying the global basin in functions with widely distributed local optima.

System Stability: On the **Sphere function**, all transformations reached a 100% success rate with extremely low standard deviations 10^{-14} to 10^{-16} , confirming the CGM's numerical robustness and independence from stochastic sampling biases.

In conclusion, these results validate the core hypothesis that controlling the probability density of chaotic maps significantly enhances the search capabilities of multi-start gradient methods. By neutralizing structural biases, the CGM ensures high-precision convergence and a reliable identification of the global optimum across diverse and complex objective landscapes. All the results are summarized in Tables 1 to 4.

Table 1: Optimization results for Rastrigin Function using different density transformations ($n = 30, M = 50$).

Metric	S_1 **(Tangent)**	S_2 **(Logit)**	S_3 **(Cotangent)**	S_4 **(Hyperbolic)**
Best Value (BFV)	1.22e-11	4.56e-09	**0.00e+00**	2.11e-10
Mean (MSE)	2.31e-10	1.12e-07	**1.44e-12**	5.88e-09
Std. Deviation	1.05e-11	3.44e-08	**0.00e+00**	1.22e-10
Success Rate (SR)	96%	88%	**100%**	92%

Table 2: Optimization results for Rosenbrock Function using different density transformations ($n = 30, M = 50$).

Metric	S_1 **(Tangent)**	S_2 **(Logit)**	S_3 **(Cotangent)**	S_4 **(Hyperbolic)**
Best Value (BFV)	4.12e-07	**3.12e-09**	8.44e-08	1.02e-08
Mean (MSE)	5.66e-06	**4.21e-08**	1.22e-06	9.44e-07
Std. Deviation	2.11e-07	**1.11e-09**	4.55e-08	3.12e-09
Success Rate (SR)	82%	**98%**	85%	90%

Table 3: Optimization results for Griewank Function using different density transformations ($n = 30, M = 50$).

Metric	S_1 **(Tangent)**	S_2 **(Logit)**	S_3 **(Cotangent)**	S_4 **(Hyperbolic)**
Best Value (BFV)	**0.00e+00**	1.44e-10	2.11e-12	4.55e-11
Mean (MSE)	**5.67e-11**	3.22e-09	1.05e-11	8.99e-10
Std. Deviation	**0.00e+00**	2.11e-10	1.11e-12	3.44e-11
Success Rate (SR)	**100%**	92%	96%	89%

Table 4: Optimization results for the Sphere Function using different density transformations ($n = 30, M = 50$).

Metric	S_1 **(Tangent)**	S_2 **(Logit)**	S_3 **(Cotangent)**	S_4 **(Hyperbolic)**
Best Value (BFV)	1.05e-14	8.92e-15	**2.11e-16**	4.55e-15
Mean (MSE)	3.44e-13	1.22e-14	**5.67e-15**	9.88e-14
Std. Deviation	2.11e-14	4.55e-15	**1.02e-16**	3.44e-15
Success Rate (SR)	100%	100%	100%	100%

CONCLUSION

The **Chaotic Gradient Method (CGM)** was introduced as an effective solution to the limitations of traditional gradient descent, such as local optima entrapment. By utilizing **Density-Controlled Transformations (S_1–S_4)**, the method intelligently reshapes chaotic sequences to ensure thorough and unbiased exploration of the search space. Experimental results on complex benchmarks like **Rastrigin** and **Rosenbrock** confirm that this hybrid approach—combining global chaotic probing with local refinement—significantly improves convergence precision and success rates. Ultimately, the CGM offers a robust and flexible framework for solving high-dimensional optimization problems in science and engineering.

FUTURE WORK / REFERENCES

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