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Modeling a multihost epidemic with seasonal forcing and stochasticity.

SANNAKY Ibtissam*, EL BERRAI Imane, ADNAOUI Khalid
Laboratory of Analysis, Modeling and Simulation (LAMS), Faculty of Sciences Ben M'Scik, Hassan II University of Casablanca, Casablanca, Morocco
** Corresponding author email: bissamsannaky@gmail.com*

INTRODUCTION & AIM

Every year, the silent parasite ***Echinococcus granulosus*** impacts thousands of lives, and causes devastating global economic losses exceeding $3 billion annually. The disease**, *Cystic Echinococcosis (CE)***, spreads from dogs to livestock to humans through contaminated soil, going undetected until advanced stages.

So what drives disease persistence, and under what conditions can it go extinct?

To answer this, we combine deterministic and stochastic frameworks **(ODE, CTMC, and Multitype Branching Processes).** This allows us to identify the weakest links in the transmission chain, and determine exactly the optimal seasonal windows where interventions hit hardest.

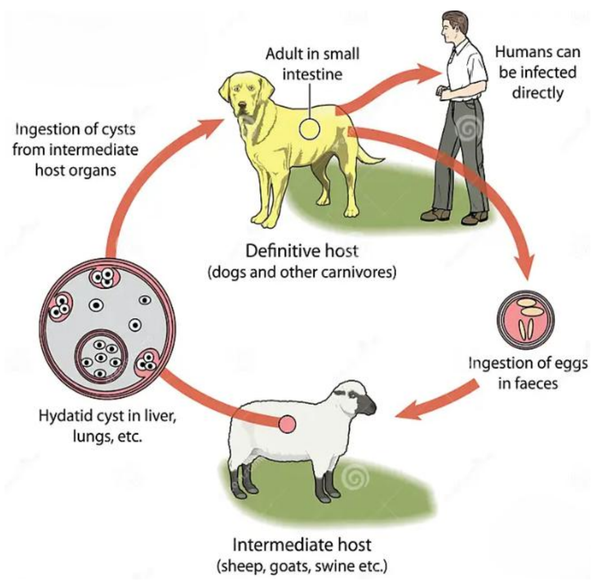
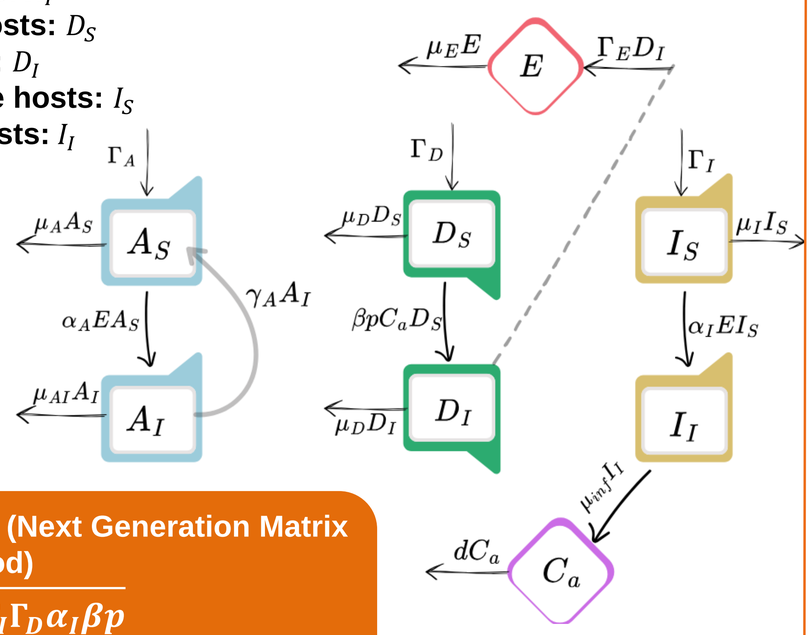


Figure 1: Transmission cycle of Cystic Echinoccocosis among different types of hosts

METHODS

Deterministic ODE model

- **Susceptible accidental hosts:** A_S
- **Infected accidental hosts:** A_I
- **Susceptible definitive hosts:** D_S
- **Infected definitive hosts:** D_I
- **Susceptible intermediate hosts:** I_S
- **Infected intermediate hosts:** I_I
- **Infected carcasses:** C_a
- **Parasite eggs density:** E



Basic reproduction number (Next Generation Matrix Method)

$$R_0 = \sqrt[4]{\frac{\Gamma_E \Gamma_I \Gamma_D \alpha_I \beta p}{\mu_D^2 \mu_E \mu_{inf}\, d}}$$

- The squared dog mortality rate $\mu_D{}^2$ in the denominator means that targeting dogs has a large effect on reducing R_0.
- The disease persists in the hosts populations if $R_0 > 1$ and is eliminated otherwise.

CTMC stochastic extension

Markov Chain properties

- Time homogeneous
- Markov property
- One event per a unit of time Δt

Why stochastic?

- Infection occurs in integer events not continuous flows
- Captures demographic stochasticity
- Helps to quantify extinction and outbreak probabilities

State vector: $\vec{Y} = [A_S, A_I, D_S, D_I, I_S, I_I, C_a, E]$

Events and transition rates modelled:

- Recruitment
- Infection
- Death
- Slaughter
- Carcass decay
- Egg shedding

Events examples	Transitions $\Delta\vec{Y}(t)$	Transition rates
Definitive hosts infection	$(0,0,-1,1,0,0,0,0)$	$\beta p C_a D_S$
Defacation by infected definitive hosts	$(0,0,0,0,0,0,0,1)$	$\Gamma_E D_I$
Decay of infected carcasses C_a	$(0,0,0,0,0,0,-1,0)$	dC_a

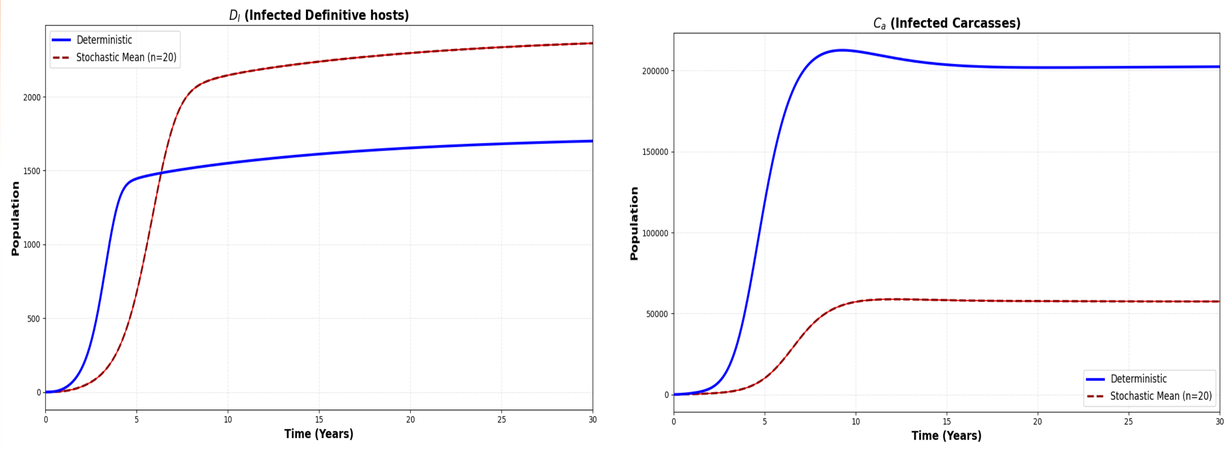
Multitype branching process

Applied to infectious classes: A_I, D_I, I_I, C_a, E ⟶ **A five-type branching process** employed under the assumption that susceptible populations are sufficiently large at the disease-free state.

The accidental hosts pgf $f_1(u) = 1$ for $u \in [0,1]$ → Humans are a demographic absorbing state → NO MATTER HOW MANY HUMANS ARE INFECTED THEY GENERATE ZERO SECONDARY INFECTIONS.

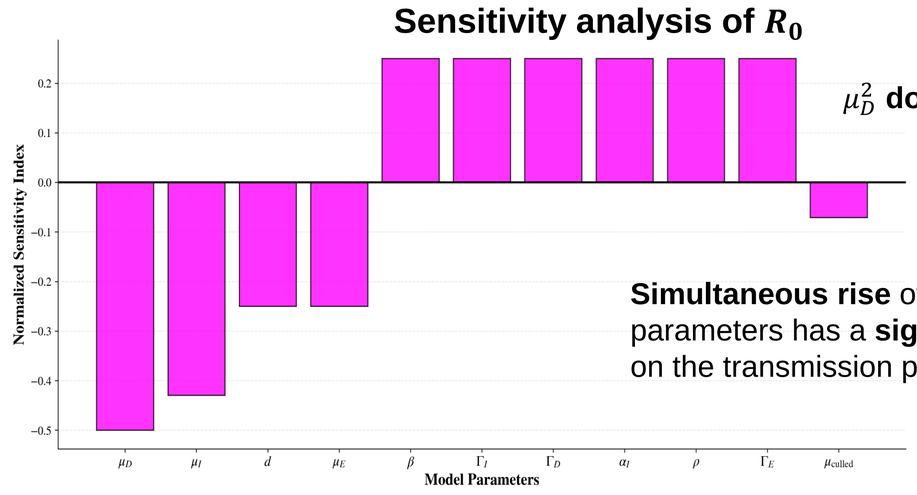
RESULTS & DISCUSSION

ODE vs CTMC simulations



- The deterministic models can systematically underestimate the true infection burden.
- Discrete transmission events amplify the persistence of infected definitive hosts.
- Infected carcasses are short-term transmission bridges rather than long-term reservoirs.

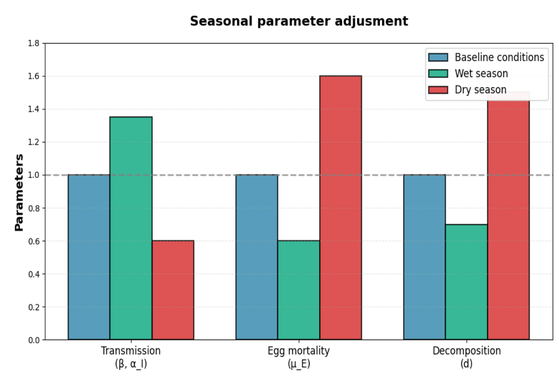
Sensitivity analysis of R_0

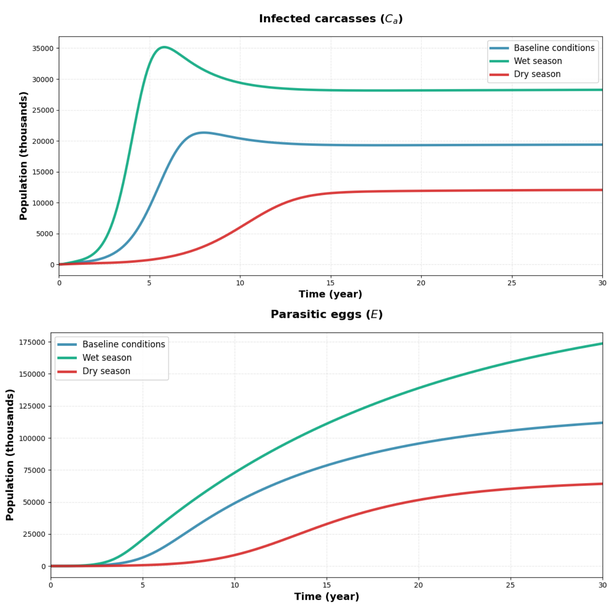


μ_D^2 **dominates** R_0.

Simultaneous rise of other parameters has a **significant impact** on the transmission process.

Impact of seasonality





Dry and hot seasons naturally minimize disease transmission → **Intensifying control measures during this period could BREAK the transmission cycle and AMPLIFY intervention effectiveness.**

Extinction and outbreak probabilities

i_1, i_2, and i_5 are the initial sizes of accidental and definitive hosts, and E. granulosus eggs, respectively.

A single infected dog is sufficient to trigger a 96% probability of outbreak
→ DOG CONTROL IS NOT OPTIONAL IT IS CRITICAL.
→ Control strategies must focus on the proper disposal of infected carcasses to BREAK THE CARCASS-DOG TRANSMISSION LINK.

CONCLUSION

- **The livestock-carcass-dog cycle drives CE persistence** (confirmed by R_0 structure and extinction probability analysis).
- **Humans are mathematically proven dead-end hosts**, so focusing control strategies only on humans will not break the transmission cycle of CE.
- A single infected dog yields a 96% outbreak probability, so **dog control and carcass disposal are the critical interventions.**

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