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Exact Traveling Wave Solutions of Nonlinear Coupled Wave Systems via Analytical Methods

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INTRODUCTION & AIM

Nonlinear partial differential equations (NLPDEs) play a central role in modeling complex phenomena in physics, and engineering. In fluid dynamics, the coupled nonlinear wave equations presented here provides a robust models for describing nonlinear wave propagation in modeling interacting waves, formulated as the coupled KdV equations and the coupled Hirota–Satsuma system:

$$\begin{cases} u_t - uu_x - vv_x - u_{xxx} = 0 \\ v_t + uv_x + 2v_{xxx} = 0 \end{cases}, \quad \begin{cases} u_t + 3uu_x - 6vv_x - \frac{1}{2}u_{xxx} = 0 \\ v_t - 3uv_x + v_{xxx} = 0 \end{cases}.$$

The aim is to derive analytical solutions that provide physical insight into wave interactions and nonlinear phenomena.

METHODS

In this work, we apply two effective analytical methods—the tanh–coth method and the generalized $\exp(-\varphi(\xi))$-expansion (GEE) method—to obtain exact solutions.

A traveling wave transformation $\xi = x - \omega t$ reduces the NLPDEs into nonlinear ODEs. Specific expansion assumptions transform the differential equations into nonlinear algebraic systems, which are solved symbolically using Maple 25.

- **The tanh–coth Method**

We propose a solution of the form

$$U(\xi) = \sum_{i=0}^{m} a_i Y^i + \sum_{i=1}^{m} b_i Y^{-i}, V(\xi) = \sum_{i=0}^{n} c_i Y^i + \sum_{i=1}^{n} d_i Y^{-i},$$

where $Y = \tanh(\mu\xi), Y^{-1} = \coth(\mu\xi)$.

The coefficients $a_i,\ b_i,\ c_i,$ and d_i are unknown constants, while m and n are positive integers to be determined by balancing the dominant nonlinear term with the highest-order derivative term in the reduced ordinary differential equations.

Procedure:

1. Substitute the above expressions for $U(\xi)$ and $V(\xi)$ into the reduced system.
2. Express all terms as powers of $\tanh(\mu\xi)$.and $\coth(\mu\xi)$,
3. Collect like powers of $\tanh(\mu\xi)$ and $\coth(\mu\xi)$.
4. Set the coefficients of each power equal to zero.

This yields a system of algebraic equations for the unknown parameters $a_i,\ b_i,\ c_i,\ d_i,\ \mu$, and ω.

- **The GEE Method**

We assume solutions of the form

$$U(\xi) = \sum_{i=0}^{m} \alpha_i S^i(\xi), \qquad V(\xi) = \sum_{j=0}^{n} \beta_j S^j(\xi),$$

where $S(\xi) = \exp(-\phi(\xi))$, and $\phi(\xi)$ satisfies the auxiliary equation $\phi'(\xi) = p\exp(-\phi(\xi)) + q\exp(\phi(\xi)) + r$,

with p, q, and r being arbitrary constants.

Procedure:

1. Substitute the assumed forms of $U(\xi)$ and $V(\xi)$ into the reduced system.
2. Use the auxiliary equation to express all derivatives in terms of powers of $S(\xi)$.
3. Collect the resulting terms as polynomials in $S(\xi)$.
4. Set the coefficient of each power of $S(\xi)$ equal to zero.

This process leads to a system of algebraic equations for the unknown parameters $\alpha_i,\ \beta_j,\ p,\ q,\ r,\ and\ \omega$.

RESULTS & DISCUSSION & VISUALISATION

Using the balancing procedure, we obtain $m = n = 2$. Therefore, the solutions are assumed by the Tanh–Coth Method in the form

$$\begin{cases} U(\xi) = a_0 + a_1\tanh(\mu\xi) + a_2\tanh^2(\mu\xi) + b_1\coth(\mu\xi) + b_2\coth^2(\mu\xi), \\ V(\xi) = c_0 + c_1\tanh(\mu\xi) + c_2\tanh^2(\mu\xi) + d_1\coth(\mu\xi) + d_2\coth^2(\mu\xi), \end{cases}$$

where a_i ,b_i ,c_i ,d_i ,and μ are constants to be determined. Substituting these expressions into the reduced ordinary differential equations and equating the coefficients of like powers of $\tanh(\mu\xi)$ and $\coth(\mu\xi)$ yields a system of algebraic equations for the unknown parameters.

Similarly, balancing the highest-order derivative terms with the dominant nonlinear terms gives $m = n = 2$. Hence, the solutions are sought by the GEE Method in the form

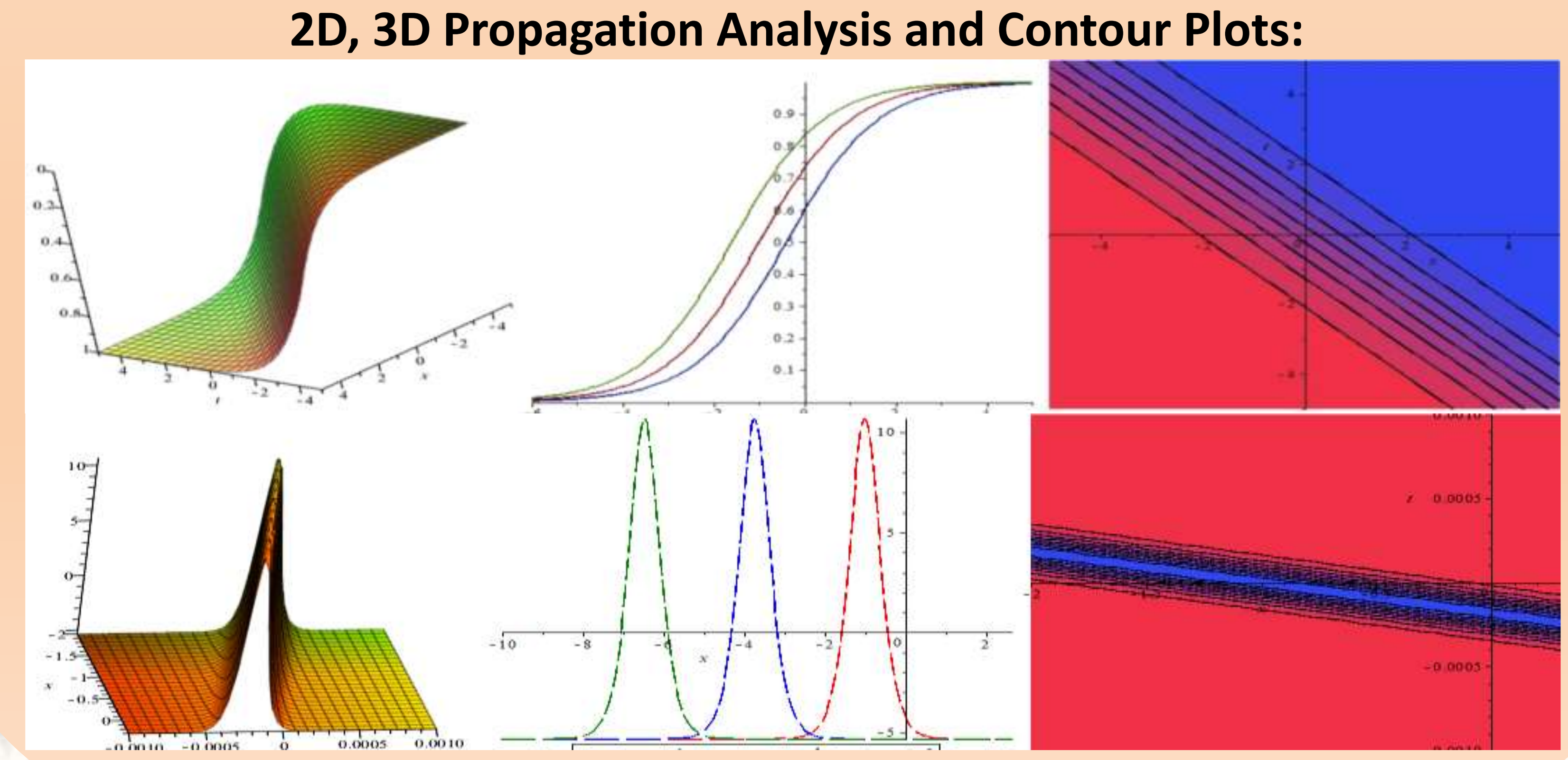
$$\begin{cases} U(\xi) = \alpha_0 + \alpha_1 e^{-\phi(\xi)} + \alpha_2 e^{-2\phi(\xi)}, \\ V(\xi) = \beta_0 + \beta_1 e^{-\phi(\xi)} + \beta_2 e^{-2\phi(\xi)}, \end{cases}$$

Substituting the assumed solutions into the reduced system and collecting coefficients of equal powers of $e^{-\phi(\xi)}$ leads to a system of algebraic equations for α_i ,β_i ,p ,q ,r ,and ω.

The application of both methods yields a broad family of exact traveling wave solutions, including: Solitary-wave solutions, Kink-type wave structures, Periodic trigonometric solutions, Rational wave profiles.

The tanh–coth method proves particularly effective for constructing soliton and kink solutions, whereas the GEE method generates a richer variety of wave structures, providing a more diverse set of exact analytical solutions for the nonlinear system under consideration.

2D, 3D Propagation Analysis and Contour Plots:



CONCLUSION

This work highlights the effectiveness of the tanh–coth and GEE methods in solving coupled nonlinear wave systems. The use of symbolic computation via Maple 25 significantly simplifies the derivation of exact solutions. These approaches serve as powerful tools in the analysis of nonlinear dynamics and contribute to a deeper understanding of wave phenomena in physical systems.

FUTURE WORK

- Extend the methods to higher-dimensional systems.
- Study fractional nonlinear equations.
- Investigate numerical stability of the obtained solutions.

REFERENCES

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