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Computational Modeling and Convergence Analysis of Generalized 1-D Linear Evolution Equations

Douha Saadi, Ahcene Merad
System Dynamics and Control Laboratory, Department of Mathematics and Computer Science, University of Oum El Bouaghi, Algeria
douha.saadi@univ-oeb.dz

Introduction & Mathematical Model

•The telegraph equation models wave propagation, vibrations, and damped signal transmission in a bounded domain with initial and boundary conditions.
•Classical numerical methods such as FDM and FEM are computationally expensive due to mesh generation requirements.
•This work applies a mesh-free RKHS approach to the generalized 1-D linear telegraph equation:

$$u_{tt} + au_t + bu = cu_{xx} + f(x,t)$$

u(x,0)=g(x), $u_t\,(x,0) = E(x)$,u(0,t)=q(t),u(1,t)=p(t),
•A homogenization transformation is used to convert the problem into an equivalent system with homogeneous conditions, enabling efficient and accurate solution construction.

Main Steps of the RKHS Method

1. Establish the differential operator. Show that the operator L is linear and bounded

$$\|\mathcal{L}u\|_{W_2^{(1,1)}(\Omega)} < C\|u\|_{W_2^{(3,3)}(\Omega)}.$$

2. Select a dense set of points. Choose a dense sequence of points $\{(x_i,t_i)\}_{i=1}^{\infty} \subset \Omega$
3. Construct the basis functions. Using the reproducing kernel K (x, t), define

$$\phi_i(x,t) = \mathcal{L}_{(y,s)} K_{(y,s)}(x,t)\Big|_{(y,s)=(x_i,t_i)}.$$

4. Apply Gram–Schmidt orthogonalization. Generate an orthonormal basis $\{\overline{\phi}_i(x,t)\}$ by

$$\overline{\phi}_i(x,t) = \sum_{k=1}^{i} \beta_{ik}\phi_k(x,t).$$

5. Obtain the exact solution representation. Express the exact solution as

$$u(x,t) = \sum_{i=1}^{\infty}\sum_{j=1}^{i} \beta_{ij} F(x_j,t_j)\overline{\phi}_i(x,t).$$

6. Construct the numerical approximation. Truncate the series after N terms:

$$u_N(x,t) = \sum_{i=1}^{N}\sum_{j=1}^{i} \beta_{ij} F(x_j,t_j)\overline{\phi}_i(x,t).$$

7. Truncated approximation and convergence

$$u_N(x,t) \to u(x,t) \quad as\ N \to \infty$$

Numerical Results

Parameters: a= 1, b = 25, c = 20, Ω = [0, 1] × [0, 1]
Exact solution:

$$u(x,t) = e^t x^2 (1-x)^2$$

Error Comparison at: T = 1, Δx = Δt = 0.1:

Method	$L_\infty Error$	$L_2 Error$
Fourth-order compact difference	1.7556×10^{-3}	3.3633×10^{-3}
MLRPI	3.2881×10^{-4}	5.1019×10^{-4}
RKHS (present method)	3.2751×10^{-5}	3.7505×10^{-5}

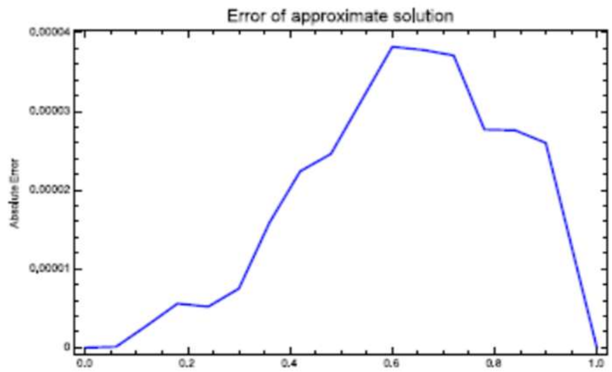

Figure1: Absolute error distribution be-tween the RKHS solution (u_{81}) andthe exact solution for Example

Discussion and Conclusion

1. The RKHS method solves the generalized linear telegraph equation accurately without meshgeneration or discretization.
2. Convergence analysis and numerical experiments confirm the reliability and superior accuracyof the proposed approach.
3. The method is simple, efficient, and can be extended to nonlinear and higher-dimensional prob-lems.

FUTURE WORK / REFERENCES

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