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Reaction–diffusion model of infection propagation with influence of resident macrophages

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INTRODUCTION & AIM

Respiratory viral diseases are still an urgent challenge of world health care. The understanding of the immune dynamics of the propagation of viral infection allows one to find possible therapy targets, choose appropriate treatment strategies, propose new mechanisms for virus elimination by drugs, etc.
While the virus infects the target cells the mechanisms of immune response, innate and adaptive, are activated. Nowadays, the inflammation component, which in narrow sense is the part of innate immune response, attracts attention of specialists. In particular, the role of resident macrophages, which are macrophages permanently placed in tissues, in different diseases is under active investigation.
Distribution of infection in tissues has spatial component, and it can be described by reaction-diffusion waves. The aim of this work is to investigate the influence of resident macrophages on the severity and infectivity of the respiratory virus infection.

METHOD

To investigate the basic characteristics of the respiratory virus infection propagation and their dependence on the dynamics of tissue resident macrophages, we have constructed the following reaction-diffusion model with integral terms:

$$\frac{\partial E}{\partial t} = -a_1 EV, \qquad \frac{\partial F}{\partial t} = a_1 EV - k_1 FS - \sigma_1 F,$$

$$\frac{\partial M}{\partial t} = -a_2 MV, \qquad \frac{\partial N}{\partial t} = a_2 MV - \sigma_2 N,$$

$$\frac{\partial V}{\partial t} = D\frac{\partial^2 V}{\partial x^2} + bF - \sigma_3 V,$$

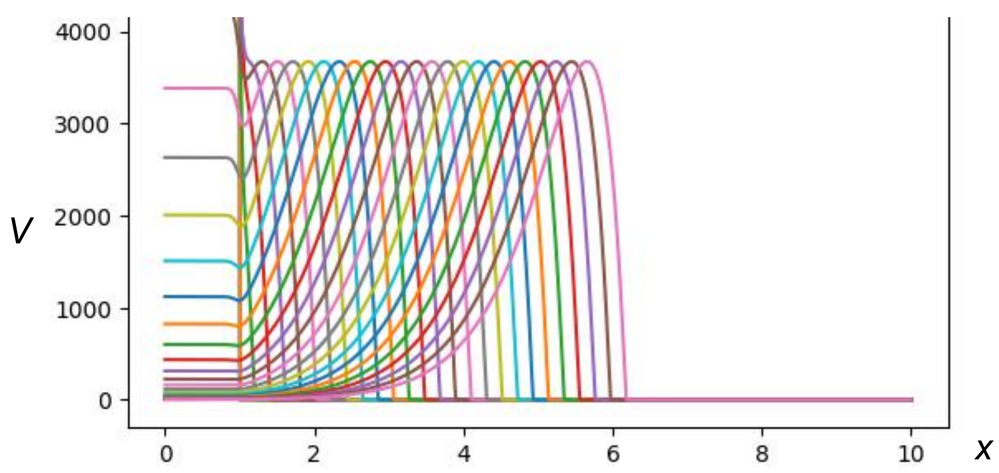
$$\frac{dS}{dt} = a_3 J(F) + dJ(N) - k_2 J(F)S - \sigma_4 S,$$

$$J(F) = \int_{-\infty}^{+\infty} F(x,t)dx\,, \qquad J(N) = \int_{-\infty}^{+\infty} N(x,t)dx\,,$$

where

- *E(x,t)* is the concentration of uninfected cells;
- *F(x,t)* is the concentration of infected cells;
- *M(x,t)* is the concentration of inactivated resident macrophages;
- *N(x,t)* is the concentration of activated resident macrophages;
- *V(x,t)* is the concentration of viral particles in the extracellular space;
- *S(t)* is the concentration of proinflammatory cytokines.

The model can admin the wave solution, i.e., continuous bounded solution propagating in time with the constant speed $c > 0$ and which has the same form in space:



Here, different curves correspond to consecutive times.

RESULTS

In this system, we have determined the following basic characteristics:

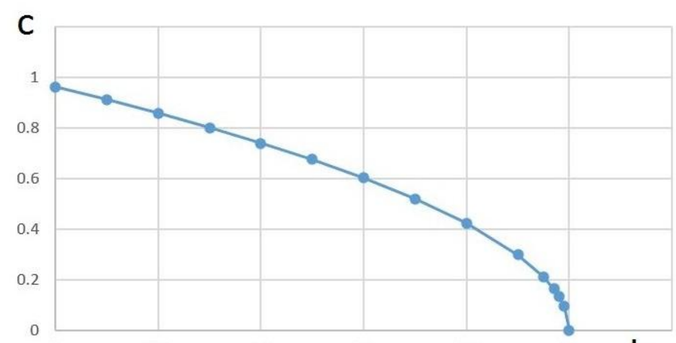
- **Virus replication number R_v**. It characterizes the ability of the virus to produce infection and corresponds to the number of newly infected cells by one infected cell

$$R_v = \frac{a_1 b\, U_0}{\sigma_1 \sigma_3};$$

- **Immunity effectiveness number P**. It characterizes the strength of immunity against the virus infection. For $\sigma_4 = 0$ it has the form

$$P = \frac{k_1 a_3 + k_1 d\, R}{k_2}, R \approx \frac{k_1 m_0 a_3 + \sigma_1 m_0 k_2}{k_2 \sigma_2 u_0 - k_1 d\, m_0} > 0$$

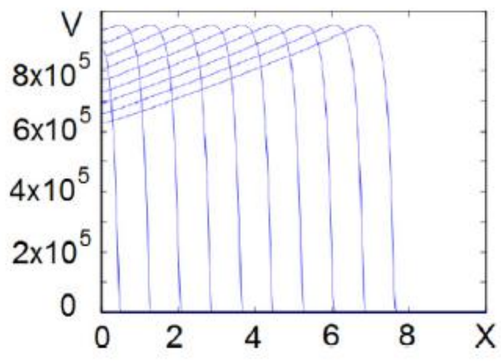
It reduces the speed c of infection propagation:



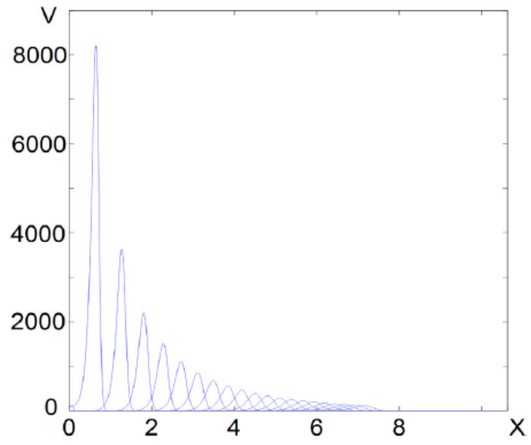
More active functioning of resident macrophages increase the immunity effectiveness number P thereby decreasing the severity and infectivity of the respiratory virus infection.

The development of infection is determined by the following conditions:

- If $R_v < 1$ the infection does not develop.
- If $1 < R_v < 1 + P$ the infection develops, but immune response suppresses it;
- If $R_v > 1 + P$ the infection develops



$R_v > 1 + P$



$1 < R_v < 1 + P$

CONCLUSION

Influence of resident macrophages can eliminate the emerging infection. New regime when the infection can be suppressed by the immunity is obtained in the model. Characteristics governing the infection propagation are determined and their explicit representations are obtained. The model considers the respiratory virus infection, like COVID-19 or influenza, but also can be applied for other infections.

FUNDING & REFERENCES

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M. Bouzari, L. Ait Mahiout, A. Mozokhina, V. Volpert Infection propagation in a tissue with resident macrophages. *Mathematical Biosciences*. 2025; 381: 109399. https://doi.org/10.1016/j.mbs.2025.109399