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Nonlinear Wave Propagation in an Extended (3+1)-Dimensional Calogero-Bogoyavlenskii-Schiff equation

M. Hemapriya

Department of Mathematics, Bharathiar University, Coimbatore-641 046, India.

INTRODUCTION & AIM

The (3+1)-dimensional extended variable-coefficient Calogero–Bogoyavlenskii–Schiff (CBS) equation models nonlinear wave propagation in non-homogeneous media where physical parameters vary in space and time, making it more realistic than constant-coefficient models for applications in ocean dynamics, plasma physics, and turbulent flows. Despite their physical significance, obtaining exact analytical solutions to such high-dimensional variable-coefficient nonlinear PDEs remains a substantial mathematical challenge. The Variable Coefficient Generalized Abel Equation Method (VCGAEM) addresses this by providing a systematic framework for deriving rich families of exact traveling wave solutions, revealing the intrinsic wave structures of complex nonlinear systems.

Aim: To apply the Variable Coefficient Generalized Abel Equation Method to construct exact traveling wave solutions for the variable-coefficient CBS equation and visualize their spatio-temporal dynamics through 3D surface plots.

METHOD

- The Extended (3+1) dimentional Calogero-Bogoyavlenskii-Schiff fluid equation is given by,

$$u_{xt} + \alpha(u_{xxxx} + 6u_x u_{xx}) + \beta(u_{xxxy} + 4u_x u_{xy} + 2u_{xx}u_y) + \gamma(u_{xxxz} + 4u_x u_{xz} + 2u_{xx}u_z) + au_{xx} + bu_{xy} + cu_{xz} = 0 \qquad (1)$$

- The traveling wave transformation,

u(x,y,z,t)=U(ξ), ξ=kx+ly+mz-nt is used to reduce the PDE (1) into the following ODE

$$(\alpha k + \beta k^2 l + \gamma k^2 m)(U'')^2 + (ak + bl + cm - n)(U')^2 + 2(\alpha k^2 + \beta kl + \gamma km)(U')^3 = 0. \qquad (2)$$

- The solution of equation (2) is assumed to be as the Abel equation,

$$\frac{d}{d\xi}U(\xi) = \omega_2(\xi)U^2(\xi) + \omega_1(\xi)U(\xi) + \omega_0(\xi), \qquad (3)$$

Where ω_0, ω_1 *and* ω_2 represents the arbitrary functions to be determined.

- Substituting equation (3) into equation (2) yields a polynomial in terms of U(ξ), collecting the like powers of U(ξ) and equating them to 0 a system of algebraic equations is obtained. By solving this system the solution families are obtained.
- Among those families we choose 3 families

1. $n = (a + C_1)k + bl + cm, \omega_1 = 0, \omega_2 = 0, \omega_0 = -\frac{\left(\tanh\left(\frac{\sqrt{C_1(lk\beta+km\gamma+\alpha)}(C_2+\xi)}{2(lk\beta+km\gamma+\alpha)}\right)^2 - 1\right)C_1}{2(\alpha k+\beta l+\gamma m)}.$

2. $m = \frac{-\alpha k-\beta l}{\gamma}, \ \omega_1 = 0, \omega_2 = 0, \omega_0 = \frac{exp\left(\frac{\sqrt{\gamma k\alpha(k-1)(k+1)(k\gamma a-k\alpha c+\gamma lb-\beta lc-\gamma n)}C_1}{\gamma k\alpha(k^2-1)}\right)}{exp\left(\frac{\sqrt{\gamma k\alpha(k-1)(k+1)(k\gamma a-k\alpha c+\gamma lb-\beta lc-\gamma n)}\xi}{\gamma k\alpha(k^2-1)}\right)}.$

3. $m = \frac{-\alpha k-\beta l}{\gamma}, n = \frac{(-\alpha k^3 C_1{}^2 + (C_1{}^2\alpha + a)k + bl)\gamma - c(\alpha k+\beta l)}{\gamma}, \omega_0 = \frac{C_3 \exp(C_2C_1)\exp(C_1\xi)}{-1+\exp(C_2C_1)\exp(C_1\xi)}, \omega_0 = \frac{C_1 \exp(C_2C_1)\exp(C_1\xi)}{-1+\exp(C_2C_1)\exp(C_1\xi)}.$

RESULTS & DISCUSSION

1. $u_1(x,y,z,t) = U_1(\xi)$

$$U_1(\xi) = \frac{C_1(lk\beta+km\gamma+\alpha)\tanh\left(\frac{\sqrt{C_1(lk\beta+km\gamma+\alpha)}(C_2+\xi)}{2(lk\beta+km\gamma+\alpha)}\right)}{(\alpha k+\beta l+\gamma m)\sqrt{C_1(lk\beta+km\gamma+\alpha)}} + C_3,$$

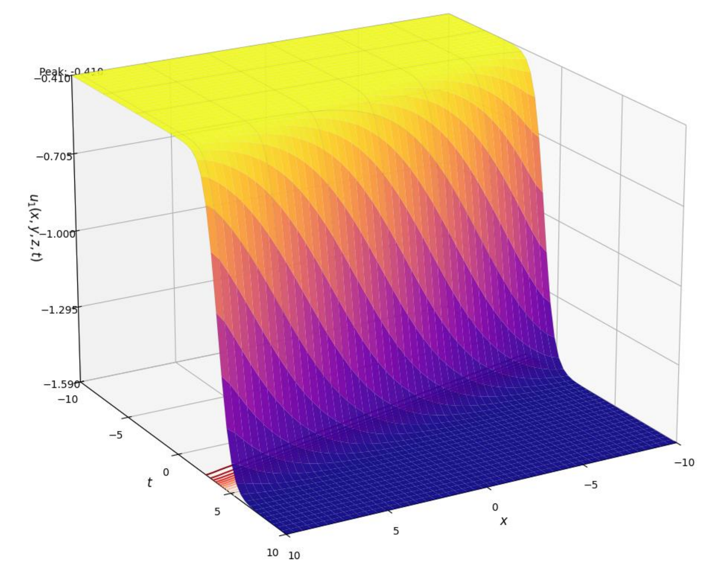
2. $u_2(x,y,z,t) = U_2(\xi)$

$$U_2(\xi) = -\frac{\sqrt{\gamma}k\alpha(k^2-1)exp\left(\frac{\sqrt{\alpha k(\gamma ka-\alpha kc+\gamma lb-\beta lc-\gamma n)(k^2-1)}C_1}{\sqrt{\gamma}k\alpha(k^2-1)}\right)}{\sqrt{\alpha k(\gamma ka-\alpha kc+\gamma lb-\beta lc-\gamma n)(k^2-1)}exp\left(\frac{\sqrt{\alpha k(\gamma ka-\alpha kc+\gamma lb-\beta lc-\gamma n)(k^2-1)\xi}}{\sqrt{\gamma}k\alpha(k^2-1)}\right)} + C_2,$$

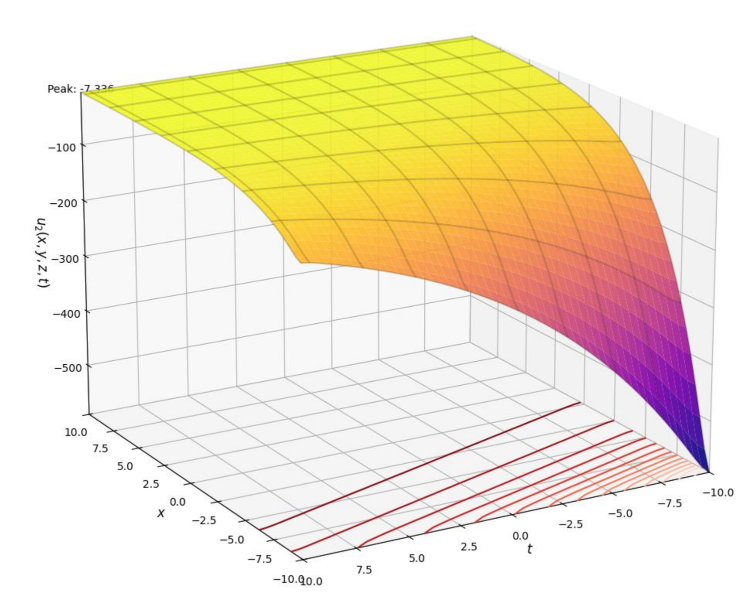
3. $u_3(x,y,z,t) = U_3(\xi)$

$$U_3(\xi) = -\frac{C_3}{C_1} + C_4(\exp(C_1C_2)\exp(C_1\xi) - 1),$$

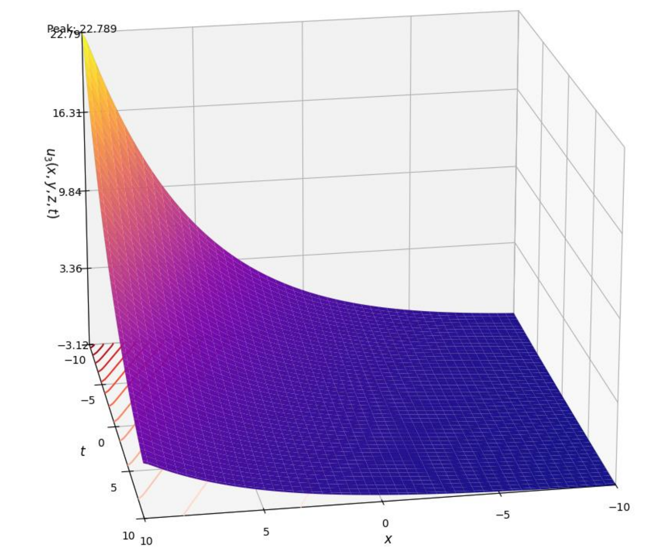
where C_1, C_2, C_3 *and* C_4 are arbitrary constants with ξ=kx+ly+mz-nt.



Surface plot of $u_1(x,y,z,t)$ representing a kink soliton solution for the parameters α=2, β=3, γ=1.5, a=1, b=0.5, c=0.5, k=1, l=1, m=0.5, $C_1 = 2, C_2 = 5$ *and* $C_3 = -1$.



Surface plot of $u_2(x,y,z,t)$ representing a singular traveling wave (exponential front) solution for the parameters α=2, β=1, γ=4, a=1, b=-1, c=0.3, k=3, l=1.5, n=0.5, $C_1 = 3$ *and* $C_2 = -7$.



Surface plot of $u_3(x,y,z,t)$ representing a singular exponential front for the parameters α=1, β=0.5, γ=2, a=0.2, b=1, c=0.4, k=1, l=0.5 and $C_1 = 1.2$

CONCLUSION

The VCGAEM successfully constructed exact traveling wave solutions for the variable-coefficient CBS equation, providing deeper insight into nonlinear wave behavior in ocean dynamics, plasma physics and turbulent flows.

FUTURE WORK / REFERENCES

1. M. S. Hashemi, A variable coefficent third degree generalized Abel equation method for solving stochastic Schrodinger-Hirota model, *Chaos, Solitons and Fractals* (2024) (180), 114606.
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